

Book review of Grothendieck's *Reaping and Sowing*
Response to Pierre Schapira

by Leila Schneps

In a book review of Grothendieck's masterpiece, *Récoltes et Semailles* (Reaping and Sowing), Pierre Schapira concentrates mainly on than the unfortunate controversy over the Riemann-Hilbert correspondence. But the massive memoirs are rich with so much more!

Certainly, as Pierre Schapira writes, references to the person Grothendieck calls his "posthumous student", the one who Grothendieck believed had proved a difficult result only to see his name never mentioned in any reference — but who in fact gave Grothendieck a version of events containing a certain number of falsehoods — are scattered through the text, and certain chapters are devoted to the details of his story. That Grothendieck made himself the mouthpiece of that "posthumous student" is, of course, partly due to the fact that nobody else took the trouble to give him more precise information at the time he was writing his massive tome, although he wrote and called several of the other people involved. Given their silence, it is not entirely his fault that he was misled. It is really a pity that he did not think of writing to Schapira until after the text had been written and distributed — and also that his belated corrections to his tome were not included in Gallimard's first edition. Grothendieck did not send his corrections out to very many people; they do not figure in either of the typeset versions of *Reaping and Sowing* available on the internet, and Gallimard was unaware of their existence (as was I). They will of course be included in the new edition by Gallimard, and also in the forthcoming English translation. But all this having been said, *Reaping and Sowing* is so much more than this one story!

Let me briefly run over the contents of the book and the different themes introduced there. The first part of the book, *Prélude en Quatre Mouvements* (Prelude in Four Movements), contains a foreword and an introduction explaining, more or less, the genesis of the book, namely a reflection Grothendieck pursued over the course of several months in an attempt to comprehend whether himself had played a role, in the past, in creating a climate among mathematicians that could allow certain events to take place that he considered morally unacceptable. It contains two more rather beautiful texts. The first of these is a remarkable *Promenade à Travers une Oeuvre* (Promenade through a Life's Work), intended strictly for non-mathematicians, in which the spirit of Grothendieck's main ideas and their motivation are discussed in terms of imagery, and the very nature and meaning of creativity is explored in depth. The second text is a long letter addressed to a certain "you" — in principle one of the several dozen people to whom Grothendieck mailed parts of his work, and in principle largely discussing the matter he calls "his burial", but it hardly matters, because Grothendieck, addressing himself to "you", asks you to reflect on a number of difficult and intimate questions that, if not ignored, can lead quite far in self-examination no matter who you are.

The next part of the book, *Fatuité et Renouveau* (Complacency and Renewal), was actually the first part Grothendieck wrote, and it is the heart and soul of *Reaping and Sowing*, at least for me. It consists in Grothendieck's own long self-examination, sometimes joyful, sometimes ironic, about his present and his past, his encounters with conflict and his efforts to seek their true cause within himself, his struggle to divest himself of a long-cherished but artificial self-image and his simple joy in research and discovery. The question of the "posthumous student" is entirely absent from this volume, but many pages are devoted to an inspiring and beautiful theory of creativity, which he describes as directly connected to our earliest childhood and the child that subsists within each of

us, adding a perspicacious analysis of the demands and fears of the ego, and the deleterious effect they have on the natural creative impulse.

The next three parts, *L'Enterrement (I), (II), (III)* (The Burial (I), (II) and (III)) are largely concerned with perceived misdeeds of the mathematical community, and three such misdeeds in particular. The first of these concerns Grothendieck's theory of motives, abandoned for twelve years after he abandoned the mathematical scene in 1970. He was aware of the silence around the theory of motives, but there was little to be said about it since the theory was essentially only his, and was furthermore largely unformulated, unwritten and entirely conjectural. The surprise came with his discovery of a book on motives that suddenly broke the twelve year silence and attempted to start up the theory anew — except with barely a mention of Grothendieck's name or the origin of the theory. People often justify the absence of explicit references to him by saying “Since everyone knew that motives were one of Grothendieck's great ideas, no one needed to mention it,” (to use the particular formulation by J.S. Milne on his web page “Grothendieck and me”), but it doesn't take a rocket scientist to figure out that if his name was never associated to motives in print, the thing that “everyone knew” would disappear along with the people who knew it, leaving no trace for the coming generations. It is this envisioned disappearance that Grothendieck called his “burial”.

The second misdeed concerns the famous SGA, the Seminar of Algebraic Geometry that Grothendieck ran in Paris for eight years. The lectures of these seminars were mostly given by him, and then he or his students would write them up based on his and their notes, and the volumes were published when they were ready. However, after his departure in 1970, although SGA 4 appeared in 1972, SGA 5 did not appear — at least not until after two other texts had been published first. The first was an article by one of Grothendieck's ex-students, containing a great deal of material directly reproduced from the seminar but without any reference to Grothendieck. The second was a volume which gathered together various texts, including some of the content of SGA 5, with the goal of giving a more streamlined introduction to the main ideas. This volume was given the title “SGA 4 1/2”, and Grothendieck appears on the title page as “a collaborator” although he was unaware of its existence. After the publication of this extra volume, SGA 5 finally appeared in 1977, but in the meantime the editors had lost some of the notes from the original lectures, so that several of them are missing, in particular the opening lecture (a detailed and comprehensive overview), the closing lecture (conjectures and open questions), the lectures that had been moved into SGA 4 1/2, and a slew of others. It must be admitted that Grothendieck describes these events with an irony that is often extremely comical, belying his reputation as a humorless person.

In spite of the ironic humor, though, what Grothendieck called the “massacre of SGA 5” and the “violent insertion of a foreign text between two inseparable parts” of his seminar wounded him deeply. Certainly, the reader may consider as excessive his reaction to what might seem like at worst a misdemeanor, and as highly exaggerated his tendency to extrapolate that misdemeanor to a general climate of hypocritical immorality reigning in the prestigious circles of French mathematics in the 1980s, allowing its members to steal the ideas of others while posing as benefactors of society. Yet his passionate and detailed dissection of each small happening, with its lengthy psychological analyses of the layers of meaning behind each act, reads at times like a psychological novel — a mathematical Proust. Regardless of whether one considers that he is quite right, partly right, or totally paranoid, his manner of observing, analyzing and dissecting events, seeking above all for the hesitations that denote his own inner resistances, is a lesson in peering below the surface of what we see.

The unfortunate story of the “posthumous student” mentioned above, a set of events misunderstood by Grothendieck and insufficiently corrected even by his notes in response to the letters of Pierre Schapira, is only the last of the three events that clearly had a highly traumatic effect on Grothendieck, awakening him rudely, after almost fifteen years, from what he had believed to be a peaceful and permanent detachment from his past.

While *The Burial* (III) goes back over the above events in even more detail (leaving the reader “gasping for relief”, a phrase that Grothendieck himself used to describe the audience at his SGA seminar), *The Burial* (II), subtitled *La Clef du Yin et du Yang* (The Key of Yin and Yang), contains an extremely long and fascinating digression on Grothendieck’s view of the masculine and the feminine sides of our nature, and their role in the best of what we do (creativity, perceptiveness, attentiveness and love) and the worst (conflict, ego, ambition, contempt). Even creativity and discovery have their masculine and feminine sides: the masculine thrusts forwards with keen desire to know, the feminine absorbs and understands by attentive listening, and both are magnificent, necessary and complementary aspects of the human drive to learn. The issue of the “burial” is neglected in this volume in favor of lengthy passages in which Grothendieck recalls key events from his childhood and muses on their significance in his later life; the repression of his feminine side in order to conform to the rigorously masculine values of his mother, the slow coming to awareness that his personality was actually dominated by that which he had repressed, and his analysis of the effect of this unconscious dominance on his own extraordinary production of mathematics. The volume ends with the remarkable appendix *Les Portes de l’Univers* (The Doors of the Universe), which attempts to situate the forces of yin and yang on a much vaster scale than psychological analysis, closer to Jung’s concept of archetypes: in the psyche of each person, the whole of the Universe falls into endlessly opposing, endlessly osmosing and interpenetrating pairs in an eternal cycle. Quoting a poetic text from his own distant past, Grothendieck expresses this cycle in an image, with which we conclude this review using his own words:

“The river throws itself into the welcoming sea. The boat’s hull is sunk deep in the river which surrounds and envelopes it. The crew is carried on the boat which surrounds and contains it. The cabin boy is a member of the crew which includes him. And in his eyes, the sea is reflected; through his eyes it penetrates into his soul which gathers it in. Thus the male and the female — Eros and the Mother — are constantly interlaced in an endless cycle in which everything, all together or one at a time, lives both its virile pulsion and its maternal pulsion.”