Chapter 4. International Wanderings: 1953-1955

São Paulo, 1953-1954

Faithful to his usual frugal habits, the first thing that Grothendieck did upon arriving in São Paulo to take up his new job was to move with his mother into a private house, and rent himself a piano. The house was situated on rua Oscar Freire, which is now a chic, modern and central shopping zone; number 1052 where Grothendieck lived is gone, replaced by a parking lot and a pseudo-chic clothing store.

But the room he had was described by his friends as "tiny", as all his rooms would always be: even later on, as a reputed and highly-paid professor, he would continue to pursue an incongruously modest lifestyle. In 1953, he had good reason to be economical; although he was receiving a salary, the terms of his contract allowed him to return to Paris for the entire fall and winter, probably starting in October, when seminars and lecture courses in France would typically begin. Presumably he was not paid a full salary from São Paulo during these months, and in any case, a large proportion of everything he earned went for the support of Hanka. With the exception of one visit to Brazil, during which mother and son went up to Rio and spent a few days being taken to all the best tourist attractions by Paulo Ribenboim and his wife Huguette, Hanka remained in France.

Grothendieck was immediately noticed and made much of by the São Paulo math department, which was used to receiving French celebrities and learning the most avant-garde subjects from them, and whose members themselves regularly made the arduous flight to Paris*. His reputation preceded him; the department knew all about his six important publications, each of which could have earned him a doctoral degree, about the statelessness which prevented him from holding a university position in France, and about the mother who was dependent on him. Brazil was flourishing economically at that time, and the department was only too happy to hire the young genius. Chaim Hönig**, already a professor at USP, recalls that Grothendieck always remained close to the Paris mathematical circle, exchanging letters with French mathematicians, keeping abreast of their work, and hurrying home in the fall to attend as many lectures and seminars as he could. For his own research, Grothendieck preferred to work at home, and with few

* P. Ribenboim recounts that a small airplane carried passengers from São Paulo to Rio, as there was no airport in São Paulo that could accommodate the large Constellation propeller planes used for transatlantic voyages. From there, the passengers embarked on an overnight flight which, after stopping for refueling in Recife, continued on to Dakar, where breakfast was offered in the airport. The plane then continued onwards to Lisbon (lunch), Madrid (tea) and Paris (dinner), with a dwindling number of passengers as travellers reached their respective destinations, and reached London as a final stop about 24 hours after departure.

** Professor Hönig's memories recounted here are taken from an interview organized and filmed for me by the kind efforts of F.C. Polcino Milies.

exceptions, he showed no interest in discussing anything apart from mathematics. He hardly had any choice in the matter, Professor Hönig adds: "He had to make his mark as a mathematician, otherwise his future situation would be extremely complicated". In fact, it came as a great surprise to Hönig when Grothendieck confided to him one day that as far as a choice of profession was concerned, he hesitated between becoming a mathematician or a pianist. Hönig, who had no idea that Grothendieck loved music, was startled. Although no one seems to remember ever hearing Grothendieck play, he certainly did play quite regularly (he once claimed to have worked through every single slow movement of Beethoven's sonatas), but it does not seem that there could be any possible comparison between his talent in music and his talent in mathematics.

Still, it was obvious to everyone that outside of the time when Hanka came to visit São Paulo in 1953, Grothendieck had no social life at all. In a new alimentary phase adapted to the continent, he was proud to tell people that he ate nothing but milk, bread and bananas "so as not to lose any time over it", and his (lack of) participation in his colleagues' cultured conversations revealed the depth of his ignorance of literature, in spite of his mother's being, as he always clarified, a writer. He didn't waste much time chatting with colleagues anyway, not even about his research.

He did, however, teach; in French, of course, which was understood by everyone in the mathematical milieu of the time. Hönig remembers how during one lecture, someone in the audience interrupted Grothendieck to point out the window, exclaiming "Look at the rainbow!" Grothendieck looked, remarked "Yes indeed," and continued his lecture. Grothendieck's work was very advanced, and the number of auditors was no more than four or five, but a faithful and persistent group did attend the entire course. One of them, José Barros Neto, took a set of notes called *Espaces Vectoriels Topologiques*, which eventually served Grothendieck as a first draft for the Bourbaki volume on Topological Vector Spaces.

Barros Neto is one of the only people who actually tried to make friends with Grothendieck during the two 6-month periods he spent in São Paulo. When Grothendieck first arrived there, Barros Neto was getting ready to begin a Ph.D. thesis. He already had a good background in topology, and was eager to take the course that Grothendieck announced: an introduction to topological vector spaces. At first, some 5 or 6 people followed the lectures; the department head Candido da Silva Lima Dias, who was responsible for Grothendieck's formal invitation, was interested in topological vector spaces, as was Chaim Hönig. But participation wore away as the months advanced, and Barros Neto found himself alone with Grothendieck in the lecture room on more than one occasion.

As the course moved forward, Professor Dias suggested that Grothendieck actually make written notes for it, which could be distributed to listeners. The idea suited Grothendieck, and he began to write very detailed notes, which were then typed up, rechecked by him, and mimeographed for distribution. As the notes thickened and began to turn into a book, Grothendieck realized that he needed an opening chapter containing many of the background notions of topology that he was using, and invited Barros Neto to write it for him. He didn't want to call it an "introduction", because, as he said, an introduction would have to contain some mention of what he was going to be talking about, a statement of the aims and goals and content of the course. All he wanted was

the topological definitions and statements necessary to his ulterior developments, what he ended calling a "chapter 0". In order to explain the different types of topology that can be put on the dual of a topological vector space (weak, strong etc.), for example, he needed to be able to refer to the different types of convergence that are studied in general topological spaces. He made a fairly detailed plan, and as Barros Neto wrote up sections of it, Grothendieck would read them over and make suggestions; he knew exactly what he wanted. He was very pleased to get it (Barros Neto is warmly thanked for performing such an ungrateful task), and as the two worked together over the chapter, they developed a certain camaraderie. As Grothendieck lived several blocks from campus, Barros Neto would frequently walk him home, and during these walks they talked of other things than mathematics – though it is not always very clear what. "Movies," Barros suggested*, but couldn't remember whether Grothendieck had actually gone to see any movies, although he certainly had. In any case, they did not go to movies together, nor to each other's homes. Nor did they even ever eat together; as he worked alone, Grothendieck also preferred to eat alone, and Barros also remembers him arriving at the office with a bunch of bananas under his arm. They did take those walks, though, talking about this and that. "Books, politics. Grothendieck knew a lot of things. We students at USP back then were all leftists, of course, but he hated that. He didn't like communists, he didn't like socialists. He wasn't really interested in politics."

Functional analysis in São Paulo

Grothendieck submitted numerous publications from São Paulo, but a close look at his publications from that time show that the work he did there to some extent came down to correcting, writing up or at most developing ideas that dated back to his time in Nancy. An examination of the ten articles he submitted in Functional Analysis from São Paulo (not counting the bound lecture notes from his course in São Paulo and a write-up of a conference lecture) shows that only the two papers actually sent to the Boletim da Sociedade Matemática de São Paulo, both from 1953, contain the really original work that Grothendieck probably did in his first year in Brazil**.

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* This was during the course of a very long and interesting telephone interview in 2011.

** The article in Summa Brasiliensis **3** had already been submitted to the Circolo di Palermo in 1952, whence it was withdrawn after the discovery that it was necessary to pay a membership fee in order to have the right to publish there. It was then sent to *Trans. AMS*, only to be withdrawn once again when the AMS accepted to publish the enormous Memoir - Grothendieck's thesis - but not both texts. The Ann. Inst. Fourier **6** article is an erratum of the thesis; the Bull. Math. Soc. France **84** article on Fredholm theory also dated back to 1952 and had already been accepted in a Brazilian journal in 1953, but was withdrawn due to long publication delays. Thus, these four articles represent work done before São Paulo. The Canadian J. Math. **6** article, submitted in October 1953 at the end of his first stay in São Paulo, is a short 3-page observation, and the Canadian J. Math. **7** article submitted in September 1954 is an erratum, devoted to proving a result first announced in his fourth Note aux C.R.A.S. from 1951: "Cet article est destiné surtout à donner une preuve correcte d'un des résultats annoncés dans une ancienne note

The first Boletim article, *Résumé de la théorie métrique des produits tensoriels topologiques*, begins with a dry ten page introduction to "tensor norms", preceded by the typically Grothendieckian warning "Remarquons que nous aurions pu nous dispenser de développer tout le formalisme des b-normes (§§1,2,3) pour formuler et démontrer les résultats fondamentaux du §4. Mais il me semble que, comme en mainte occasion analogue, cela aurait économisé encre et papier aux dépens de l'effort intellectuel du lecteur. En effet, ce n'est que par ces préliminaires que l'on arrive à donner les énoncés sous la forme concise et suggestive qui permet de saisir, d'un seul coup, les relations entre les très nombreuses variantes du théorème fondamental, et qu'on parvient à une compréhension véritable de la théorie." [1] The heart of the paper is a theory in which three types of spaces: Hilbert spaces, $C(K)$-spaces and L-spaces, are considered as building blocks for the entire structure theory of Banach spaces, and operators on Banach spaces are studied via their factorization through spaces of these types. One of his most interesting conclusions is the solution of one of the many problems posed by Banach and Mazur: he proves that if X is a Banach space which is both a subspace of an L-space and a quotient of a $C(K)$-space, then it is isomorphic to a Hilbert space. This work, based on the classification of operators according to the types of spaces they factor through, is a pure expression of the approach that later became one of Grothendieck's trademarks: focusing attention, in studying the structure of a given type of object (Banach spaces), on the properties that can be observed via the maps between them.

Grothendieck's second article from the Boletim, *Sur certaines classes de suites dans les espaces de Banach et le théorème de Dvoretzky-Rogers*, reflects the same approach. Already in his thesis, he had given a new proof of a theorem due to Dvoretzky and Rogers: they showed that in every (infinite-dimensional) Banach space, there exists an unconditionally convergent series which is not absolutely convergent. The new proof of this result that Grothendieck gave in his thesis is operator-theoretic, based on the study of what he called absolutely summing operators. In the second Boletim paper, he used this approach to generalize the result, proving that in every (infinite-dimensional) Banach space, there exists a weakly p-summable series which is not absolutely p-summable, for a real number $p > 1$*.

Much of his time in Brazil was spent in preparing the final text of his thesis for publication. The book, now a hefty two-part tome, underwent several misadventures; the "so-called" report by the "so-called" referee was described by Dieudonné as "a striking example of intellectual dishonesty", and together with André Weil, Dieudonné lost no time in emitting a formal complaint to MacLane, who resolved the situation by asking Dieudonné to act as referee himself. Grothendieck then struggled with typing up the manuscript according to very rigid and detailed instructions (the *Memoirs* required photocopy-ready

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(je m'étais aperçu peu après la publication de cette note que la preuve de ce résultat était incomplète)." In the two CRAS Notes submitted in June 1954, he sketches the proof that the identity map from a Hilbert space to itself is preintegral; this result had already been proved in detail in the papers submitted to the Boletim of São Paulo, which had at that time been accepted but not yet printed (the volume actually appeared in 1956).

* A deeper and more detailed expository account of the work Grothendieck did in his thesis and in São Paulo can be found in [D].

manuscripts), and then received so many criticisms and injunctions that he was on the point of giving up and trying to publish it somewhere in France. Dieudonné intervened once again and contacted MacLane who was on the point of refusing to publish it, sent in a very positive referee report, and gave his blessing to the project of having the whole thing typed up by the ultra-competent "Bastien", Bourbaki's official secretary, on her own time and paid directly by Grothendieck.

No sooner was this problem finally resolved than Grothendieck stumbled on another. He was involved in a very interesting joint book project with top young Brazilian mathematician Leopoldo Nachbin who, just 6 years older than Grothendieck, had reached sudden and international fame at age 22 with a paper establishing a bound on the growth rates of an analytic function. Those who knew Nachbin perceived him as being dynamic and highly energetic; always on the go, travelling frequently between France, the US and Brazil, writing many dozens of letters to his mathematical friends and sending postcards (often depicting charming girls) to the others, as well as participating in the role of editor in an astonishing number of journals and book series for which he tirelessly recruited articles. But Dieudonné expressed serious doubts that the joint book would ever get off the ground, because Nachbin was "Brazilian, and so inclined to indolence", as he said forthrightly, though not unkindly. Certainly Dieudonné cannot easily have found a match to his own brand of unflagging devotion to even the most thankless parts of mathematical work anywhere, but he still found Nachbin's lightheartedness exaggerated: "Schwartz remembers that in 1952, after promising to write up his lectures, he didn't even produce 3 pages! Now, you're over there, so you're the best judge of Nachbin's degree of enthusiasm, but I think it's only right to warn you." By December 1954, the plan of the book was complete, with the first part intended to introduce the classical theory that Grothendieck had explained in his two-year course, based on the copious and detailed notes for that course that had been typed and bound by the University of São Paulo (with an introductory chapter by Barros Neto), and the second meant to cover the theory of tensor products and nuclear spaces, the whole not to exceed 600 pages. Perhaps the project was too similar to work that Grothendieck had already written up extensively, or perhaps Nachbin's enthusiasm waned; in any case, Dieudonné was perfectly right, and no book was ever produced*.

From all these observations, it would appear that although Grothendieck did profound work in his first year in São Paulo, he did not undergo another flash of discovery of the vast and deep type that characterized his thesis. As for the second year he spent there, he seems to have devoted it mainly to thinking about the fundamental problem of whether every Banach space possesses the "approximation property". A space X is said to have this property if any operator on the space X can be approximated uniformly on compact sets of X by finite rank operators, and when Grothendieck went to São Paulo, it had already been known for years that all Hilbert spaces possess it. The general problem originated in a café in the Polish town of Lvov, called the Scottish Café (Kawiarnia Szkocka), which was much beloved by the group of mathematicians surrounding Stefan Banach at that time, because they could write formulas directly onto the white marble-topped tables, and the

* In a letter to Gottfried Köthe from March 1956, Grothendieck remarks: "Nachbin has unfortunately given up the idea of a joint book, after a short straw fire of enthusiasm."

waitress could remove them easily afterwards with a swipe of wet rag. In order to prevent the group from losing important results in this manner, Banach's wife presented them with a large notebook in which they recorded all sorts of solved and unsolved problems, offering prizes such as a bottle of brandy for the hardest ones*. In 1936 a student of Banach, Stanislaw Mazur, offered a live goose for the solution of the approximation problem.

In spite of his best efforts, Grothendieck did not win the goose. But he did a lot of interesting work in the subject. To start with, he reformulated the problem completely; indeed, Mazur's original question was equivalent to a question of Banach as to whether every Banach space X possesses a "Schauder basis", an infinite sequence of elements $v_n \in X$ such that every element $v \in X$ can be written uniquely as an infinite sum $v = \sum_n a_n v_n$ convergent for the norm topology. Grothendieck understood that any Banach space which has a Schauder basis has the approximation property, which is not hard, but he also showed that if every Banach space has the approximation property, then every Banach space has a Schauder basis. In this way, he focused the attention of mathematicians on the problem of finding a Banach space without the approximation property in order to solve Mazur's problem. (This was finally accomplished in 1973 by Norwegian mathematician Per Enflo, who was duly handed a goose in a basket by the now elderly Mazur.) Most of the work Grothendieck did on the subject, however, had been accomplished in his thesis. The fact is that he did not make much headway on the topic during his stay in São Paulo. The main impression left in Grothendieck's mind by that difficult period seems to have been one of frustration and failure.

Au temps où je faisais encore de l'Analyse Fonctionnelle, donc jusqu'en 1954 il m'arrivait de m'obstiner sans fin sur une question que je n'arrivais pas à résoudre, alors même que je n'avais plus d'idées et me contentais de tourner en rond dans le cercle des idées anciennes qui, visiblement, ne "mordaient" plus. Il en a été ainsi en tous cas pendant toute une année, pour le "problème d'approximation" dans les espaces vectoriels topologiques notamment, qui allait être résolu une vingtaine d'années plus tard seulement par des méthodes d'un ordre totalement différent, qui ne pouvaient que m'échapper au point où j'en étais. J'étais mû alors, non par le désir, mais par un entêtement, et par une ignorance de ce qui se passait en moi. Ca a été une année pénible – le seul moment dans ma vie où faire des maths était devenu pénible pour moi! Il m'a fallu cette expérience pour comprendre qu'il ne sert à rien de "sécher" – qu'à partir du moment où un travail est arrivé à un point d'arrêt, et sitôt l'arrêt perçu, il faut passer à autre chose – quitte à revenir à un moment plus propice sur la question laissée en suspens. Ce moment presque toujours ne tarde pas à apparaître – il se fait un mûrissement de la question, sans que je fasse mine d'y toucher par la seule vertu d'un travail fait avec entrain sur des questions qui peuvent sembler n'avoir aucun rapport avec celle-là. Je suis persuadé que si je m'obstinais alors, je n'arriverais à rien même en dix ans! C'est à partir de 1954 que j'ai pris l'habitude en maths d'avoir toujours beaucoup de fers au feu en même temps. Je ne travaille que sur

* This book was eventually edited by R.D. Mauldin and has been published as *The Scottish Book: Mathematics from the Scottish Café*, Birkhäuser, 1981. The Dvoretzky-Rogers theorem of which Grothendieck gave a new proof solved problem 122 from the Scottish book.

l'un d'eux à la fois, mais par une sorte de miracle qui se renouvelle constamment, le travail que je fais sur l'un profite aussi à tous les autres, qui attendent leur heure.[2] [RS 196]

This frustration may be at least partly the cause of the change that was wrought in Grothendieck's character during the time he spent in São Paulo. Although his relationships with other men were essentially professional, the women who were close to him could not fail to notice this change. If his personal life was "on hold" while he was in São Paulo, where camaraderie played but a small role*, he made up for this swiftly on his return to his native continent, where in less than no time his life turned into a "daily, endless hell" of love/hate relationships, recriminations, accusations, strife and rupture.

A European interlude

The first thing that happened on his return to France in October 1953 was the birth of the son, Serge, that he had fathered with Aline Driquert. At this time, Grothendieck had no ill feelings towards Aline, and at least a passing interest in the baby, whom he probably visited. But, as Aline later told Serge, Hanka was determined to detach her son from Aline and her children, and Grothendieck did not feel attached to Aline by any emotional tie or sense of duty. Hoping, perhaps, to get him away from outside influence, Hanka organized a trip to Germany in November, where she and Schurik were to meet a number of people that they cannot have seen since before the war: the Heydorn family, and several Grothendieck family relations.

From a long letter written by Hanka to Wilhelm Heydorn after this visit, dated December 3, 1953, it emerges clearly how different her relationship to Grothendieck had become since the time when they lived together in Mayrargues. Although it is certain that they quarreled vigorously even then, even occasionally breaking the china, such quarrels were something of a family habit and of a mutually recognized expression of idealism. Obviously neither Hanka nor Schurik considered them worthy of comment: indeed, Schurik's description of the years in Mayrargues are poetically nostalgic in their simplicity, and Hanka's letter to Dagmar from that time rings with praise and admiration for her son, and spontaneous joy at the closeness and depth of their relationship. As for her truthfulness, her letter to Wilhelm Heydorn five or six years later (as well as her autobiographical writings) prove beyond a doubt that although her vision of her own self may have been a bit rosy*, she had no hesitation in expressing her feelings with brutal honesty, no fear of looking bad, and definitely no particular desire to paint her situation in a flattering light.

In this letter, two painful quarrels with Schurik are described. One had taken place during the Grothendiecks' visit with the Heydorns, and had ended up with Hanka reduced to such rage that, for the first time in her life, she gave her son a resounding box on the ear. This scene must have been excruciatingly embarrassing, there in the Heydorn

* As usual, Grothendieck did find the time for a love affair here and there, but nothing that could hold his attention once he returned to Europe.

* Both she, and many years later Grothendieck, loved to talk about the immensity and depth of their self-analysis and self-comprehension, something that seems actually quite wanting viewed from the outside.

household, where under the influence of the pastor, everyone and everything was measured and reflective, consciously oriented towards a religious and humanistic vision of goodness. "I must explain something to you," she wrote to Wilhelm Heydorn about this incident, "my very sharp reaction to him while we were at your house. It's like this: he has become used to treating me like a dependent child, or a feeble-minded old lady, who has to be interrupted whenever she starts speaking unsuitably, and whose life and doings must (of course, in her "best interests") be decided for her. My cousin has seen this frequently, and has been quite annoyed by it, and I have had to endure her actually protesting about it in my presence, which is obviously not very pleasant." Although Hanka perhaps thought that she could explain away the shocking incident with these few brief lines, Wilhelm Heydorn had already drawn his own conclusions from his observations of their personalities and their relationship with each other.

On the same day as their visit to the Heydorns, Schurik had planned a party in the evening to celebrate his engagement to his cousin Hannelore, which had occurred with remarkable celerity within a few days of his arrival in Hamburg. Although according to Aline, this engagement was part of Hanka's plan to keep Schurik away from her, it was obviously not a peaceful process, and feelings were running high between mother and son. Thus, after the scene in which Grothendieck's ears were boxed, he demanded that she stay away from his engagement party, and she fully agreed that her attendance would be impossible under the circumstances. That morning, it had been decided that as they were to return to France the next day, it would be best for Schurik to send a telegram to Maidi, who was then living with her child in La Haute Maison, the CIMADE residence for refugees in the village of Sucy-en-Brie.

Upon arriving in France, it emerged that this telegram had never been sent, whereupon a tremendous family scene erupted once again. Upon being asked why he had not sent the telegram, Schurik announced that he didn't have the address, and that this was entirely Hanka's fault, because she had not come to his engagement party the previous evening (when he was supposed to have sent the telegram) "on purpose to seem like a martyr". Hanka replied that he knew the telephone number, and that the Heydorns had told him to send the telegram by telephone. Upon this, Schurik burst into a tirade of insults and called her a liar. She demanded that he ask the Heydorns whether what she remembered was right or wrong – she declared that he owed her at least that – he replied that he owed her nothing – she ordered him out of the room – he stormed out forthwith. The purpose of Hanka's letter to Wilhelm was a plea (laced with complaints) that he should write to Schurik and tell him that she was right, that she remembered correctly, that he had been told to telephone, and if possible, to exhort him to a better treatment of his mother (although, as she warns, "please drop any attempt to appeal to his conscience, as that would only drive him further into obstinacy").

If the letter only described this incident, it would be merely sad and a little petty. But it also contains an expression of Hanka's deeper fears, and her despair at the change she perceived in her son, and in their relations. Even if the letter was motivated by her own wounded pride and the need to be proven right, her observations ring true. "I can't tell you in what desperate straits I am with this boy. He used to be so upright, so honest, and now it isn't the first time that he takes facts from two or three days ago and quite simply denies

them, calling me a liar, or irresponsible, with a cynicism that takes my breath away. Stupid mother cow that I am, I still have so much trouble really realizing the terrible change in my son that I'm often tempted to just believe I remember things wrongly, except when the facts or some third person proves that I'm right, and that Schurik, whether on purpose or not, simply remodeled reality in his brain however he wanted...But whether it's just a question of bad memory or not, after this latest scene I have seriously begun to worry that he 'isn't quite normal any more', as people commonly say, and that his megalomania and infallibility complex is going to end up turning into mental illness."

Grothendieck was a physically attractive man with a powerful, overwhelming personality, and again and again in his life, women fell in love with him, only to be deeply disabused and disappointed after a relatively short time. So it was with Hannelore. She moved to Sucy-en-Brie, where Grothendieck and Hanka now joined Maidi and her small daughter Angelika, and it was decided that she would learn French while taking care of Hanka during her worst periods of illness even after Grothendieck's return in the springtime to São Paulo; presumably the financial situation was to be taken care of by Grothendieck, who always sent a large portion of his salary to Hanka. But this plan did not outlast the winter. If Maidi had already discovered that living together with Hanka and Alexandre was a hellish experience, Hannelore found it all the worse, as Hanka's health had deteriorated and her motivation to work on her epic autobiographical novel had all but disappeared, whereas Grothendieck had become not only angry and bitter, but also, as Hanka had noted, something of a megalomaniac. And this was not Hanka's perception alone: according to Hannelore*, "Grothendieck lived only for his math, and conducted himself in an aggressive, condescending manner that was quite simply mean. He was domineering, bossy, a know-it-all, and moreover unpractical". Because the Grothendiecks spoke German at home, Hannelore made no progress in French, for which Alexandre mocked her. In the end, it became impossible for the family to continue living together and all idea of an engagement was dropped. First Maidi and then Hannelore moved to Paris and found work, and Hanka left Sucy to live in a small apartment in the Parisian suburb of Bois-Colombes. Schurik returned to spend his second year in Brazil. It is not clear how much Hanka was responsible for the family breakdown and how much Alexandre, but in any case when he left for São Paulo in March 1954, between them they had managed to destroy the remnants of their relationship with each other, as well as Schurik's relationships with his sister, his fiancée and the mother of his baby.

A change of subject and a certain Jean-Pierre Serre

As noted earlier, from the point of view of research, Grothendieck's second year in São Paulo was worse than his first. It is possible that the rather frightful human wreckage he had left behind him in France was partly the cause of this, but the cause and effect may also be inverted, if he was already feeling the frustration referred to in the quotation from *Récoltes et Semailles* before he even returned to France. This impression is borne out by the fact that in a letter to the Austrian mathematician Gottfried Köthe, a specialist in topological vector spaces who had shown interest in Grothendieck's work since 1950,

* Speaking in person to W. Scharlau

Grothendieck says: "at this time I'm not working in topological vector spaces any more, and I hope that someone else will find the answer". The letter is dated December 16, 1953; he had just returned home from his first stay in South America.

What is clear is that by 1954, Grothendieck was really just trying to tie up a few loose ends in functional analysis before orienting himself towards a new subject. Indeed, José Barros Neto was surprised to see him in the university library one day, checking out an enormous pile of books, contrary to his usual habits. When asked what on earth he was doing, he responded that he was interested in the Riemann-Roch theorem, so he had to learn some topology*. In another letter to Köthe, dated a year after the first (December 25, 1954), Grothendieck speaks with satisfaction of two recent *Notes aux Comptes Rendus* in which he had solved some questions that had been bothering him for over two years, adding "So now I can finally leave the field of topological vector spaces with no regrets, and start seriously working on algebraic topology."

BOURBAKI

Je pourrais dire, en exagérant à peine, qu'entre le début des années cinquante jusque vers l'année 1966, donc pendant une quinzaine d'années, tout ce que j'ai appris en "géométrie" (dans un sens très large, englobant la géométrie algébrique ou analytique, la topologie et l'arithmétique), je l'ai appris par Serre, quand je ne l'ai pas appris par moi-même dans mon travail mathématique. Cest en 1952 je crois, quand Serre est venu à Nancy (où je suis resté jusqu'en 1953), qu'il a commencé à devenir pour moi un interlocuteur privilégié – et pendant des années, il a été même mon **seul** *interlocuteur pour les thèmes se plaçant en dehors de l'analyse fonctionnelle. La première chose je crois dont il m'ait parlé c'était les Tor et les Ext, dont je me faisais un monde et pourtant, regarde donc, simple comme bonjour..., et la magie des résolutions injectives et projectives et des foncteurs dérivés et satellites, à un moment où le "diplodocus" de Cartan-Eilenberg n'était pas publié encore. Ce qui m'avait attiré vers la cohomologie dès ce moment, c'étaient les "théorèmes A et B" qu'il venait de développer avec Cartan, sur les espaces analytiques de Stein – j'en avais déjà entendu parler je crois, mais c'est par un ou deux tête à tête avec Serre que j'ai senti toute la puissance, la richesse géométrique que recelaient ces énoncés cohomologiques tellement simples. Ils m'avaient d'abord totalement passé par dessus la tête, avant qu'il ne m'en parle, à un moment où je ne "sentais" pas encore la substance géométrique dans la cohomologie faisceautique d'un espace. J'étais enchanté au point que pendant des années j'avais l'intention de travailler sur les espaces analytiques, dès que j'aurais mené à bonne fin les travaux que j'avais encore en train en analyse fonctionnelle, où décidément je n'allais pas m'éterniser! Si je n'ai pas vraiment suivi ces intentions, c'est parce que Serre entretemps s'était tourné vers la géométrie algébrique et avait écrit son fameux article de fondements "FAC", qui rendait compréhensible et hautement séduisant*

* When Barros Neto arrived in Paris in the fall of that same year, he found that no one was working on topological vector spaces any more. "The open problems that Grothendieck left behind are all very difficult," Schwartz advised him, "you'd better change subject. Malgrange is working on partial differential equations; why don't you look at those?" It seems that the open questions that Grothendieck had left behind were too hard for everyone at that time, including Grothendieck himself.

ce qui auparavant m'était apparu rébarbatif au possible – si séduisant même que je n'ai pas résisté à ces charmes, et me suis dirigé alors vers la géométrie algébrique, plutôt que vers les espaces analytiques.[3] [RS 503]

Lawrence, Kansas: spring 1955

News announcement from the K.U. News Bureau, January 11, 1955:

Alexandre Grothendieck, brilliant young French mathematician, has arrived at the University of Kansas for a six months assignment as a visiting research associate professor.

Grothendieck is an expert on the theory of linear topological spaces. A large research project, partially government sponsored, has been in progress at K.U. in that field for several years.

"We sought Professor Grothendieck's services a year ago as he was highly recommended by the best French mathematicians," said Dr. G. Baley Price, department chairman. "However he was still on a 2-year engagement in São Paulo, Brazil."

Dr. Nachman Aronszajn, formerly of France, is in his fourth year as visiting professor here heading up special research, and teaching in topological problems.

A funny anecdote is still told in the math department at the University of Kansas about Grothendieck's arrival in Lawrence. Although he was formally invited by the department, the person who specifically requested his visit was Nachman Aronszajn, who knew Grothendieck already from having encountered him in Paris, and who was keenly interested in functional analysis. G. Baley Price, who was department chair and always eager to hire new talents, drove to the airport to welcome Grothendieck, and asked Aronszajn to accompany him. But the moment Grothendieck and Aronszajn saw each other, Price's role was reduced to that of taxi driver, as the two foreigners hugged, climbed into the back seat together, and spent the rest of the ride chatting enthusiastically to each other in French.

An excellent mathematician and a wonderful and devoted chairman, Price was as influential as Aronszajn in making the Kansas math department into one of the top research centers in the world. A list of the visitors that they brought in during the 1950's, mostly for the express purpose of working with Aronszajn, reads like a 'Who's Who' of young international talent: Gustave Choquet and Jacques-Louis Lions from France, Lars Gärding and Lars Hörmander from Sweden, and the Israeli Avner Friedman as well as the Hungarian Peter Lax, who both visited from other positions within the United States. Prestigious conferences were organized there in fields that interested Aronszajn, particularly partial differential equations, and attended by mathematicians from around the world. As for permanent members of the department with mathematical interests close to Grothendieck's, there was Robert Schatten, who even before Grothendieck's thesis had studied one of the two tensor products investigated there, and who, like Aronszajn, was a Polish Jew who had lost his entire family in the war.

Nachman Aronszajn was an incredibly friendly and energetic force of nature. Born in Warsaw in 1907, he studied there through his Ph.D., which he obtained in 1929*. That year, he moved to Paris, where he began working with Fréchet. Supported by a scholarship from the CNRS, the same institution that had funded Grothendieck as a doctoral student, Aronszajn remained in Paris for ten years, until the outbreak of World War II.

During the war, Aronszajn joined the Polish army in exile, fighting in France and England, where he learned to play bridge while convalescing in a military hospital. While most of his family perished in the Warsaw ghetto, Aronszajn worked for the British Navy; he also managed to become acquainted with G.H. Hardy, and maintained contact with him for the rest of his life. Aronszajn returned to France after the war, but in 1949 he left to spend a year at Harvard, and subsequently decided to seek a job in the States. He was hired by Oklahoma A&M, the Stillwater branch of Oklahoma State University, but two years into his position, he found himself obliged to resign for moral reasons. This event is recounted colorfully by John Isbell*, who came to Oklahoma shortly afterwards, and who recalls that period of the 50's as being like a "Shakespearean play".

This was Joe McCarthy's second act, on a Shakespearean plan of five acts. Act III would be when Eisenhower held his nose and rode McCarthy's train to the White House. Well, Northern and California universities were firing leftists; Oklahoma was firing Quakers. To hold a faculty position in a state school there you had to sign an oath that you would shoot commies. Ainsley Diamond had been a noncombatant officer in the Air Force in World War II, but he wouldn't sign to shoot them. (It didn't say 'shoot', it said 'bear arms'.) I didn't mind that for myself, but when Oklahoma A&M fired Diamond, Nachman Aronszajn quit too, and they moved to the University of Kansas...The best graduate students at A&M...left to go to Kansas with Aronszajn.

When Ainsley Diamond was fired from Oklahoma, Aronszajn resigned with him; he could not really have signed the oath in any case, as he was a French and not yet an American citizen. G. Baley Price heard of the situation, saw his chance and reacted fast, bringing the whole group to Kansas. Aronszajn settled in extremely well, and remained there happily until his retirement at the age of 70.

Equipped with boundless energy and very high standards for himself and for those who surrounded him, Aronszajn loved to play bridge and chess, both very well, and to socialize. He threw numerous parties which are still remembered fondly in the department, and thought of himself as rather a famous person around town. Charles Himmelberg, who came to Kansas in 1959, recalls how Aronszajn enjoyed getting to know the most important people in Lawrence, and how disappointed he was once, when upon finding himself to be sitting next to the coach of the Kansas University football team in an airplane, he introduced himself only to discover that the coach had never heard of him.

But most of all, he devoted himself to mathematics. According to Pawel Szeptycki, "Mathematics was for Aronszajn a profession, a religion and a passion. During academic

* This biographical information Aronszajn comes from the article *Nachman Aronszajn 1907-1980* by P. Szeptycki, originally published in Polish in *Wiadomosci Matematyczne* XXV, 88-100.

* K.D. Magill, Jr., An interview with John Isbell, *Topological Commentary* **1** (2), 1996.

years he felt obliged to work intensively on his long-term projects related to eigenvalues approximation. During vacations he would think, for a change, about problems in various other fields of mathematics." He preferred research to teaching, although he did run a weekly seminar that was reputed to be very difficult for graduate students, and was also considered to have "a great appreciation for new and arising talents". Because Aronszajn remained in contact with Fréchet and with French mathematics in general, visiting the country regularly, he must have run into someone who recommended Grothendieck to him, perhaps at the Bourbaki seminar: it could easily have been Schwartz or Dieudonné. The idea of hiring Grothendieck would have appealed to Aronszajn both for his talent and devotion to mathematics and because of his research subject in functional analysis. In any case, they first became acquainted in Paris, and were very pleased to meet again in Lawrence.

The money that was available to Aronszajn to invite talented young foreigners came mostly from military sources: a contract with the Office of Naval Research and another with the Air Force Office of Scientific Research. There was also, however, a special grant from the National Science Foundation for promoting "Research Project on the Geometry of Function Space"*; the project was apparently given a meaningless title on purpose to leave the department plenty of leeway to invite whom they wished. This last grant is the one that supported Grothendieck.

One of the novelties that Aronszajn brought to Kansas was the intensive seminar that everyone in his group was expected to attend religiously: three hours on Tuesdays and two hours on Fridays. His group consisted of the many visitors that he brought to Kansas with money from his various grants, as well as certain faculty members and graduate students whose summer salaries were covered by him. The members of the "Aronszajn group" had lighter teaching loads than other faculty members, and this caused a certain resentment in the department. But the ever wise G. Baley Price was eventually able to turn the situation to advantage, by using this resentment and the privileges of the group to leverage lower teaching loads for the entire department.

When people were actually collaborating on research with him, Aronszajn liked to invite them to work in his home. This would go on for hours, after which drinks would appear, and often an invitation to dinner, prepared by his wife Sylvia whose reputation as an excellent cook was well known in the department. It is likely that Grothendieck drew some real enjoyment from these sessions; he certainly later followed the same method with many of his own graduate students (minus the parties).

On top of the ideal working conditions, Grothendieck was well-paid in Kansas. As always, he wanted to send a large part of the money back to Paris for Hanka. His only friend in Paris at that time was José Barros Neto, who had left São Paulo for Paris in the fall of 1954 at the same time as Grothendieck. Grothendieck asked Barros if he could send him some checks in dollars through the mail, for Barros to get changed into francs at the best possible rate and bring directly to Hanka.

Being paid in dollars himself at that time, via checks from the Brazilian Science

* The information about the Kansas mathematics department comes from a typed *History of the Department of Mathematics of the University of Kansas, 1866-1970*, written in 1976 by G. Baley Price.

Foundation, Barros knew all about the best way to change dollars into francs on the black market. Banks, of course, were to be avoided at all costs; unofficial changing booths were better, but in the end Barros actually became acquainted with a Brazilian employee at the Brazilian embassy who was happy to change dollars to francs at a comfortable rate from the privacy of his luxurious apartment. Barros went there regularly with his own checks, and the first time he received one from Kansas, he took it to his helpful friend as well.

Grothendieck had impressed upon him how much Hanka was in need of money, so the moment he had the cash in hand, Barros hurried into the metro, went straight to Hanka's apartment and loudly rang the doorbell. It was a Sunday morning, and it never occurred to the friendly and unceremonious Brazilian that in Europe, people usually announce their visits by telephone before showing up at the door. After waiting several minutes, the door opened a crack and a shocked and disheveled Hanka peered out. Embarrassed, Barros tried to hand forth the money with a confused and stammered explanation. To his shock and astonishment, Hanka refused it outright. It was a whole European education: South American pragmatic warm-heartedness suddenly and unsuccessfully encountering the rigid European attitudes to privacy, to money, to honor*. Barros went away gloomily, money in hand, and informed himself about how to deposit it directly into Hanka's post-office account. He never saw Hanka again.

Homological algebra: evolution or revolution?

About his early work in measure theory and then functional analysis, and his radical change of subject in 1954-1955 to homological algebra and algebraic geometry, Grothendieck writes

Mes premières et solitaires réflexions, sur la théorie de la mesure et de l'intégration, se placent sans ambiguïté possible dans la rubrique "grandeur", ou "analyse". Et il en est de même du premier des nouveaux thèmes que j'ai introduits en mathématiques (lequel m'apparaît de dimensions moins vastes que les onze autres). Que je sois entré dans la mathématique par le "biais" de l'analyse m'apparaît comme dû, non pas à mon tempérament particulier, mais à ce qu'on peut appeler une "circonstance fortuite": c'est que la lacune la plus énorme, pour mon esprit épris de généralité et de rigueur, dans l'enseignement qui m'était proposé au lycée comme à l'université, se trouvait concerner l'aspect "métrique" ou "analytique" des choses.*

L'année 1955 marque un tournant crucial dans mon travail mathématique: celui du passage de l'"analyse" à la "géométrie". Je me rappelle encore de cette impression saisis-

* Barros recounted another amusing anecdote, concerning the Brazilian mathematician Nachbin, who was staying in Paris and living in the same street as Henri Cartan. Once, Barros overheard Nachbin's maid, who had accompanied him from Brazil, chatting in the street with Cartan's maid, who was Portuguese. "What does your boss do?" Nachbin's maid asked. "I don't know," replied Cartan's, "but he can't do it very well, because he spends most of his time in his office tearing up paper."

* Topological tensor products and nuclear spaces.

sante (toute subjective certes), comme si je quittais des steppes arides et revêches, pour me retrouver soudain dans une sorte de "pays promis" aux richesses luxuriantes, se multipliantes à l'infini partout où il plait à la main de se poser, pour cueillir ou pour fouiller... Et cette impression de richesse accablante, au delà de toute mesure, n'a fait que se confirmer et s'approfondir au cours des ans, jusqu'à aujourd'hui même.

C'est dire que s'il y a une chose en mathématique qui (depuis toujours sans doute) me fascine plus que toute autre, ce n'est ni "le nombre", ni "la grandeur", mais toujours la forme. Et parmi les mille-et-un visages que choisit la forme pour se révéler à nous, celui qui m'a fasciné plus que tout autre et continue à me fasciner, c'est la structure cachée dans les choses mathématiques.[6] [RS Promenade P27]

In his list of the major themes he introduced in mathematics, Grothendieck cites that of "continuous and discrete duality" (via derived categories and his theory of six operations) as the first theme he introduced following topological tensor products and nuclear spaces. His development of the new theme carried over many years and was constantly nourished by the other themes that he developed and explored concurrently (schemes, topos, cohomology). But this grand panoply of ideas did not really emerge in Grothendieck's work until 1956. In São Paulo in 1954, he was still learning what he needed in order to explore the world of algebra and geometry**. During the eight or nine months he spent in Lawrence, he worked out a complete theory of fiber spaces with structure sheaf, writing up a 104-page fascicule of introductory lecture notes on the subject, and concentrated on mastering the subject of homological algebra in depth.

Homological algebra was a subject of great attention in Paris at that time, with the new book by Cartan and Eilenberg, still unpublished but whose manuscript was being circulated from hand to hand amongst friends and colleagues of the authors. In the Bourbachic spirit of the times*, Cartan and Eilenberg had understood that the cohomological methods which had already given such promising results in the domain of topology, but which had been introduced and used disparately and independently in all kinds of different situations – groups, Lie algebras, associative algebras – needed a unified and functorial approach. In their book, working within the context of the category of modules over a ring, equipped with module homomorphisms and the tensor product, they gave a general

** Grothendieck was already fully familiar with the notions of sheaves and cohomology, which he had first seen in 1948-1949 at the Cartan seminar and in Leray's course at the Collège de France. The fact that he manipulated these concepts with ease is visible from a short paper written and submitted to *Summa Brasiliensis* in 1953, but withdrawn and published in Bull. Soc. Math. France **84** in 1956 due to the long delay in publication. Naturally enough, the specific examples of sheaves considered in this short note are *topological vector sheaves*, namely sheaves of vector spaces whose spaces of sections are topological vector spaces, and *sheaves of type (F)*, whose spaces of sections are (F)-spaces. But the theorem proved there is completely general, and produced as a corollary the following result, proved earlier that same year by Cartan and Serre: *The cohomology groups of a compact holomorphic variety with coefficients in a coherent analytic sheaf are finite-dimensional.*

* One hears this observation made again and again in all kinds of contexts right through the 1950's, including, frequently, by Grothendieck.

definition of homology and cohomology theory, and established in this framework many of the results that had already been found in specific cases.

Far off in the United States without a copy of the book, Grothendieck, having heard on and off about parts of its contents from seminars in Paris during the fall of 1954 and exchanges of letters with his friends in Paris, decided to prepare a series of lectures along what he "imagined to be" the outline of the famous book. Although it took him nearly two years to complete, he was certainly thinking intensively about this during his time in Lawrence. Perhaps his most striking contribution to the theory of homological algebra, the definition of abelian categories, arose from his desire to "imagine" Cartan and Eilenberg's approach; his brain, more adapted to considering objects by the definition of their properties rather than considering well-known types of objects such as modules, attempted to situate the whole context of derived functors (containing Ext, Tor, cohomology etc. etc.) in a category that would be defined by precisely the right axioms to make those notions work: a category that would be "very like" the category of modules over a ring in the sense that it would have all the "important" properties of that category, but that would be defined purely abstractly. The search for the right definition of such a category led him, already in February 1955, to the realization that by working in sufficient generality, cohomology with coefficients in a sheaf could also be realized as a derived functor: "Je me suis aperçu qu'en formulant la théorie des foncteurs dérivés pour des catégories plus générales que les modules, on obtient à peu de frais en même temps la cohomologie des espaces à coefficients dans un faisceau: on prend la catégorie des faisceaux sur l'espace donné X, on y considère le foncteur $\Gamma_\Phi(F)$, à valeurs dans les groupes abéliens, et on prend les foncteur dérivés. L'existence résulte d'un critère général, les faisceaux fins joueront le rôle des modules "injectifs". On obtient aussi les suites spectrales fondamentales comme cas particuliers de délectables et utiles suites spectrales générales...D'ailleurs, probablement tout ça se trouve plus ou moins explicitement dans le bouquin Cartan-Eilenberg, que je n'ai pas encore eu lheure de voir encore." [Letter to Serre, February 26, 1955]

Grothendieck was wrong; the search for the most general category in the context of which to study derived functors was not in Cartan and Eilenberg's book, although according to Serre's answer to Grothendieck's letter, they were aware of the fact that sheaf cohomology, like other cohomologies, must be nothing more than yet another special case of derived functors.

Four months later, Grothendieck seems to have figured out the situation to his satisfaction, by defining what he called "classes abéliennes" (later "catégories abéliennes"), abelian categories, defined by axioms precisely calibrated to make sure the category had all the needed properties: the much-considered category of modules over a ring now became nothing but an example of this definition. Sending Serre a typed copy of what must have been an early draft of the famous paper affectionately referred to as *Tohoku**, Grothendieck seems very pleased with the state of his understanding: "J'ai déjà la conviction que la façon bourbachique de faire de l'algèbre homologique, c'est de changer de classe abélienne à tout instant, comme on change le corps des scalaires, ou la topologie en Analyse Fonctionnelle".

Serre took this draft to a Bourbaki meeting in July, and reported back to Grothendieck that "ton papier sur l'Algèbre homologique a été lu soigneusement, et a converti tout

* Because it eventually appeared, in 1957, in the Japanese *Tohoku Mathematical Journal*

le monde (même Dieudonné, qui semble complètement fonctorisé!) à ton point de vue. Sammy a décidé de faire une rédaction dans ce sens (pour Bourbaki), avec pour Chapitre I la théorie générale de l'homologie dans les classes abéliennes, Chapitre II l'application aux modules, Chapitre III l'application aux faisceaux. Il se mettra en rapport avec toi pour les questions de rédactions et de démonstrations." Dieudonné himself agreed enthusiastically: "Serre a dû vous donner des nouvelles du Congrès Bourbaki, où vos classes abéliennes ont soulevé l'enthousiasme! On compte sur vous pour le Congrès d'Octobre et on espère que vous nous ferez un rapport rupinant sur les fonctions analytiques."

If Eilenberg immediately understood the scope of Grothendieck's generalization of his book with Cartan, he was nonetheless worried about one thing:

FINISH

The reluctant exile

Multiple testimonies from letters sent and received by Grothendieck reveal how troubled he was at being kept far from France by financial necessity. The richness of mathematical life in France (in spite of Godement's observations, Harvard and Princeton did not seem to represent much of a temptation for Grothendieck in comparison with Paris), the working conditions, libraries, seminars and easy communication with colleagues, Bourbaki and the increasingly intense communication with Serre, as well as the warmth of his familiar circle of friends, particular Laurent Schwartz and his family, and the necessity of being near Hanka, who for health reasons was not able to globe-trot, aroused in Grothendieck a real longing to return to Paris and remain there. But he was determined not to yield up his stateless condition, which clearly represented what he felt himself to be; he would not ask for French nationality, nor for any other.

In a letter to G. Köthe from December 25, 1954, after the completion of his final year in São Paulo, he says "Ich hatte die Absicht, jetzt ein bis zwei Jahre in Frankreich zu bleiben. Leider erhoben sich unvorhergesehene Schwierigkeiten (z.B. die steigende Inflation in Brasilien, di mir unmöglich machte, etwas zu ersparen), so dass ich dieses Jahr wieder reisen muss."

Indeed, Grothendieck had been aware since the previous July that his application for the only position possible for him in France – another temporary stint with the CNRS – had been rejected. Dieudonné was worried about the situation ("à l'heure actuelle il n'y a plus beaucoup de chances pour que vous fassiez une carrière en France!"[4]), and anxious to help. "Puisque le CNRS vous a refusé une maîtrise de recherche (je crois que vous êtes hélas victime de la réaction anti-Bourbaki!), et que je présume que vous n'avez pas l'intention de vous incruster au Brésil, envisageriez-vous maintenant de venir aux USA? Il y a naturellement l'obstacle du voyage de votre mère, mais Schwartz me dit que vous ne l'avez pas emmenée avec vous cettefois, d'où je présume que vous accepteriez peut-être de venir dans les mêmes conditions aux Etats-Unis?"

Word-of-mouth led to Aronszajn's invitation to Kansas, and Grothendieck was not unhappy to go there: "Diesmal gehe ich nach Kansas, als "Associate Professor", unter ganz günstigen Bedingungen (z.B. werden keine Vorlesungen oder Konferenzen von mir

verlangt)", but in spite of his first rejection, he wasn't giving up on Paris. In spite of his rejection by the CNRS, and in spite of Dieudonné's doubts, he even seemed to feel confident that a second application for a one or two-year position would work better, although he knew he would have to be thinking about the problem of finding a permanent job after that. "So hoffe ich, wenigstens nächstes Jahr für ein Jahr oder zwei in Paris bleiben zu können, wo die Arbeitsbedingungen unvergleichlich gut sind...Mit diesem Jahr habe ich nun das Herumreisen endgültig und gründlich satt, und ich möchte wenn irgend möglich nach meinen letzten Jahren im CNRS (Herbst 56 oder 57) eine feste Stellung in einen Land finden, wo die Mathematik besser repräsentiert ist als in Südamerika. Die USA gehen leider nicht, weil meine Mutter aus Gesundheitsgründen keine Einreiseerlaubnis bekommt. Vielleicht können Sie mir sagen, was für Möglichkeiten in Deutschland wären? Man sagt allerdings, es wäre schwierig, dort einen vernünftigen Posten zu bekommen (so soll Stein z.B. bis vor kurzen nur Assistent gewesen sein). Würde ich rechnen können, nicht mehr Arbeit aufgebürdet zu kriegen als was ich in höchstens drei Arbeitstage stecken kann (so dass drei Tage für persönliche Arbeit bleiben)? Vergessen Sie nicht, dass ich nicht Deutscher, sondern staatenlos bin (und es bleiben will); ergibt das eine Schwierigkeit?"[5]

Just a few weeks later, in February 1955, he wrote to Serre from Kansas: "J'ai appris qu'on créait en France une centaine de postes de professeurs associés, et qu'ils étaient aussi accessibles aux étrangers. Es-tu renseigné si la chose a fait du progrès, si éventuellement il y aurait une chance pour moi de décrocher un poste, et comment et quand on pose sa candidature? Ça m'intéresse énormément, car il ne peut être question pour moi de rester aux États Unis, et je préfère nettement rester en France que d'aller en Allemagne ou même en Amérique du Sud*!"[6] Serre sent back a tactful and kindly answer: "Nous n'avons aucun détail sur la question des professeurs associés. Combien y en aura-t-il pour toute la France? Mystère. A Paris, Cartan a un candidat tout trouvé: Chevalley** (confidentiel!). Y aura-t-il des postes en province? En tout cas, tu peux être tranquille, s'il y a une possibilité pour toi, on bondira dessus (les gens de Strasbourg avaient d'ailleurs plus ou moins pensé à toi, à un moment où ils se figuraient avoir un poste)."

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* Grothendieck had certainly changed his mind about this since the time in 1949 when he had stated his preference for settling in any country in the world, even Ethiopia, rather than France. Presumably, the change occurred because he had discovered that where he really wanted to be was in one of the intense mathematical centers of the world, of which Paris and a few places in America were the main ones at that time. It is not clear why Grothendieck considers it impossible that he should settle in the U.S.; to Köthe, he says it is because of Hanka's health, but the "no question" seems stronger than that – yet later on, after her death he did consider the possibility once again.

** Chevalley, who had been in Princeton at the outbreak of WWII, had spent the rest of the war in America and taken American nationality. Later, when he wished to return to France, his application for a chair at the Sorbonne was humiliatingly rejected by the French scientific establishment, which mobilized a group of university veterans and former resistance fighters to protest against his return – in spite of the fact that he was by far more qualified for the chair than any of the other candidates. Incensed by the situation, Chevalley's friends in France were determined to help him return, both for the sake of the man himself and for the better mathematical glory of the country.

The permanent positions for foreigners did not materialize (even Chevalley did not manage to return to France until 1957), but Grothendieck's optimism with respect to the CNRS turned out to be justified. The good news arrived in Kansas in the month of May, with a letter from Schwartz, letting him know that his second application for a position at the CNRS was going to be accepted. Grothendieck's pleasure at the news was mixed with justified suspicion: "Merci pour la bonne nouvelle du CNRS: à vrai dire, je ne compterai avec que quand j'aurai une lettre officielle du CNRS m'instituant maître de recherches, mais il semble qu'il y ait bon espoir." But the letter did arrive in due course, and Grothendieck packed his bags for a return to Paris that he meant to last for at least two years, and which ended up lasting fifteen.

Translations of the foreign language quotations

[1]Note that we could have avoided developing all of the formalism of b-norms (§§1,2,3) to state and prove the fundamental results in §4. But it seems to me that, as on many other analogous occasions, this would simply have saved ink and paper at the price of the reader's intellectual effort. Indeed, it is only through these preliminaries that one can give the statements in the concise and suggestive form which makes it possible to grasp in one go the relations between the very numerous variations of the main theorem, which leads to a real comprehension of the theory.

[2]At the time when I was still working on Functional Analysis, so until 1954, it happened to me to keep on obstinately and endlessly working on a problem that I couldn't solve, even when I had no more ideas and was just going in circles through the old ideas which clearly weren't "biting" any more. This happened for an entire year, with the "approximation property" in topological vector spaces, which was only solved twenty years later by methods of an entirely different kind, which could not but escape me at the point where I was then. I was moved, not by desire, but by a stubbornness and an ignorance of what was going on inside me. It was a painful year – the only moment in my life in which doing mathematics had become painful for me! I had to have this experience in order to understand that there is no use in "drying up" – that at the moment when one's work has ground to a halt, as soon as that halt is recognized, one should go and work on something else, coming back if necessary at some more propitious time to the question left hanging. That time usually doesn't take long to arrive – the question ripens, without my even having the intention of touching it, just by virtue of work done with enthusiasm on problems that seemingly have nothing to do with it. I am convinced that if I become obstinate at these times, I would not succeed in doing anything even in ten years! It was starting from 1954 that I got into the habit in math of always having several irons in the fire at the same time. I never work on more than one or two of them at a time, but by a sort of miracle that keeps on occurring, the work I was doing on one of them helps all the others that are quietly awaiting their hour.

[3] I could say, almost without exaggerating, that from the early 50s until about 1966, so for a good fifteen years, everything I learned in "algebraic geometry" (in a very wide sense containing algebraic and analytic geometry, topology and arithmetic), I learned it from Serre, except when I learned it myself during my mathematical work. It was in 1952, I think, when Serre came to Nancy (where I stayed until 1953), that he started to become, for me, a privileged interlocutor – and for years, he was even my **only** interlocutor for themes outside of functional analysis. The first thing he told me about, I think, were the Tor and Ext, which I had thought were ?? (dont je m'etais fait un monde) and yet, just look, how simple they really are... and the magic of injective and projective resolutions, and derived functors and all the related stuff, at a time when Cartan and Eilenberg's "diplodocus" hadn't been published yet. What attracted me towards cohomology already then were the "theorems A and B" that he had just developed together with Cartan, on Stein analytic spaces – I had already heard about them, I think, but it was only after one or two conversations alone with Serre that I started to feel the power, the geometric richness

hidden within these seemingly so simple statements. At first they had gone right over my head, before he talked to me about them, at a time when I didn't yet "feel" the geometric substance in the sheaf cohomology of a space. I was charmed to the point that for years I meant to start working on analytic spaces, as soon as I would have finished the work I was doing in functional analysis, where I absolutely didn't intend to spend all eternity! If I didn't actually follow through on this intention, it's because in the meantime, Serre turned to algebraic geometry and wrote his famous foundational article "FAC", which made the things that had seemed impossibly forbidding before now seem highly comprehensible and attractive, so attractive that I couldn't resist its charms, and turned towards algebraic geometry myself, rather than analytic spaces.

[4]At the present time, there isn't much more chance that you can have a career in France.

[5]So I hope, at least next year, to be able to spent a year or two in Paris, where the working conditions are incomparably good...This year I finally really have enough of traveling around, and if only it's possible, after my last years in the CNRS (fall of 56 or 57), I'd like to find a fixed position in some country in which mathematics is better represented than in South America. The U.S. is unfortunately not possible, since my mother will not receive permission to travel there for reasons of health. Perhaps you can tell me what possibilities exist in Germany? People say that in any case it would be difficult to find a reasonable position there (for example, Stein was apparently merely an Assistant until recently). Could I count on not being burdened with more work than I could accomplish in at most three working days (so that three days would remain for personal work)? Don't forget that I am not German, but stateless (and want to remain so); would that be a problem?

[6]My first, solitary reflections, on the theory of measure and integration, belonged without any doubt to the domain of "size", or "analysis". And the same is true of the first of the new themes that I introduced in mathematics [topological tensor products and nuclear spaces] (which appears to me to be of less vast dimensions than the eleven others). The fact that I entered mathematics by way of analysis seems to me to be due, not to my particular temperament, but to what could be called a "chance circumstance": the fact that the most gaping hole, for my spirit in love with generality and rigor, in the teaching that was offered to me in high school and at university, happened to be in the area of the "metric" or "analytic" aspect of things.

The year 1955 marked a crucial change in my mathematical work: the passage from "analysis" to "geometry". I still recall the striking (but of course entirely subjective) impression of leaving rude and arid steppes, to find myself suddenly in a sort of "promised land" filled with luxuriant riches, multiplied everywhere to infinity, so that it suffices to lay your hand anywhere you like to gather or explore...And this impression of overwhelming riches beyond all measure was confirmed and deepened as the years passed, even until today.

This is to say that if there is one thing in mathematics that (from the start, no doubt) fascinates me more than any other, it is neither "number" nor "size", but always "shape". And amongst the thousand-and-one faces that shape chooses to reveal itself to us, the one

that fascinates me more than any other and continues to fascinate me is the structure hidden within mathematical things.