Chapter 1. The Great Turning Point

In 1970 an event occurred which Alexander Grothendieck later often referred to as the "great turning point" in his life. Everything changed. He began to withdraw from mathematical research as well as from the community of mathematicians, he gave up his research professorship, his family life went into a crisis and all his relationships with colleagues, friends and students disintegrated. He set himself new goals, completely different from those which he had pursued up to then, goals which increasingly estranged him from society. And while he himself was convinced that this new path would lead him to greater maturity, inner freedom, independence and true spirituality, it seemed to the people around him as if he was throwing away everything that had made of him one of the greatest magicians of mathematics: his incomparable creativity and his almost unique capacity get to the bottom of complex phenomena and reveal their essential principles.

If one had to name the day on which the great turning point (*le grand tournant*) occurred, one would most likely choose 25 May 1970. On this day he informed the members of the *Comité Scientifique* of the *Institut des Hautes Etudes Scientifiques* (IHES) of his resignation, planned at the latest for 1 October 1970. On 9 June followed a letter of confirmation to the director of the IHES, Léon Motchane:

Massy le 9.5.1970

Monsieur le Directeur,

Suite à ma lettre au Comité Scientifique, je vous confirme par la présente mon départ de l'IHES à partir du 1. Octobre 1970.

Veuillez agréer, Monsieur le Directeur, l'expression de ma considération distinguée

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Dear Sir,

Following my letter to the Scientific Committee, I herewith confirm my departure from the *IHES* starting 1 October 1970.

Please accept, Sir, the expression of my distinguished consideration.

A. Grothendieck

Grothendieck was barely 42 years old. He had worked for all of eleven years and eight months at the IHES, an unbelievably short time when one considers how much he achieved in those years. And although in the spring of 1970 the consequences of his decision to resign were not fully clear even to him, this was the beginning of a permanent leave-taking not only from the world of science, but from everything else as well.

Before discussing *why* Grothendieck arrived at this great turning point, it would be appropriate to indicate in a general way *where* this turning point was leading. This subject concerns the whole of the third volume of his biography, and at this point only the most important stages of his "professional" activity shall be mentioned. Although from 1970 on - apart from short periods - mathematics was not the main focus of Grothendieck's life, he naturally had to earn his living in the following years, and he did in fact hold positions as university lecturer and scientist until the year of his retirement in 1988. In order to gain a better idea we shall list them here.

After leaving the *IHES* he held a guest professorship at the *Collège de France* for two years, a post arranged for him by his closest mathematical discussion partner Jean-Pierre Serre. (Serre himself worked at this institution from 1956 to 1994.) There, Grothendieck gave lectures that were invariably followed by lengthy political discussions.

He next obtained a temporary position at the University of Orsay¹ where he gave a one year course in 1972/73. During these three years he also travelled frequently, especially in North America.

Subsequently Grothendieck was given an independent professorship (*professeur à titre personnel*), a position that he could exercise at any French university. Beginning in the fall of 1973 he chose Montpellier, where he taught

¹ For readers who are unfamiliar with the universities in the suburbs of Paris, one should note that Orsay is very close to Bures-sur-Yvette where the IHES is located.

courses as well as having a series of undergraduate and graduate students. In the long term, however, he was not completely satisfied with this activity and with the less talented and enthusiastic students with whom he had to deal. By the end of the 1970's he applied for a research position at the institution where his professional life had begun, namely the CNRS (*Centre National de la Recherche Scientifique*). This application or applications, as the case may be, were initially refused, since it was unclear if he actually really wanted to do mathematical research. In 1984 he applied again with a research program that eventually gained a certain fame, the *Esquisse d'un programme*. In the last years of his professional life, starting in October 1984, his salary was in fact paid by the CNRS, although he held a formal professorship in Montpellier. At this time he did not give any more lectures and rarely visited the university. In 1988, at the age of sixty, he retired.

What became the focus of his life now that he had largely given up mathematics? For more than two years he devoted himself with the same energy, enthusiasm, and seemingly inexhaustible capacity for work to two movements triggered by the 1968 student revolution, the peace movement and the budding ecology movement. Three almost indistinguishable fields of endeavor were at the center of his activity: the fight against militarism and armament, in particular nuclear armament and the war in Vietnam; the commitment to what today we would call environmental protection, i.e. the preservation of the ecological basis of society from the threat of major technological projects, particularly the use of nuclear energy; and finally, the demand for a new social order based on a self-determined life-style, such as that propounded by the protagonists of the student revolution and the American hippies. In order to reach these goals and to attract fellow campaigners, he founded a group called *Survivre* ("Survive", later *Survivre* et Vivre, "Survive and Live"), to which he devoted almost all of his energy (and financial means!) from the middle of 1970 to around the middle of 1972.

After this short overview, let us return to the "great turning point".

There can be no doubt that a variety of reasons were responsible for the "great turning point", reasons that both complement and contradict each other. Some appear obvious, others lie hidden in the depths of Grothendieck's existence and his past, and can only be brought to light with great difficulty. The enigma persists, and one never has the feeling that one truly "understands", or even that "it is possible to understand" such a radical act. Colleagues, students, and friends of Grothendieck have all asked themselves what the causes and incentives were which led to this step. No one has been able to give a definitive answer.

It is equally certain, however, that any attempt to write Grothendieck's biography must raise this issue. As one of the major events in his life, even his work as researcher and mathematician must be viewed in the light of his

subsequent radical rejection of mathematics. Our first step will be an attempt to find out the causes of the "great turning point", although many questions may remain open. The author wishes to emphasize that the following attempt at an explanation reflects his personal opinion. We shall begin with what seems to be the most obvious cause.

It has frequently been said that the decisive reason for Grothendieck's break with the IHES was that part of the budget of the IHES (about 5%) came from the French Ministry of Defense. Indeed, this was the only reason mentioned by Grothendieck in his letter of resignation dated 25 May. In it, he explains that the *Conseil d'Administration* of the IHES for the year 1970 had again agreed to a grant from the Ministry of Defense, that the institute's position on this matter was going to lead to internal strife, and that this was neither under his control nor that of the *Comité Scientifique*. As a consequence, he intended to leave the institute on October 1 at the latest, and would not maintain any scientific relationships with it for as long as the situation continued.

Grothendieck himself confirmed this rationale many times in later years, i.e. in his memoir *Récoltes et Semailles*². In this respect we can assume to a certain degree that it is the authentic, "official" explanation.

The opinion of the author is, however, that this explanation is hardly adequate, and not even especially plausible. It is certainly true that funding by the Ministry of Defense was unacceptable to Grothendieck as it was impossible to reconcile this with his pacifist, anarchist and radical leftist political convictions. But Grothendieck did not only give up his position at the IHES, he also increasingly turned away from mathematics and later on from the mathematical community and from human society altogether. If funding for the IHES had been the only problem, there were plenty of places the world over which were more in line with his political and ethical convictions. He would have been welcome to continue his research at those places, and many students would have followed him.

Even the situation at the IHES did not preclude a solution to the problem. There had been many discussions on the theme of financing among the members of the administration and the permanent members of the IHES, and the tenured faculty largely supported Grothendieck. With a little good will the problem could certainly have been solved. This good will, however, was not available on either side: neither that of the founder and all-powerful director Motchane, nor Grothendieck's. As will be demonstrated in more detail, both actually desired the separation.

At this point one must inevitably speak of the personal relationship between

² This text, to which reference shall often be made, will occasionally be designated *ReS*.

Grothendieck and Motchane, a relationship that was difficult and strained from the start. The reason for this almost certainly stemmed above all from the fact that Grothendieck did not recognize any authority. This major personality trait was the legacy of his parents, in particular his mother, but also of the years which he had spent living with the Heydorns, in internment camps, and in Le Chambon-sur-Lignon.

Very soon after Grothendieck's appointment at the institute, the first tensions surfaced, and Motchane had on occasion to instruct Grothendieck on the basics of collaboration in a spirit of trust. Grothendieck had worked less than half a year at the IHES when on 12 June 1959 Motchane wrote:

Sir, I was most astonished to receive your letter, which was unacceptable either in form or content. [...] I am convinced that in the interest of our institute, which is not a personal concern, but which belongs to all those who work together here in any way, it is important to clear up the reasons for any potential misunderstandings. This makes it possible to establish a climate of mutual trust, support and sympathy, without which it would be impossible to create anything long-term.

Later on there were repeated disagreements, where Grothendieck, as a rule, would cling to his position with a somewhat grating self-righteousness.³ Around the middle of the winter of 1969/70 the relationship between the two had for various reasons become so completely damaged that consensual action was no longer possible. The events and the reasons for Grothendieck's break with the IHES and particularly with Motchane have been extensively examined and analyzed by David Aubin in his thesis. For more details on the conflict we refer to this study.⁴ Although the main subject of the thesis is not Grothendieck, but René Thom, and it deals with chaos and catastrophe theory rather than algebraic geometry, the conflict between Motchane and Grothendieck receives considerable attention. Allyn Jackson has also provided a very informative portrayal of the critical years 1968 - 1972.⁵

As already stated, we do not wish to enter into the details of the conflict between Grothendieck and Motchane at this point. But we will nevertheless give a short account of the decisive meeting of the scientific committee of the *IHES*,

³ The Grothendieck-Motchane correspondence is kept in the archives of the IHES. I am grateful to the institute for their permission to consult these documents.

⁴ David Aubin; A Cultural History of Catastrophes and Chaos: Around the Institut des Hautes Etudes Scientifiques, France 1958 - 1980, Ph.D. thesis (Princeton University).

⁵ Allyn Jackson: *Comme Appelé du Néant - As if summoned from the Void: The Life of Alexander Grothendieck*, Notices of the AMS, 51 (2004). We will frequently quote from this biographical article.

which took place on 6 April 1970, and during which the "explosion" occurred.

Grothendieck had for some time been getting interested in biology, while his colleagues had their doubts as to how intensive and how long-lasting this interest was going to be. At this meeting, Grothendieck pushed vigorously for an invitation to a personal friend of his, the Romanian molecular biologist Mircea Dumitrescu. In the end, in spite of some reservations, all the members of the committee including Motchane agreed to the visit, with the exception of Peierls. However, Motchane stipulate that the financing of the invitation was to function in "rigorous" accordance with the statutes of the institute. Formally one could hardly criticize this position; in case of doubt, however, it afforded Motchane the possibility of ultimately torpedoing Grothendieck's suggestion. In response, Grothendieck proposed registering Dumitrescu's name in the category "other invitations", in order to avoid attracting the attention of inspectors from the Ministry of Finance. "You are suggesting that I cheat [tricher]," Motchane retorted. This led to a fierce exchange of words which, in spite of Ruelle's attempts at mediation, culminated in the following exclamation on Grothendieck's part: "I have always considered you a arrant liar [fiéffé menteur]!" and Motchane declaring: "This is unacceptable. Sirs, you are witnesses," and turning to Grothendieck: "If you are looking for a serious conflict, you have one now."

When reading the concise protocol of this 2009 meeting forty years later, one cannot help feeling that Grothendieck did not in fact desire an agreement. If he had been genuinely and unconditionally interested in having Dumitrescu invited, if this had been his main concern, he could have agreed to the proposed manner of proceeding. Instead one is left with the impression that he had subconsciously already traversed the "great turning point", and that fundamentally he desired to leave the IHES.

Viewed this way, the (serious) conflict about military funding appears in a completely different light. Motchane and a few other full-time professors at the institute knew that no matter how small, funding from the French armed forces was unacceptable to Grothendieck. And this ultimately afforded Motchane the opportunity to get rid of the - from his point of view - paranoid troublemaker Grothendieck. In the spring of 1970 things had gone so far that Motchane didn't even think of trying to find a compromise, but instead used this opportunity to provoke Grothendieck's resignation. Perhaps he did not really have a choice; it is possible that the permanent members Thom and Michel would have left the IHES if Grothendieck had remained. David Ruelle, who participated in and witnessed the whole affair goes so far as to suspect that even if Motchane had not received any funding from the Ministry of Defense he would have hidden this fact from Grothendieck in order to induce him to resign.

Perhaps this removes some of the sting from the much-discussed clash. In fact, Grothendieck experienced the IHES as a "golden cage" from which he longed to escape. His interests were turning to other things. He was profoundly shaken by the 1968 revolution (or "revolt", depending on one's point of view and assessment). Motchane, on the other hand, felt increasingly that collaboration with Grothendieck was becoming more and more problematic, and that he was gradually evolving into a foreign element at the institute. Both of them wanted the dissociation, neither possessed truly "rational" reasons for this, and thus only a proper row could end the conflict.

Additionally, in a further letter to Motchane dated 9 June 1970, although Grothendieck admittedly expressed regret for his choice of words, he did not really apologize, but rather deemed that since in any case he would be leaving the IHES, the matter should be considered closed.

We have already underlined the fact that Grothendieck not only gave up his position at the IHES, but over the course of the next few years turned away from mathematics altogether.

Almost fifteen years later, in *Récoltes et Semailles*, Grothendieck looked back upon his time as a mathematician. In many respects this text is a reckoning with former colleagues and students, with the institutions with which he had dealt, with mathematics and mathematical research generally. (*Récoltes et Semailles* will be thoroughly discussed in Chapter 23.) One might have expected Grothendieck to name and explain his motives there, but in fact he does not refer to them.

Jean-Pierre Serre, Grothendieck's most important discussion partner throughout the whole of his mathematical life, addresses the crucial point with his unique precision in a letter commenting on *Récoltes et Semailles*. Serre stresses the point that Grothendieck never felt the need to do the one thing which all the world expected of him, namely to provide, at some point in the one thousand six hundred written pages of text, a truly conclusive explanation (see the Grothendieck-Serre correspondence, letter of February 8, 1986):⁶

But you don't ask yourself the obvious question, the one which every reader expects you to answer: Why did you yourself abandon your work?

And a few lines later Serre tries to answer the question himself:

I have the impression you were simply tired, in spite of your well-known energy, because of the enormous labor that you had undertaken.

⁶ P. Colmez, J.-P. Serre (Eds.); Correspondance Grothendieck - Serre, Paris 2001.

Serre later reiterated his opinion in other letters and conversations:

I don't know anything definite. As I say in my "Correspondence," I suppose that he simply became tired. But that is only a supposition, and I am sure that he would deny it. His mental energy was extraordinary.

If one considers that for twenty years Grothendieck - as one of his colleagues put it - did math for twelve or more hours a day, seven days a week during twelve months of the year, it is not difficult to agree with this view. However, even this does not really resolve the conundrum. Many scientists (or artists) give up a project they have begun because their creativity and productive energy wanes, but they remain within the community as respected members.

Serre speaks simply of "fatigue" or perhaps "exhaustion". Others express themselves in a similar fashion, but perceive more profound causes and also cite as "disappointment" as a reason. According to Helmut Koch, Igor Shafarevitch thinks that Grothendieck's decision to begin his work on *Elements de Géométrie Algébriques* was ill-fated. He should have devoted his creativity to the "big problems" and not to the construction of a gigantic theory in complete detail. The physicist David Ruelle, a colleague of Grothendieck at the IHES, expressed himself similarly. Grothendieck must have realized after superhuman efforts that he would never be able to complete the work he had begun. It was as if he had taken it into his head to build a cathedral single-handedly. When the walls were just two meters high he was forced to abandon the task.

It seems to the author that all three, Serre, Shafarevitch and Ruelle, grasp important points, but ultimately not decisive ones. These can explain (in part!), why Grothendieck gave up mathematics, but not why he altered his whole life, or why he withdrew from society. Originating in the very depths of his personality, this transformation is doubtless a far more profound and more emotional event than the mere abandonment of his profession or his withdrawal from scientific research. The thought again suggests itself that it could be the other way around: because Grothendieck - for whatever reasons - no longer could or would remain in the society in which he had lived from 1950 to 1970, he was obliged to give up mathematics.

Grothendieck's old friend and colleague Pierre Cartier⁷ undertakes to offer a more complex explanation. Without underestimating the significance of the financing problem at the IHES, and while acknowledging the crisis in Grothendieck's mathematical work, he also explains that the rupture in

⁷ P. Cartier: *A mad day's work: From Grothendieck to Connes and Kontsevich - The evolution of concepts of space and symmetry*, Bull. Am. Math. Soc. 38, 389-408, 2001.

Grothendieck's life had far deeper causes:

I would like to try and analyze the causes for the abrupt end to a so astonishingly fertile career at the age of forty-two. The discovery that the Ministry of Defence had contributed financial support to the institute was given as the reason for this[...] In order to understand the strength of his reaction one must take into account his past and the political situation of that time. He is the son of a militant anarchist, who had devoted his whole life to the revolution. He knew very little of his father directly. Most of what he knew came from the idolizing stories told by his mother. He lived his entire childhood as an outcast, and held the status of displaced person for many years [...] He always felt uneasy when visiting "upper class" localities, and was far more at ease among the poor and disadvantaged. Solidarity with outcasts aroused in him a strong feeling of empathy. He lived according to his convictions and his house was always open to the stranded. Ultimately he reached a point where he experienced Bures as a golden cage that kept him away from real life. Added to this, there were psychological crises and doubts as to the worth of his scientific activities. As early as 1957, during a Bourbaki conference, he confided these doubts to me and told me that he was thinking about other activities than mathematics.⁸ Perhaps one should also take into account the well-known Nobel Prize syndrome. After the Moscow conference where he received the Fields Medal, he was working on the last (decisive) step of the proof of the Weil conjectures, and he perhaps began to realize that it would take Deligne to complete the program which he had set himself to finish by 1974, and maybe the fatal thought crossed his mind that at the age of forty mathematical creativity slackens, that he had passed his zenith and that he could only repeat himself with decreasing efficiency.

There was also the influence of the zeitgeist. The disaster of the Vietnam War from 1963 to 1974 had been a wake-up call to the conscience of many.

All in all Cartier's analysis seems broadly correct, and all that is left is to elaborate on a few points and elucidate them more fully. If it has been maintained that the dispute about funding was not a truly defining issue, that does not mean that political, or more exactly, socio-political reasons did not play any role in all these decisions. On the contrary, they were of great significance, as Cartier so aptly demonstrates. To illustrate this more clearly we shall return to Grothendieck's political activities again in Chapter 3.

These actions must be considered against the backdrop of his life and that of his parents. Cartier is no doubt absolutely correct in underlining the fact that his parents were always present in his mind as role models. His father spent his entire life fighting against the powerful of this world for freedom and self-determination.

⁸ In a short and very cryptic note about the spiritual stages of his life Grothendieck mentions for 1957 "vocation and infidelity". This probably means that he had felt the pull of a different vocation, but failed to follow it.

Grothendieck's sympathies were always with the poor, the persecuted, the oppressed, and those who stood in the shadows; he always had leftist, liberal, and also anarchist political convictions. For many years however, these convictions did not lead to any political action. It goes without saying that he was opposed to the French war in Algeria, and although he did not participate in open political action, as did many of his closest colleagues such as Schwartz, Chevalley, Samuel, Godement or Cartier, he did take the issue seriously enough to consider emigrating to the United States in 1961/62. He wrote letters to various colleagues (i.e. Hirzebruch in Bonn) and also to the director of the institute, Motchane. The reason for which these plans were abandoned is not revealed in the correspondence. It is possible that his wife's illnesses (she suffered from tuberculosis and also from psychological problems) played a role.

We shall postpone any further description of Grothendieck's "political" activities at the end of the sixties to Chapter 3 and return instead to his budding interest in biology, which was, after all, the catalyst for the final split with Motchane. His interest in biology was possibly inspired by Mircea Dumitrescu, whom he met on his second(?) trip to Romania (see Chapter 5), and later again at the house of their common friend Valentin Poenaru. Dumitrescu was originally a doctor, but had subsequently taken up molecular biology. Having recently fled Romania, he was seeking a position. In the winter of 1969/1970 he held a series of lectures at the IHES and dealt with subjects on genetics such as DNA and the genetic code. These lectures were attended by, among others, Grothendieck, Poenaru and Egbert Brieskorn. At that time Grothendieck lacked elementary basic knowledge on the subject; for instance, according to Poenaru, he did not possess even the slightest inkling about chemistry. Thus, his first task was to fill in the gaps in his own education. With this in mind he organized the acquisition of some elementary textbooks for the library of the IHES. To all appearances however, his preoccupation with biology does not seem to have lasted long.

Brieskorn recalls that Grothendieck was at that time of the opinion that the genetic code should make it possible to understand and predict phylogenesis and the evolution of species in a fully deterministic manner. Thom contradicted this vehemently. In any case Thom and Grothendieck each denied that the other could even be considered a mathematician.

The meeting of April 1970 apparently dealt with an extension of Dumitrescu's invitation. He did subsequently remain in France, but his scientific career did not meet with much success. Poenaru recounts that he amassed considerable debts with all sorts of friends, which he now and again reimbursed by creating new debts with other friends. He kept up a loose but friendly contact with Grothendieck until his death in 1988. Grothendieck apparently visited him that year, shortly before his

death, in order to provide spiritual assistance as the end drew near.

The manner in which the "great turning point" has been described shows it to be a profoundly earnest question of morals, responsibility, firmness of principle, and scientific creativity: a matter in which almost no word is too weighty. To get a balanced picture, one must add that also Grothendieck's everyday life was altered by the "great turning point." From then on he no longer inhabited the bourgeois world where in any case he had always felt himself to be a stranger, and in which he had never really been more than a guest. When he spent the night with friends, he refused to sleep in a bed, but instead rolled himself up in a sleeping bag which he would spread out on the stone floor. Even at home he did not sleep in a bed, but on a door removed from its hinges - he said that this reminded him of his youth in the internment camp. For health reasons he fasted one day a week, and often twice in order to express solidarity, for example with miners in Chile. In any case he lived on a highly reduced diet. During meals with friends and acquaintances he annoyed the wives by lecturing on the chemicals with which their food was probably contaminated. He trudged through the snows of the Canadian winter barefoot in sandals. When a serious accident on a motorbike rendered an operation necessary he refused anesthesia and opted for acupuncture. His atavistic "Jewish avarice" (his own words) broke through, but he dispensed gifts of money right and left with the greatest generosity. Money meant nothing to him, and he reduced his own needs to almost nothing. He switched off the electricity and there was no running water in his house, but it stood open to all and sundry. His cars were the oddest of vehicles; one was a hearse that had served its time, another one had holes in the bottom plate, so that one had to be careful not to let one's feet poke through onto the road. He failed his driving test nine times and drove for years without a license.

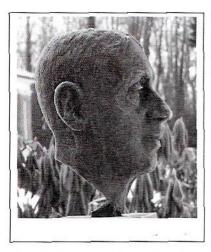
Let us give one of the tales about him verbatim:⁹

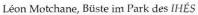
Legend has it (everything is legend) that, out at the farm, he would save his shit in buckets, then go around - ever the good ecologist - to the local farmers trying to sell it. What would be your reaction to being offered the shit of a Fields medalist at bargain prices?

Overtad from:

⁹ Quoted from; Roy Lisker: Visting Alexander Grothendieck, www.fermentmagazine.org. A few lines later in this text Lisker gives some advice to potential biographers, on how they might go about their work: My feeling is that you have got three choices: (1) You can ask him for permission to write about him. After he refuses, which is certain, you can write it anyway. (2) You can interview him, then write the article without his permission: or (3) You can just invent everything from beginning to end, without even bothering to go see him. - The results will be equally incredible.

Grothendieck lived in another world - in the no man's land beyond the border. And through a long and painful process, he was compelled to realize that he lived there quite alone.







Jean-Pierre Serre, etwa 1975







Jean Dieudonné