

Chapter 1. Mathematics and creativity

Grothendieck and generality

When asked to describe the nature of Grothendieck's mathematical contribution – not the results, but the style, the approach, the twist that made his work uniquely recognizable – the first thing mathematicians usually mention is the absolute generality visible in his approach to any question. Confronted with a specific problem, Grothendieck would tend to perceive it as a special case of some vast, far-reaching situation, and the cornerstone of his mathematical philosophy was that a general description of the vast situation would, once completed, naturally and effortlessly yield the properties of the situation, which would then trickle down to give a solution to the particular problem. Whereas mathematicians intent on proving a given result will often add to the initial situation the extra hypotheses which, while restricting the scope, will allow them to use arguments not valid in the general case, Grothendieck's approach led him inversely to prove results with the least possible hypotheses on the objects he considered.

What mathematicians ordinarily refer to as generalization is not precise or well-defined, although most mathematicians would agree without the slightest dispute when something has been generalized. One way to generalize is to consider a specific, more or less well-understood statement on some set of objects, and prove that in fact it, possibly in some modified form, applies to a larger set of objects. Another type of generalization is to take a result which, in order for the proof to succeed, necessitated a certain number of hypotheses at the start, and prove that the statement still holds when these hypotheses are removed. Accomplishing something of this kind is perceived as adding something to the original, less general starting point, and the proof often appears more difficult in the general situation, which is wider and broader. The perceived difficulty of generalization lies partly in the unfamiliarity or abstraction of the generalized objects compared to the specific ones of the starting point, which being more classical or more familiar, provide a more thorough and accurate intuition and possibly a greater range of techniques.

In later chapters, we will discuss Grothendieck's generalizing approach to such topics as the Riemann-Roch theorem, the development of schemes, topos theory, etc. That Grothendieck approached almost any new mathematical topic with the attitude that it was too specific and must be generalized is famous, one of the trademarks of his style. He developed this tendency towards maximal generalization of every situation extremely early in his mathematical career, and it shows up in virtually every aspect of his research. A very colorful example is given in the following anecdote from the pen (and the memory) of Prof. Jaap Murre of Leiden, who worked with Grothendieck in the 1950s and 60s.

It happened precisely on the 18th of April, 1959.

Nico Kuiper had invited Grothendieck to give a lecture in Wageningen in the Nether-

lands. Nico Kuiper was then professor at the Agriculture University in Wageningen (we do have such a university in Holland); later Nico became professor at the University of Amsterdam and after that he became – as you undoubtedly know – director of the IHES). He was a friend of Grothendieck; I think they knew each other from the Arbeitstagung in Bonn.

There were (except Nico) almost no mathematicians in Wageningen, but people had come from other places because Grothendieck was already well-known (Edinburgh 1958 for instance!).

After the lecture in the morning (on cohomology of sheaves) and after lunch, we went to Nico's house. I think I was, except for Grothendieck, the only algebraic geometer present there (unbelievable now!), so I did have ample opportunity to discuss with him.

At that time there was a very important unsolved question 'in the school of Weil' (to which I belonged), namely the behavior of the Picard **variety** if the original variety X_t , say, was varying in a system T . In the beginning of the 50s Matsusaka, Weil himself and Chow had constructed algebraically the Picard variety, but how did it vary when the variety moved in a system and moreover – and worse – in characteristic $p > 0$? Igusa had discovered all kinds of "pathological behavior", around 1955. This was something which was not understood at all, neither from the point of view of the school of Weil, nor from the somewhat different point of view of Chevalley (and his school: Seshadri) who worked on this Picard theory a little later (see for instance *Sém. Chevalley 1958/59*).

So, during that discussion I asked Grothendieck whether he could explain this behavior of the Picard variety.

He said that he had not yet studied that question carefully, because this would be in chapter 12 of EGA. But he said that he would certainly settle that question and explain this behavior. He said that his feeling was that "those people" made too strict assumptions and tried to prove too little. He intended to make less assumptions and to prove a more precise theorem.

I said something like: "Oh!, that sounds interesting" (but I was very sceptical!).

However then, in 1962, Grothendieck completely **solved the question** (not in chapter 12, of course, but in his two Bourbaki seminars).

With his theory of the Picard functor and the Picard **scheme** he proved a stronger theorem and explained precisely what was going on; moreover things were not pathological at all from his point of view but natural.

I was at the IHES at that time (by the way the IHES was still in Paris then); I attended his Bourbaki lectures, and needless to say, I was very impressed !

Of course, what was going on was that in characteristic $p > 0$, even if one started with nice varieties (i.e., reduced schemes), it could happen that the Picard functor was not representable by a variety but only by a scheme.

Undoubtedly, people did see in the mid 50's that one could generalize a lot of things to schemes, but Grothendieck saw that such a generalization was not only possible and natural, but **necessary** to explain what was going on, even if one started with varieties.

This episode illustrates Grothendieck's approach in a nutshell – his seemingly more complicated and more general constructions were actually intrinsically necessary to com-

prehend situations, and thus in fact simplified them*.

Taken altogether, Grothendieck's body of work is perceived as an immense *tour de force*, an accomplishment of gigantic scope, and also extremely difficult both as research and for the reader, due to the effort necessary to come to a familiar understanding of the highly abstract objects or points of view that he systematically adopts as generalizations of the classical ones. All agree that the thousands of pages of his writings and those of his school, and the dozens and hundreds of new results and new proofs of old results stand as a testimony to the formidable nature of the task.

In this initial essay, however, without looking in detail at actual mathematics, we want to make a very important point: namely, that the idea, the concept, or even the act of generalization can be viewed in very different ways, and Grothendieck's own view of what he was doing, described in his own words, is totally different from the impression depicted above. Grothendieck certainly did not feel that he was attempting to use powerful techniques in order to obtain stronger results by generalizing. What he perceived himself as doing was *simplifying* situations and objects, by extracting the fundamental essence of their structure. What makes this seem so difficult for many people is a certain feeling of blindness in front of the new definitions, which by their abstraction and unfamiliarity deprive us of intuition**.

Apparently Grothendieck had no need for internal mental recourse to a concrete picture or a well-thumbed example. His famous response 'You mean like 57?' to somebody's suggestion 'Pick a prime number' is often taken to illustrate this. Michel Demazure recounts how with irrepressible optimism Grothendieck would summarize his very first perception of a new situation with a conjecture, – which Serre would frequently observe was false for the most elementary example – and J. Giraud [G] also describes the contrast between Serre's precautionary style and Grothendieck's habit of tossing out ideas with a casual "Ça doit être vrai!"

* Another example: "L'an dernier encore, j'ai vu reprocher à Contou-Carrère de ne pas s'être borné dans sa thèse à se placer sur un corps de base au lieu d'un schéma général – tout en lui concédant quand même la circonstance atténuante que c'était sûrement sur les instances de son patron de circonstance qu'il avait dû s'y résoudre. Celui qui s'exprimait ainsi était pourtant suffisamment dans le coup pour savoir que même en se bornant au corps des complexes, les nécessités de la démonstration forcent la main pour introduire des schémas de base généraux..." (RS 279)

** For instance, Grothendieck gives this description of his efforts to reach a concrete understanding of an interesting situation illustrated by one example: "Je pense notamment, dans le contexte justement de la cohomologie des variétés algébriques, à la découverte par Griffiths de la fausseté d'une idée séduisante qu'on avait eu longtemps sur les cycles algébriques, à savoir qu'un cycle homologiquement équivalent à zéro avait un multiple qui était algébriquement équivalent à zéro. Cette découverte d'un phénomène tout nouveau m'avait alors assez frappé pour que je passe bien une semaine de travail pour essayer de bien saisir l'exemple de Griffiths, en transposant sa construction (qui était transcendante, sur le corps \mathbf{C}) en une construction "aussi générale que possible", et valable notamment sur des corps de caractéristique quelconque. L'extension n'était pas tout à fait évidente, à coups (si je me rappelle bien) de suite spectrales de Leray et de théorème de Lefschetz." (RS 289)

Grothendieck's independence from the concrete mathematical object was well-known; as Jean-Pierre Serre, a lover of small and charming special cases, observed, it was not until his interest was sparked by *dessins d'enfants* that he finally understood the value and usefulness of actual examples. It would seem clear that this mental faculty of his to think and reason independently of the *object* is the sign of a much more general mental state studied by certain psychiatrists (such as J. Lacan), and not unrelated to his attitude towards his own body and physical comfort. Grothendieck's independence from the concrete objects of daily life was as well-known to his contemporaries as his mathematical style; his diet of milk, cheese and bananas, his rudimentary house and clothing, his sleeping on boards instead of a bed, his insistence on being operated without anesthesia: all these idiosyncrasies have become part of his legend. Yet the two different forms of independence from the concrete have not really been perceived as signs of one and the same mental state.

Because he saw the special features of specific mathematical objects and situations as needlessly complicating the issue, and general situations as being much simpler, he repeatedly described his approach to mathematics as 'childish' ('*enfantine*'), even 'silly' or 'babyish' ('*bébête*'), and assumed that other people did not adopt it because they would not stoop to something so absurdly simple. These feelings about his own approach are expressed in his writings in many different places. For example, on his work on schemes, he writes:

*L'idée même de schéma est d'une simplicité enfantine – si simple, si humble, que personne avant moi n'avait songé à se pencher si bas. Si “bébête” même, pour tout dire, que pendant des années encore et en dépit de l'évidence, pour beaucoup de mes savants collègues, ça faisait vraiment “pas sérieux”! Il m'a fallu d'ailleurs des mois de travail serré et solitaire, pour me convaincre dans mon coin que “ça marchait” bel et bien – que le nouveau langage, tellement bête, que j'avais l'incorrigible naïveté de m'obstiner à vouloir tester, était bel et bien adéquat pour saisir, dans une lumière et avec une finesse nouvelles, et dans un cadre commun désormais, certaines des toutes premières intuitions géométriques attachées aux précédentes “géométries de caractéristique p”. C'était le genre d'exercice, jugé d'avance idiot et sans espoir par toute personne “bien informée”, que j'étais le seul sans doute, parmi tous mes collègues et amis, à pouvoir avoir jamais idée de me mettre en tête... (RS 51)**

And to explain the introduction of topos theory, his new conception of topological space, he writes:

Voici donc l'idée nouvelle. Son apparition peut être vue comme une conséquence de cette observation, quasiment enfantine à vrai dire, que ce qui compte vraiment dans un espace topologique, ce ne sont nullement ses “points” ou ses sous-ensembles de points, et les relations de proximité etc. entre ceux-ci, mais que ce sont les faisceaux sur cet espace, et la catégorie qu'ils forment. Je n'ai fait, en somme, que mener vers sa conséquence ultime l'idée initiale de Leray – et ceci fait, franchir le pas.

* The initials RS refer to Récoltes et Semaines, and all page numbers correspond to the typed version available on the Grothendieck circle website.

Comme l'idée même des faisceaux (due à Leray), ou celle des schémas, comme toute "grande idée" qui vient bousculer une vision invétérée des choses, celle des topos a de quoi déconcerter par son caractère de naturel, d'"évidence", par sa simplicité (à la limite, dirait-on, du naïf ou du simpliste, voire du "bébête" – par cette qualité particulière qui nous fait nous écrire si souvent: "Oh, ce n'est que ça!", d'un ton mi-déçu, mi-envieux; avec en plus, peut-être, ce sous entendu du "farfelu", du "pas sérieux", qu'on réserve souvent à tout ce qui déroute par un excès de simplicité imprévue. A ce qui vient nous rappeler, peut-être, les jours depuis longtemps enfouis et reniés de notre enfance... (RS 56)

Given the usual impression one has of Grothendieck as a mathematician at the summit of his profession, surrounded by admiring colleagues and students, one may wonder who were those 'well-informed people' and 'knowledgeable colleagues' who 'judged' his efforts to be 'idiotic' and 'hopeless'. The name of André Weil springs to mind. Weil was undoubtedly somewhat allergic to Grothendieck's style of doing mathematics: '...je me réjouis fort de voir cette "hypercohomologie" donner des résultats tangibles en géométrie algébrique – Weil sera furieux!' Serre wrote to Grothendieck in February 1956, and '...cette difficulté que j'avais à m'insérer dans le travail commun [à Bourbaki], ou les réserves que j'ai pu susciter pour d'autres raisons encore à Cartan et à d'autres, ne m'ont à aucun moment attiré sarcasme et rebuffade, ou seulement une ombre de condescendance, à part tout au plus une fois ou deux chez Weil (décidément un cas à part)' (RS 141). But the feeling that in generalizing, he was simplifying, and that this was not understood by almost any of his colleagues, even the most willing, goes deeper than the pinch of resentment which transpires here, and must have contributed to the touch of complacency expressed in these remarks (something felt strongly by Jean-Pierre Serre, and denied with absolute sincerity by a surprised Grothendieck, cf. their correspondence).

Beyond this, there is also present the echoes of a circle of deeper ideas about doing mathematics, and about creativity in general, which he took up in other places and explored in detail. Indeed, it is natural to ask why Grothendieck was so easily able to think in these general terms, and what it was about his approach that differed from his colleagues and made things that were obvious to him seem difficult to them? Because this biography is an attempt to create a portrait of a mathematician, combining into a coherent whole the psychological traits of the individual and his astounding professional accomplishments, this question is a key one, and its answer necessarily possesses manifold aspects, some of which we hope to illustrate both from Grothendieck's point of view and from that of an outside observer.

Grothendieck and the rising sea

In one of the most frequently cited passages from *Récoltes et Semailles*, Grothendieck compares his approach to solving a mathematical problems as one of letting the obstacle become slowly absorbed, soaked and weakened by the imperceptibly rising sea.

La mer s'avance insensiblement et sans bruit, rien ne semble se casser rien ne bouge l'eau est si loin on l'entend à peine...Pourtant elle finit par entourer la substance rétive,

celle-ci peu à peu devient une presqu'île, puis une île, puis un îlot, qui finit par être submergé à son tour, comme s'il s'était finalement dissous dans l'océan s'étendant à perte de vue...[...]

C'est 'l'approche de la mer', par submersion, absorption, dissolution – celle où, quand on n'est très attentif, rien ne semble se passer à aucun moment: chaque chose à chaque moment est si évidente, et surtout, si naturelle, qu'on se ferait presque scrupule souvent de la noter noir sur blanc, de peur d'avoir l'air de bombiner, au lieu de taper sur un burin comme tout le monde...C'est pourtant là l'approche que je pratique d'instinct depuis mon jeune âge, sans avoir vraiment eu à l'apprendre jamais. (RS 502)

Qualifying this approach as 'feminine', or 'yin', identifying as the passive 'acts' (or perhaps states) of waiting, watching, observing, letting ripen, gestating, he clearly contrasts this approach with another, perhaps more widespread perception of mathematics as a quintessentially active (and thus masculine): activity, consisting of 'solving problems', 'penetrating the unknown', 'unveiling secrets', acts which necessarily carry within them a seed of violence.

*Prenons par exemple la tâche de démontrer un théorème qui reste hypothétique (à quoi, pour certains, semblerait se réduire le travail mathématique). Je vois deux approches extrêmes pour s'y prendre. L'une est celle du **marteau et du burin**, quand le problème posé est vu comme une grosse noix, dure et lisse, dont il s'agit d'atteindre l'intérieur, la chair nourricière protégée par la coque. Le principe est simple: on pose le tranchant du burin contre la coque, et on tape fort. Au besoin, on recommence en plusieurs endroits différents, jusqu'à ce que la coque se casse – et on est content[...]*

Je pourrais illustrer la deuxième approche, en gardant l'image de la noix qu'il s'agit d'ouvrir. La première parabole qui m'est venue à l'esprit tantôt, c'est qu'on plonge la noix dans un liquide émoullent, de l'eau simplement pourquoi pas, de temps en temps on frotte pour qu'elle pénètre mieux, pour le reste on laisse faire le temps. La coque s'assouplit au fil des semaines et des mois – quand le temps est mûr, une pression de la main suffit, la coque s'ouvre comme celle d'un avocat mûr à point! (RS 501)

These descriptions of his own work date from the 1980's, and thus come much later than the work itself, but they are nonetheless very accurate. There was a consistent slowness, a refusal to hurry or to seek directly for specific results in his approach. The anecdote by Murre recounted above illustrates this perfectly; results will ripen on the tree that he is patiently gardening, and fall off at the right time of their own juicy weight.

This attitude, this incredible confidence that his rising sea would eventually englobe and yield (easy, natural) proofs of all the open questions in the domain, was perceived by many as one of those superhuman traits that distinguished him from the common run of mathematicians; an incredible confidence and ability to correctly predict his future ability to prove something. Jean-Pierre Serre scoffs at such an idea: 'He wasn't predicting anything at all, he knew how to prove those things already!' Steve Kleiman points out, however, that Grothendieck announced the future purpose of étale cohomology in Edinburgh in 1958, at a time where he certainly did not yet know how to prove all the necessary results. Either way, Grothendieck never intended to write up a proof until its

proper place in the grand written work covering the entire theory had been reached, meaning that many of his theorems had to wait years before appearing in print, or else be written up and published by others, such as when Borel and Serre wrote up his proof of the Grothendieck-Riemann-Roch theorem.

One might summarize the quotations above by the remark that Grothendieck's approach to the mathematical object was not 'figure out what's going on' but 'observe it deeply until its true nature is slowly revealed' – an activity in which *naming* the newly emerging objects, rather than exploring their properties, played a fundamental role.

*Dans une telle situation, quand les choses elles-mêmes nous soufflent quelle est leur nature cachée et par quels moyens nous pouvons le plus délicatement et le plus fidèlement l'exprimer, alors que pourtant beaucoup de faits essentiels semblent hors de la portée immédiate d'une démonstration, le simple instinct nous dit d'écrire simplement noir sur blanc ce que les choses nous soufflent avec insistance, et d'autant plus clairement que nous prenons la peine d'écrire sous leur dictée! Point n'est besoin de se soucier de démonstrations ou de constructions complètes – s'encombrer de telles exigences à ce stade-là du travail reviendrait à s'interdire l'accès de l'étape la plus délicate, la plus essentielle d'un travail de découverte de vaste envergure – celle de la naissance d'une vision, prenant forme et substance hors d'un apparent néant. Le simple fait **d'écrire**, de **nommer**, de **décrire** – ne serait-ce d'abord que décrire des intuitions évasives ou de simples "soupçons" réticents à prendre forme – a un **pouvoir créateur**. C'est là l'instrument entre tous de la passion de connaître, quand celle-ci s'investit en des choses que l'intellect peut appréhender. Dans la démarche de la découverte en ces choses-là, ce travail en est l'étape créatrice entre toutes, qui toujours précède la démonstration et nous en donne les moyens – ou pour mieux dire, sans laquelle la question de "démontrer" quelque chose ne se pose même pas, avant que rien encore de ce qui touche l'essentiel n'aurait été formulé et vu. Par la seule vertu d'un effort de formulation, ce qui était informe prend forme, se prête à examen, faisant se décanter ce qui est visiblement faux de ce qui est possible, et de cela surtout qui s'accorde si parfaitement avec l'ensemble des choses connues, ou devinées, qu'il devient à son tour un élément tangible et fiable de la vision en train de naître. Celle-ci s'enrichit et se précise au fil du travail de formulation. Dix choses soupçonnées seulement, dont aucune n'entraîne conviction, mais qui mutuellement s'éclairent et se complètent et semblent concourir à une même harmonie encore mystérieuse, acquièrent dans cette harmonie force de vision. Alors même que toutes les dix finiraient par se révéler fausses, le travail qui a abouti à cette vision provisoire n'a pas été fait en vain, et l'harmonie qu'il nous a fait entrevoir et qu'il nous a permis de pénétrer tant soit peu n'est pas une illusion, mais une réalité, nous appelant à la connaître. Par ce travail, seulement, nous avons pu entrer en contact intime avec cette réalité, cette harmonie cachée et parfaite. (RS 286)*

This attachment of his to the 'true nature' of things was another typical feature of his approach. Demazure likens his attitude towards the contemplation of mathematical results or objects or examples to that of a geologist picking up rocks at random and informing the ignorant and astounded bystander that whereas this one means nothing at all, that other one indicates the existence of some profound phenomenon a million years ago. 'When Grothendieck looked at examples, he was searching for hints about the true reality of

things, “nature’s plan” – or “God’s plan”. As for motives, it was the idea of perceiving in something that everyone else thinks is perfectly anodine, the sign of a much deeper, hidden structure.’

If the ‘true nature’ of things was a notion that had a clear meaning for Grothendieck, it left some other mathematicians – those with a more democratic attitude towards objects, proofs and results – cold. Jean-Pierre Serre, for example, is ready to admire a difficult proof containing unexpected twists and tricks, whereas such proofs made Grothendieck profoundly uncomfortable, as containing something unnatural, forced, or incomplete. This difference in style was clearly noticeable already in 1961, when over an argument about the inclusion of valuations into a Bourbaki volume, Serre wrote to Grothendieck: ‘Je suis beaucoup moins “entier” que toi sur ces questions (je n’ai pas de prétention à connaître “l’essence” des choses)...’ a sentence which does denote a certain impatience with Grothendieck’s attitude.

Pierre Cartier observed that when Grothendieck took interest in some mathematical domain that he had not considered up till then, finding a whole collection of theorems, results and concepts already developed by others, he would continue building on this work ‘by turning it upside down’. Michel Demazure described his approach as ‘turning the problem into its own solution’. In fact, Grothendieck’s spontaneous reaction to whatever appeared to be causing a difficulty – nilpotent elements when taking spectra or rings, curve automorphisms for construction of moduli spaces – was to adopt and embrace the very phenomenon that was problematic, weaving it in as an integral feature of the structure he was studying, and thus transforming it from a difficulty into a clarifying feature of the situation.

All of the aspects of Grothendieck’s mathematical approach discussed here: the slow, broad approach, the search for the essence, the embrace without reticence of a problem as its own solution – all will be illustrated by numerous cases and examples from his research in the coming chapters.

One might ask what it is that makes it so hard, or so rare, for other mathematicians to react this way. Grothendieck suggests that the answer is undoubtedly a form of fear; fear that the unfamiliar will not bend docilely to the mathematician’s will, fear that confronting the unknown will lead to unfortunate mathematical accidents such as error or total lack of progress, fear of not obtaining recognizable results. If there is one feature of Grothendieck’s personality to which he attributes his ability to have explored and constructed so much that no one else had done before him, it is a total lack of this kind of fear.

Grothendieck and fearlessness

There is no doubt that ordinary mortals are psychologically equipped with a complex combination of more or less unconscious or unrealized fears and inhibitions. These play a protective role in human development (as a fear of heights will protect someone from venturing to dangerous places) and most certainly, they play a role in the development of the individual into a member of society with a sense of social behavior and a realization that other people actually exist.

For reasons which may be partly inborn but are certainly partly due to family influ-

ence, it seems that Grothendieck was to all intents and purposes deprived of these fears and inhibitions, a phenomenon which accounts both for his extreme social inadaptation and his absolutely free and uncomplexed approach to mathematical research.

By all accounts, Grothendieck was pleasant, well-mannered, never unkind, generous, and already at twenty at ease with everyone, devoid of any sense of awe or complex. He spoke to the most prestigious mathematicians as equals (Roger Godement says that had he encountered Hilbert personally, he would have cheerfully said ‘How are you?’) and at the same time was particularly at ease with all people from the simplest social class. There are multiple anecdotes attesting this; his attachment to the domestic servant who lived with the Schwartz family, his kind attentiveness when Bill Messing’s absence from a seminar reminded him that the Messings were expecting a baby, his endless devotion to and generosity with his students, his suave, gentle speaking voice. To be sure, when he did become angry, he was occasionally tempted to resort to his fists; Roger Godement recalls that he claimed to be an excellent boxer, having had long practice with other boys at school, and tells how he was once summoned to the local police station in Nancy after having roundly smacked a boy in the street who responded to a scolding by calling him ‘sale Boche’. Serre and Raynaud distinctly recall seeing his fists clench and feeling a real moment of tension one day at the Institut Henri Poincaré, when Serre teased him about some mathematical assertion or other, and David Ruelle recalls that once, as he was driving Grothendieck down a rainy road away from the IHES, they were honked at and insulted by the driver just behind them; Grothendieck leaped out of the car and in a brief moment of panic Ruelle thought that he was about to witness a massacre (fortunately he was able to calm the situation down). Grothendieck’s girlfriend of a later period, Justine, mentions his actually attacking police officers, and even yielding to violent impulses with her, which actually led to the couple’s early separation.

Grothendieck’s profound social inadaptation, however, is visible from other and subtler signs than these. One of the most telling is that he felt that he had a biological rhythm that was different from the ordinary 24 hour cycle, and this led him, throughout his life, to sleep, eat and work at strange, variable hours, unrelated to what the rest of the world was doing. That he should pursue his own schedule in his own home would be normal enough, but in fact he attempted to compel his wife and children to follow it with him, and as for inviting him for a meal, Claudine Schwartz recalls that her parents Laurent and Marie-Hélène had to calculate and negotiate at length to find one day and one hour when Grothendieck’s eating schedule would coincide with theirs. Laurent Schwartz, according to Claudine, was quite fascinated with the biological phenomenon, but at the same time, he sermonized Grothendieck, and tried unsuccessfully to persuade him to adapt himself to family life and to explain to him that society existed and that he was cutting himself off from it.

When asked what Grothendieck was like as a young student in Nancy, when he visited the Godements home regularly, Roger Godement states simply, ‘Il était un sauvage.’ According to Jean-Pierre Serre, ‘J’ai toujours senti qu’il était comme une centrale nucléaire, si on disait quelque chose qui ne lui allait pas, il pouvait exploser, et ce serait grave, il valait mieux éviter.’ The impression of those who knew him best was that in spite of a pleasant exterior and what soon became a highly respected professional position, there was some-

thing deeply abnormal and disquieting about him, in his total lack of real comprehension of other people (quote from a 1995 letter: ‘Je ne m’attendais pas à recevoir encore de signe de vous, et me réjouis de m’être encore trompé, comme c’est d’ailleurs pratiquement toujours le cas quand je me hasarde à quelque prévision concernant un de mes semblables’) and in his excessive devotion to his mathematical studies, which occupied him for some sixteen hours every day, which prevented any of the people who surrounded him from approaching him close enough to form an intimate and lasting friendship. It is telling that although in *Récoltes et Semailles* and his other writings, Grothendieck very frequently refers to his friends, few or none of those people actually now describe themselves as having truly been friends with him, in the deep sense of the word friendship. Somewhere, Grothendieck must have been aware of this, since sadly but not surprisingly, he writes as a description of his entire life until the moment in 1976 in which he suddenly discovered meditation: ‘Je me sentais spirituellement absolument seul de mon espèce, et n’arrivais à me reconnaître dans aucun groupe humain, ni dans aucun autre être’ (Clef 104).

If Grothendieck did show kindness and good manners, if he did say ‘bonjour’ and ‘merci’, it is very probably because of the five years he spent living with the Heydorn family. Certainly the bringing up he received from his parents before the age of five, and the period he spent living with his mother from 1939 to 1948 were powerful influences in driving him away from what his parents considered the bourgeois aspects of society. Transgression of traditional social rules and boundaries, viewed by Grothendieck’s parents (and in general, by all who shared these early anarchist ideals which developed into the movement associated with ‘the 60’s’ in the United States and ‘May 1968’ in France) as the noblest rebellion in the history of mankind, may have been the most important and visible legacy that Sascha Tanaroff* and Hanka Grothendieck left their son.

Grothendieck’s father Sascha was a political anarchist, a true one. His mother was less political, but her temperament was violent, tempestuous and difficult, and by nature more anarchist than any theoretical anarchist. No political or social stream could contain her. She adopted the anarchist hatred of authority and capitalism, but had no real interest in their wider goals of creating a society of mutual support and friendship. In fact, she was perhaps more of a nihilist than an anarchist; rejection of bourgeois values of respectability and comfort, rejection of social values of politeness and respect, rejection of any consciously imposed values at all; profound admiration of passion, love, freedom, strength, will-power, genius. During her lifetime, she purposely violated every boundary she encountered; broke every spoken and unspoken rule with a kind of vicious pleasure, destroying her own life and the lives of everyone around her in the process. The picture of her life which she somehow imposed on both her children for many decades, was that of passionate lover to her anarchist hero Sascha and heroic mother to extraordinary children. The reality was a scene of ceaseless conflict and quarreling with her lover, the abandonment of her children, and finally, when reunited with them, the absolute and shocking rejection of every aspect of motherhood. Grothendieck described living with her during the last years of her life (she died in 1957) as hell. And yet, there is no doubt that her influence and her powerful personality left him something besides a decades-long deviant admiration for

* as Grothendieck’s father was known and identified on his papers; Grothendieck, however, asserts that this was a false name and that his father’s name at birth was Shapiro

purely masculine values, which corresponded to her character rather than to his. It could well be her transgressive influence that produced a child, then a young man, and an adult, so entirely free from fear and from any sense of limits, of boundaries, of ‘what others might think’ and of ‘when to stop’. Grothendieck most probably owes the extreme aspects of his nature, and the total freedom from fear and complex, what he describes as the complete absence of an inner division, to his parents.

In one of the most fascinating passages of *Récoltes et Semailles*, Grothendieck recounts how he himself came to realize this feature of his nature, and to begin to analyse its possible origin.

Il y a quatre ans, j’ai pour la première fois senti et mesuré la portée d’une chose dans ma vie à laquelle je n’avais jamais songé, qui toujours m’avait semblé aller de soi: c’est que mon identification à mon père, dans mon enfance, n’a pas été marquée par le conflit – qu’en aucun moment de mon enfance, je n’ai craint ni envié mon père, tout en lui vouant un amour sans réserve. Cette relation-là, la plus profonde peut-être qui ait marqué ma vie (sans même que je m’en rende compte avant cette méditation d’il y a quatre ans), qui dans mon enfance a été comme la relation à un autre moi-même à la fois fort et bienveillant – cette relation n’a pas été marquée par le sceau de la division et du conflit. Si, à travers toute ma vie bien souvent déchirée, la connaissance de la force qui repose en moi est restée vivante; et si, dans ma vie nullement exempte de peur, je n’ai pas connu la peur ni d’une personne ni d’un événement – c’est à cette humble circonstance que je le dois, ignorée encore jusqu’au delà de mes cinquante ans. Cette circonstance a été un privilège sans prix, car c’est la connaissance intime de la force créatrice en sa propre personne qui est aussi cette force, qui lui permet de s’exprimer librement selon sa nature, par la création – par une vie créatrice. (RS 376)

This description is all the more remarkable in that Grothendieck’s family (his parents, his older sister Mairi, and himself) was by all accounts torn by perpetual quarrelling and conflict throughout the five years during which they actually lived together, a state of affairs witnessed by Hanka’s writings and Mairi’s memories (as recounted by her daughter Diana) as well as by Grothendieck himself more than once:

*Les trois êtres les plus proches, qui ensemble ont constitué comme la matrice de mes premières années, étaient déchirés par le conflit, opposant chacun d’eux et à lui-même, et aux deux autres: conflit insidieux, au visage impassible entre ma mère et ma sœur, et conflit aux violents éclats entre mon père et ma mère d’un côté, ma sœur de l’autre, qui chacune pour son propre compte (et sans que personne du vivant de mes parents ait jamais fait mine de s’en apercevoir...) le faisait marcher à sa façon. La chose mystérieuse, extraordinaire, c’est qu’entouré ainsi par le conflit en ces années les plus sensibles, les plus cruciales de la vie, celui-ci soit resté **extérieur** à moi, qu’il n’ait pas vraiment “mordu” sur mon être en ces années-là et ne s’y soit installé à demeure.... Là, je reviens à cette “chose mystérieuse”, **l’absence de division en moi**, en ces premières années de ma vie.*

*Peut-être le mystère n’est plus pour moi en cette absence, mais plutôt en ceci: que mes parents, mon père comme ma mère, m’aient chacun alors **accepté dans ma totalité**, et totalement: dans ce qui en moi est “viril”, est “homme”, et dans ce qui est “femme”.*

Ou pour le dire autrement: que mes parents, déchirés l'un et l'autre par le conflit, reniant chacun une partie essentielle de leur être – incapable chacun d'une ouverture aimante à lui-même et à l'autre, comme d'une ouverture aimante à ma sœur...que néanmoins ils aient trouvé une telle ouverture, une acceptation sans réserve, vis à vis de moi leur fils. (RS 450)

That Grothendieck's approach to life, to work, to society, to other people, and to mathematics is absolutely whole and entire, monolithic, inaccessible to influence or modification from the outside, is obvious. Whether this indivision is really due to the loving acceptance of his parents during his earliest childhood, or whether contrarily he managed to preserve deeply loving, happy memories of his first five years in a conflict-ridden family, and to describe himself as having been able to remain outside these conflicts, thanks to some inborn native essential wholeness, is certainly not clear. And memories and impressions are subjective and may also change over time. However, the strong feeling he expresses of his own integral nature and lack of fear is borne out by his behavior throughout his life, both negatively and positively, with respect to his relationship with people and with mathematics.

When discussing the issue of fear and inhibition in mathematics, Grothendieck perceives and describes with frightening clarity its destructive effects on creativity.

Craindre l'erreur et craindre la vérité est une seule et même chose! Celui qui craint de se tromper est impuissant à découvrir. (RS 129)

For him, however, the fear of observation and discovery forms a vicious circle with the power of vanity which, building an admirable self-image, represses the fear but simultaneously intensifies it, as the possibility of discovering something unpleasantly different from the reassuring self-image becomes increasingly dangerous. This circle is one of the essential topics throughout his writings.

*Mon propos dans Récoltes et Semailles a été de parler de l'un et de l'autre aspect – de la pulsion de connaissance, et de la peur et de ses antidotes vaniteux. Je crois “comprendre”, ou du moins **connaître** la pulsion et sa nature. (Peut-être un jour découvrirai-je, émerveillé, à quel point je me faisais illusion...) Mais pour ce qui est de la peur et de la vanité, et les insidieux blocages de la créativité qui en dérivent, je sais bien que je n'ai pas été au fond de cette grande énigme. (RS 28)*

According to his analysis, the role of the ego is to avoid any appearance of failure in the eyes of the individual himself, and the fear of failure is above all the fear of tarnishing the self-image, and thereby losing control over the perception of the self.

Souvent une peur secrète fait barrage au “plaisir” même que l'on croit rechercher, effrayé qu'on est par la présence toute proche d'une force inconnue et redoutable, qui risque (si on n'y veille...) de balayer comme fétu de paille Celui en nous qui à tout prix tient à garder “le contrôle”. (RS 479)

Thus, for Grothendieck, fear of the collapse of one's stable and reassuring vision

(whether of the self or of the outer world, or of a mathematical situation) is what paralyzes creativity and prevents profound observation; it is a purely negative and absolutely destructive phenomenon, and its absence – a kind of confidence or self-assurance – is the necessary precondition, and seemingly the only one, for true creative work.

*Cette assurance-là est l'une des faces d'une disposition intérieure, dont l'autre face est une **ouverture au doute**: une attitude de curiosité excluant toute crainte, vis à vis de ses propres erreurs, qui permet de les dépister et de les corriger constamment. La condition essentielle de cette double assise, de cette **foi** indispensable pour accueillir le doute comme pour découvrir, est l'absence de toute peur (qu'elle soit apparente ou cachée) au sujet de ce qui "sortira" de la recherche entreprise – de toute peur, notamment, que la réalité que nous nous apprêtons à découvrir bouscule nos certitudes ou convictions, qu'elle ne désenchante nos espoirs. Une telle peur agit comme une paralysie profonde de nos facultés créatrices, de notre pouvoir de renouvellement. (RS 430)*

There is a great deal of truth in this vision. What is striking, though, is that Grothendieck seems to totally lack any understanding of the protective reasons underlying this psychological structure that he calls 'fear and vanity', which is so common and so widespread that humanity has known only a handful of exceptions (a list of some of these 'mutations' – Darwin, Freud, Walt Whitman, Gandhi – is given in the 'Mutants' section of the Notes to La Clef des Songes). While clearly perceiving the human need for a positive self-image, for reassuring and settled views, and for a certain stability in one's image of self and the world, he qualifies them contemptuously as nothing more than our 'sempiternelle fringale de certitudes et de sécurités, l'instinct du troupeau à la recherche du berger' (Clef 161) which constitutes a fundamental repressive barrier to creativity in every individual, the necessary condition for creativity being an openness to complete, profound and continual renewal.

Similarly, Grothendieck was entirely unable, as he showed again and again throughout the years of his ecological activities, to comprehend the capacity of ordinary people to remain optimistically indifferent to the terrifying danger, perceived by some as imminent, of human destruction by nuclear weapons, not even to mention popular indifference to the actual ills and evils of the world at the present moment: wars, genocide, famine, starvation all taking place more or less under our eyes and forced on our attention on a daily basis by newspapers and information, as we go about our lives. This attitude, this indifference, appear to him to be quite mad (and once his point of view has been made clear and absorbed, a part of oneself does undoubtedly come to share it). Grothendieck's eventual reclusion was in part due to the barrier created between himself and the rest of humanity by this difference, after many years of unsuccessfully attempting to share his vision, first by political and ecological activity, then by prophetic communications, none of which encountered any real wave of sympathy or success; in a 1995 letter, he wrote 'vous êtes dans la Maison des Fous, que vous imaginez encore (comme moi-même naguère) être "le monde", et me dites ce que "les gens" y pensent, et y font. Je suis dehors, "au grand large" ...'

The psychological safeguard mechanisms which place a veil of distance between ordinary people and the horrors that surround them, which allow them to continue seeking

their own comfort in their daily lives (even while paying lip service to the right feelings) are apparently totally lacking in Grothendieck's mental makeup. The humble consolations of the ego: 'the world is a dreadful place, but at least I have succeeded in avoiding the worst of the suffering, in accomplishing something that will last, in providing a happy and healthy home for my children' – these appear to Grothendieck as no more than a disastrously destructive set of blinders that we voluntarily wear out of the sheer fear of *looking at things the way they are*, whether within ourselves or outside in the larger world. To him, the way things are, the natural way in which they occur – even the worst horrors that individual and collective humanity has perpetrated – have their own necessary meaning, significance and harmony, and the essential creative act lies in taking the time to seek for this, to look, to watch, to observe, to feel and see, all the while patiently and unceasingly peeling away from our overactive minds the illusions that spring from its excessive need to interfere with observation by conscious thought. And in fact, a great deal of his later 'prophetic' writings and recordings of religious communications are concerned precisely with the cosmic explanation for the – otherwise absolutely intolerable – existence of evil.

The child-explorer and the child-builder

In one of the most important passages in *Récoltes et Semailles*, one which introduces the key image of Grothendieck's conception of creativity, he introduces the theme of the 'child'; the actual child, as the example of creativity given free rein, and the image of the child within the adult, embodying its continuation throughout life.

*La découverte est le privilège de l'enfant. C'est du petit enfant que je veux parler, l'enfant qui n'a pas peur encore de se tromper, d'avoir l'air idiot, de ne pas faire sérieux, de ne pas faire comme tout le monde. Il n'a pas peur non plus que les choses qu'il regarde aient le mauvais goût d'être différentes de ce qu'il attend d'elles, de ce qu'elles devraient être, ou plutôt: de ce qu'il est bien entendu qu'elles **sont**. Il ignore les consensus muets et sans failles qui font partie de l'air que nous respirons – celui de tous les gens sensés et bien connus comme tels. Dieu sait s'il y en a eu, des gens sensés et bien connus comme tels, depuis la nuit des âges!*

Nos esprits sont saturés d'un "savoir" hétéroclite, enchevêtrement de peurs et de paresseuses, de fringales et d'interdits; d'informations à tout venant et d'explications pousse-bouton – espace clos où viennent s'entasser informations, fringales et peurs sans que jamais ne s'y engouffre le vent du large. Exception faite d'un savoir-faire de routine, il semblerait que le rôle principal de ce "savoir" est d'évacuer une perception vivante, une prise de connaissance des choses de ce monde. Son effet est surtout celui d'une inertie immense, d'un poids souvent écrasant. (RS 127)

For Grothendieck, the ultimate creative being is the child, and this is not a metaphor. Believing as he does that the essential creative force exists in every person, his view of creativity is not to be confused with production of any actual outcome, but expresses an absolutely unfettered ability to observe and feel the flow of the natural world. The spontaneous response of the child to this flow is to thrust his hands into it and begin exploring,

touching, feeling and forming. Although Grothendieck does not explicitly mention beaches, having perhaps never spent any time on one, any tiny child on a beach provides an ideal illustration of his image, the infinitely supple plasticity of the wet sand fitting perfectly with the varying imagination of the child, engaged in an activity of which digging, shaping, destroying, wallowing, building, and simply grovelling are all merely different aspects, of equal value to the child: ‘l’enfant-qui-aime-à-explorer-les-choses, à aller fouiner et s’enfourer dans les sables ou dans les vases boueuses et sans nom, les endroits les plus impossibles et les plus saugrenus...’ (RS 63)

Perhaps the very crux of the matter is that creativity, for Grothendieck, is identified with the accepting and comprehending observation of the mysterious ways of nature – observation made with respect and love, absolutely devoid of judgment – *this is creativity*, and it is completely independent of whether anything material is actually produced.

In a footnote full of charm, Grothendieck describes the enchantment of his tiny daughter as a quintessential moment of discovery:

Elle devait avoir un an ou deux, quelqu’un venait de jeter des granulés dans un bocal de poissons rouges. Les poissons s’empressaient à qui mieux mieux de nager vers eux, la gueule grande ouverte, pour ingurgiter les minuscules miettes jaunes en suspension qui descendaient lentement dans l’eau du bocal. La petite ne s’était jamais rendue compte avant que les poissons mangeaient comme nous. C’était en elle comme un éblouissement soudain, s’exprimant en un cri de pur ravissement: “Regarde maman, ils mangent!” Il y avait de quoi s’émerveiller en effet – elle venait de découvrir en un éclair subit un grand mystère: celui de notre parenté à tous les autres êtres vivants... (RS 200)

With all this, however, the productive aspect of creativity is of course not absent from Grothendieck’s view. He perceives it as another facet: as a complement to ‘l’enfant-qui-explore’, he introduces ‘l’enfant-qui-bâtit’. The first, initially introduced as a masculine type (‘le pionnier, l’explorateur’) is transformed, as he examines more closely the activity he is trying to describe, into a feminine personage (‘une sœur des mares, de la pluie, des bruines et de la nuit, silencieuse et quasiment invisible à force de s’effacer dans l’ombre’ (RS 64)), so that the two facets of creativity together form one of the yin-yang pairs in which both are necessary to achieve a harmonious balance.

Quand je construis, aménage, ou que je déblaie, nettoie, ordonne, c’est le “mode” ou le “versant” “yang”, ou “masculin” du travail qui donne le ton. Quand j’explore à tâtons l’insaisissable, l’informe, ce qui est sans nom, je suis le versant “yin”, ou “féminin” de mon être.

Il n’est pas question pour moi de vouloir minimiser ou renier l’un ou l’autre versant de ma nature, essentiels l’un et l’autre – le “masculin” qui construit et qui engendre, et le “féminin” qui conçoit, et qui abrite les lentes et obscures gestations. Je “suis” l’un et l’autre – “yang” et “yin”, “homme” et “femme”. (RS 63)

Yet Grothendieck admits that he himself devoted the greatest part of his energy during his established professional career to the building rather than the exploring aspect of research.

Mon approche des mathématiques, depuis l'âge de dix-sept ans quand j'ai commencé à m'y investir à fond, a été de me poser des grandes tâches. C'étaient toujours, dès le début, des tâches de "mise en ordre", de grand nettoyage. Je voyais un apparent chaos, une confusion de choses hétéroclites ou de brumes parfois impondérables, qui visiblement devaient avoir une essence commune et receler un ordre, une harmonie encore cachée qu'il s'agissait de dégager par un travail patient, méticuleux, souvent de longue haleine. C'était un travail souvent à la serpillère et au balai-brosse, pour la grosse besogne qui déjà absorbait une énergie considérable, avant d'en venir aux finitions au plumeau, qui me passionnaient moins mais qui avaient aussi leur charme et, en tous cas, une évidente utilité[...]La fidélité à mes "tâches" m'interdisait d'ailleurs des échappées trop lointaines, et je ronguais mon frein dans une impatience d'être arrivé au bout de toutes et m'élancer enfin dans l'inconnu, le vrai – alors que la dimension de ces tâches était devenue telle déjà, que pour les mener à bonne fin, même avec l'aide de bonnes volontés qui avaient fini par arriver à la rescousse, le restant de mes jours n'y aurait pas suffi! (RS 201)

In certain passages, the feeling of impatience he mentions here is expressed as something much stronger; his compulsion to terminate planned tasks instead of exploring unknown territory is expressed as a heavy burden.

Souvent je ronguais mon frein d'être retenu ainsi, comme par un poids tenace et colant, avec ces interminables tâches qui (une fois vu l'essentiel) s'apparentaient plus pour moi à de l'"intendance", qu'à une lancée dans l'inconnu. Constamment je devais contenir cette pulsion de m'élancer de l'avant – celle du pionnier ou de l'explorateur, parti à la découverte et à l'exploration de mondes inconnus et sans nom, m'appelant sans cesse pour que je les connaisse et les nomme. Cette pulsion-là, et l'énergie que j'y investissais (comme à la dérobée, quasiment!), étaient constamment à la portion congrue. (RS 61)

And in the famous last sentence from the *Esquisse d'un Programme*, he calls himself literally a prisoner of this compulsion, and expresses his release from it as a blessed escape.

Aujourd'hui je ne suis plus, comme naguère, le prisonnier volontaire de tâches interminables, qui si souvent m'avaient interdit de m'élancer dans l'inconnu, mathématique ou non. Le temps des tâches pour moi est révolu. Si l'âge m'a apporté quelque chose, c'est d'être plus léger.

Jean-Pierre Serre has pointed out that one of the dominating features of the impression Grothendieck made on others during the 15 or 20 years of his established professional career was of being impelled to do mathematics, not by a sense of joy, delight, or beauty, but by a sense of duty. Grothendieck would not fully agree; if he did not always succeed in communicating his powerful sense of the search for beauty in mathematics, he most certainly felt it deeply within himself and considered it as his guide: 'C'était un sens aigu de la "beauté", sûrement, qui était mon flair et ma seule boussole' (RS 202). The sense of beauty, however, is a very subjective thing; he describes his as 'la recherche constante d'une cohérence parfaite, d'une harmonie complète que je devinais derrière la surface turbulente des choses', which is very different from the kind of delight in mathematical objects or arguments which can be perceived as pretty ('jolis'), unexpected or charming that Serre

himself might evince. Grothendieck's description of his own sense of beauty in mathematics already provides the beginning of an explanation of his compulsive devotion to completing gigantic panoramic tasks. He himself made a conscious effort to understand why he followed this tendency in himself to the point of restraining himself from the kind of free exploration which gave him as much joy or more. Attributing it to egotistic forces – his giving greater value to the 'masculine' values of completed published work than to the 'feminine' values of risky and uncertain exploration – he contrasts it with the attitude of other mathematicians, who work for the sheer joy of the thing, happily lacking this binding sense of responsibility:

Ma relation à la mathématique (et surtout, à la production mathématique) était fortement investie par l'ego, et ce n'était pas le cas chez Mike [Artin]. Il donnait vraiment l'impression de faire des maths comme un gosse qui s'amuse, et sans pour autant oublier le boire et le manger. (RS 161)

The personal value system which encouraged 'achievement' over 'exploration' that Grothendieck followed for the first part of his life also appeared to him to correspond to the values of his milieu:

Les consensus en vigueur m'encourageaient à investir le plus clair de mon énergie dans l'autre versant, dans celui qui s'incarne et s'affirme dans des "produits" tangibles, pour ne pas dire finis et achevés – des produits aux contours bien tranchés, attestant de leur réalité avec l'évidence de la pierre taillée. Je vois bien, avec le recul, comment ces consensus ont pesé sur moi, et aussi comment j'ai "accusé le poids" – en souplesse! La partie "conception" ou "exploration" de mon travail était maintenue à la portion congrue jusqu'au moment encore de mon départ. (RS 63)

There was, however, a generosity in his attitude even if he simultaneously felt that he had been impelled by forces of ego or vanity. At the time of his work, he very clearly felt that it was destined for the common good; an absolutely necessary piece of work that simply must be done by anyone who could; by himself alone if no one else was available to help. In a famous letter to Henri Cartan, from October 1961, protesting the necessity for graduate students in mathematics to leave for the mandatory two years' military service, he wrote: 'J'ai péniblement décroché pour mon séminaire de géométrie algébrique à l'IHES quatre ou cinq ex-normaliens, qui commencent à avoir de vagues lueurs, et dont un ou deux semblaient sur le point de démarrer sur du travail utile, voire urgent, savoir Verdier et Giraud. Bernique, sauf erreur tous les deux, et en tous cas Verdier, devançant l'appel, et si ça se trouve quelqu'un d'autre fera le boulot à leur place (moi-même s'il le faut).' He perceived his planned work as absolutely necessary; not his own personal research, but foundational work essential to the further development of a subject in need of renewal. In August 1959, still in the early flush of starting the EGA's, he wrote a wildly optimistic plan to Serre:

J'espère arriver dans l'année prochaine à une théorie satisfaisante du groupe fondamental, et achever la rédaction des chapitres IV, V, VI, VII (ce dernier étant le groupe fondamental), en même temps que des catégories. Dans deux ans résidus, dualité, intersections, Chern, Riemann-Roch. Dans trois ans cohomologie de Weil, et un peu d'homotopie

si Dieu veut. Et entre-temps, je ne sais quand, le “grand théorème d’existence” avec Picard etc., un peu de courbes algébriques, les schémas abéliens. Sans difficultés imprévues ou enlisement, le multiplodoque devrait être fini d’ici 3 ans, ou 4 ans maximum. On pourra commencer à faire de la géométrie algébrique!

As Demazure points out, the last sentence ‘On pourra commencer à faire de la géométrie algébrique’ must be read as it is meant: not that *Grothendieck* (and his associates) would be able to start working on algebraic geometry, but that the mathematical community as a whole would be enabled to make serious advances in the subject, thanks to the foundational work that Grothendieck and Dieudonné were undertaking in the service of all mathematicians, of mathematics itself. The strong sense of duty and public service was felt by everyone around Grothendieck*, and it is absolutely not in contradiction with the simultaneous sense of personal involvement and pride:

*La force principale, le “drive” qui était derrière l’investissement que je faisais en mes élèves en général, dans la première période des années soixante, c’était le désir de trouver “des bras” pour réaliser des “tâches” que mon instinct me désignait comme urgentes et importantes... Cette “importance” surement n’était pas purement subjective, ce n’était pas une simple question “de goûts et de couleurs”... Pourtant, pour ce qui est de ce “drive”, de cette force de motivation en moi qui me poussait vers la réalisation des tâches, ce n’était pas une certaine importance “objective” qui était en jeu – alors que “l’importance” de la conjecture de Fermat, de l’hypothèse de Riemann ou de celle de Poincaré me laissaient parfaitement froid, que je ne les “sentais” pas vraiment. Ce qui distinguait ces tâches de toutes autres, dans ma relation à elles, c’est que c’étaient **mes** tâches; celles que j’avais senties, et faites miennes... C’est le lien profond entre celui qui a conçu une chose, et cette chose... Il me paraît profondément enraciné dans la nature du “moi”, et de nature universelle. (RS 325)*

If, in the yin-yang aspects of creativity, Grothendieck chose for years of his life to privilege the yang, he does not doubt that it was at the deepest level a question of personal glory and self-aggrandizement: ‘L’investissement dans mes tâches était de nature égotique... Surement la réalisation de ces tâches était surtout, pour le “moi”, un moyen de s’agrandir, par la réalisation d’une œuvre d’ensemble aux vastes proportions... A partir d’un certain moment dans ma vie de mathématicien, il y a eu cette ambiguïté constante d’une cohabitation, d’une interpénétration étroite entre “**l’enfant**” et sa soif de connaître et de découvrir, son émerveillement en les choses entrevues et en celles examinées de près, et d’autre part le **moi**, le “**patron**”, se réjouissant de ses œuvres, avide de s’agrandir et d’augmenter sa gloire par la multiplication des œuvres, ou par la poursuite opiniâtre et incessante d’une construction d’ensemble aux grandioses dimensions!’ (RS 326) This choice, the choice that he almost unconsciously made for twenty years or more, before coming to the realization that it was not, for him, the true road to self-realization, was

* As Steve Kleiman notes: “I can attest to this statement from my own experience. Grothendieck told me to read a little EGA every day in order to acquire a sense for its contents, because he and Dieudonné were writing EGA as a service and we should appreciate this fact and take advantage of it.”

a direct consequence of the teachings and values that Grothendieck absorbed from his mother Hanka.

The image of the mother

The image of the child, free to explore and construct and discover, without pressure or prejudice or fear, is invariably accompanied in Grothendieck's mental universe by the image of the Mother, from whose womb the child springs and to which he eternally seeks to return, symbol of the psychic process of creation. The passages expressing this vision are sometimes metaphorical:

*Dans mon travail de mathématicien, je vois à l'œuvre surtout ces deux forces ou pulsions, également profondes, de nature (me semble-t-il) différentes. Pour évoquer l'une et l'autre, j'ai utilisé l'image du **bâtitteur**, et celle du **pionnier** ou de l'explorateur... Ces deux pulsions qui m'apparaissaient comme "de nature différente" sont finalement plus proches que je ne l'aurais pensé. L'une et l'autre sont dans la nature d'une "**pulsion de contact**", nous portant à la rencontre de "**la Mère**": de Celle qui incarne **et** ce qui est proche, "connu", **et** ce qui est "inconnu". M'abandonner à l'une ou l'autre pulsion, c'est "retrouver la Mère"... (RS 62)*

sometimes lyrical:

*L'Univers, le Monde, voire le Cosmos, sont choses étrangères au fond et très lointaines. Elles ne nous concernent pas vraiment. Ce n'est pas vers **eux** qu'au plus profond de nous-mêmes nous porte la pulsion de connaissance. Ce qui nous attire, c'est leur **Incar-nation** tangible et immédiate, la plus proche, la plus "charnelle", chargée en résonances profondes et riche en mystère – Celle qui se confond avec les origines de notre être de chair, comme avec celles de notre espèce – et Celle aussi qui de tout temps nous attend, silencieuse et prête à nous accueillir, "à l'autre bout du chemin". C'est **d'elle**, la Mère, de Celle qui nous a enfanté comme elle a enfanté le Monde, que sourd la pulsion et que s'élancent les chemins du désir – et c'est à **Sa** rencontre qu'ils nous portent, vers **Elle** qu'ils s'élancent, pour retourner sans cesse et s'abîmer en Elle... (RS 64)*

sometimes overtly sexual:

*Pour l'amant, l'amante est la Mère, et son élan vers elle est élan de retour vers le Giron dont il est né – dont toute chose est née... C'est l'irrésistible élan de la naissance à rebours: **retourner** dans le **Giron** accueillant de la **Mère**... Dans la mort seulement de l'amante et de l'amant l'un en l'autre, dans le Giron ruisselant de la Mère, se prépare et éclot **naissance** dans l'amante et dans l'amant. L'amant se trouve re-né dans l'amante-Mère – entre les cuisses puissantes de la Mère, reposant dans l'épuisement bienheureux de l'Accouchée. (Eloge 136)*

All of them celebrate the incredible power of the creative force, and associate it to the person of the Mother. The role of Grothendieck's own mother in his development of this

persistent metaphor for creation is a matter of interrogation and interpretation; there is no explicit answer, but the question cannot be ignored.

When Grothendieck mentions his mother Hanka in his writings, it is frequently to describe the negative influence she had over his self-development, by deluding him with her powerful personality into believing and agreeing with any number of ideas whose destruction formed one of the main ingredients of the change he underwent later in life, when he discovered meditation. Her domination, imposing on him the adoption of her own consciously held values in spite of himself, lasted for decades: ‘Il m’apparaît maintenant qu’une des forces derrière mon attitude était l’ascendant que la forte personnalité de ma mère a exercé sur moi pendant toute sa vie, et pendant près de vingt ans encore après sa mort, pendant lesquels j’ai continué à être imprégné des valeurs qui avaient dominé sa propre vie’ (RS 156). Hanka was responsible for leading him to value only the most masculine (*yang*) aspects of activity in life (her general attitude towards others being one of ‘dédain hautain et quasiment universel’ (Clef 102), for teaching him contempt for the ‘feminine’ virtues of gentleness and kindness, and for convincing him, even for decades after her death, that his parents were the most splendid of parents and she herself the most heroic of mothers – ‘elle continuait à se maintenir dans le mythe du grand et inégalable amour entre elle et mon père, et dans celui de la mère remarquable et à tous égards exemplaire qu’elle avait été,’ (Clef 106) – thereby psychologically barring him from access to deep truths concerning his own nature and development.

But there was another side to Hanka’s effect on her son which must not be underestimated. Grothendieck’s blindness to barriers and taboos of any kind, social or professional, inside or outside of mathematics, is like an echo of his mother’s violent and conscious life-long rebellion against any form of interdiction. She may have contributed, for many years at least, to his ignoring the fundamental ‘feminine’ facets of his own nature, but by her example of absolute and voluntary rejection of all taboo and social constraint, she raised him in the absolute freedom from inhibition which had such disastrous consequences on his social development, but allowed the astounding overdevelopment of his forces of pure creativity.

The very numerous references to the relation between the mother and the child and the connection of this relation to the creative impulse, while largely metaphorical, cannot be taken as entirely so; the metaphor of the mother cannot be approached psychologically entirely independently of the experience of the mother herself. As difficult, as violent and conflictual and contemptuous as she may have been, she gave him, or allowed him to preserve, the archetypal feeling of the maternal womb as the nourishing matrix of creation, and the feeling of being *whole*, of being exempt from the psychological fracture (‘coupure’, ‘division dans la personne’) caused by the ‘tabou de l’inceste qui coupe l’enfant de la mère, comme il coupe la vie de sa mère la Mort, comme il coupe aussi une génération de celle qui la précède’ (RS 474). Blind to the protective role of taboo in the preservation of a healthy collective society as he is blind to the protective role of fear, Grothendieck considers that total freedom from these constraints is a necessary condition for the release of the creative impulse. No matter what harm may be done by the transgression of natural laws, it is always good in that it leads to deeper and truer knowledge:

Je sais qu’il y a une substance nourricière dans tout ce qui m’arrive, que les semailles

soient de ma main ou de celle d'autrui – il ne tient qu'à moi de manger et de la voir se transformer en connaissance...Il n'y a ni amertume ni résignation en moi, ni apitoiement, en parlant des semailles et de la récolte. Car j'ai appris que dans la récolte même amère, il y a une chair substantielle dont il ne tient qu'à nous de nous nourrir. Quand cette substance est mangée et qu'elle est devenue part de notre chair, l'amertume a disparu, qui n'était que le signe de notre résistance devant une nourriture à nous destinée. (RS 152)

This passage is the key to Récoltes et Semailles and the key to Grothendieck's message in general. And it applies equally to his life and to his work; an expression of vast generality, in quintessentially Grothendieckian language, of the trait that Demazure evoked purely mathematically, when he described Grothendieck's approach as 'turning the problem into its own solution'.

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