

Chapter 13. Professor at Montpellier, 1973-1984

We will now return to the year 1973 and once again turn our attention to Grothendieck's professional activity.

After lengthy negotiations Grothendieck was appointed to a position as “Professeur à titre personnel” starting in the fall of 1973. The holder of such a position can basically decide at which university he or she wishes to work. (Something of this nature is probably only conceivable within the French centralized educational system.) Grothendieck decided on Montpellier, at that time still a provincial university. His intention to definitively break off with the “great world” of mathematics, and with places such as Paris or Harvard, certainly played a role in this decision. (It remains unclear however, which was the primary decision: to take up residence in the south of France or to teach at the University of Montpellier, which then necessitated the move.)

From this point onwards, Grothendieck no longer took part in mathematical conferences, did not accept any invitations to lectures or visiting positions, nor did he publish any mathematical works. Perhaps his intention in Montpellier was to exercise a perfectly “normal”, not particularly ambitious activity as professor at a rather minor university. The fact that he chose Montpellier is doubtless related to the fact that he spent three years of his youth there, and that he desired to return to his beginnings and to a familiar place. One can easily imagine that he felt the need to “come to rest” in a place which he knew well and which held no surprises for him. In any case, his memories of the city in which he had begun his studies cannot have been exclusively discouraging and negative.

His appointment to the University of Montpellier did not come through without a hitch. During the *Survivre* period, he had given his usual lecture here, too, and in all probability handed out the text which went with it. When discussing whether to hire him, the dean, a professor of chemistry, asked whether and how one could appoint someone who regarded scientific research as inherently dangerous and rejected it. (It would seem that Grothendieck never asked himself this question. Apparently he was of the opinion that people should employ him for precisely that reason.)

During term, he would appear at the mathematical institute once or, less frequently, twice a week, in order to teach his courses or seminars. During vacation he came much more erratically. Once a superintendent even tried to evict him from the campus because he was wandering around like a homeless person. In particular, his sandals made from old car tires

distinguished him from the rest of the faculty. Definitive and complete information on his teaching activities could not be obtained; in the first years it seems that he taught the following courses:

1973/74, a course on second year analytical functions

1974/75 the same course again, but this time interrupted for a lengthy period due to an accident with his motorbike

1975/76, a master's level course on introduction to topology, with particular emphasis on fundamental groups, etc.

1976/77, possibly something on number theory and/or group theory, maybe occasionally also category theory.

Information on other courses will be given later on. All in all, for a mathematician such as Grothendieck, this does not sound like an especially ambitious program, but there are many indications that he took great pains with his courses. They are reported to have been very good, at least in part, and he endeavored to create new teaching methods such as active participation of the students, although these were not always crowned with success. It is likely that many of the younger students who attended his classes were not aware that their teacher was a world-famous mathematician. Susan Holmes (later a professor of statistics at Stanford) attended his course in 1974/75. She recalls that Grothendieck dealt not only with analytical functions but with all sorts of other subjects, and that he was very friendly and helpful with the students. She also attended his course in 1977 when, in the context of group theory, he spoke among other things about the Rubik's Cube¹.

Stephanie L., who spent an academic year in Montpellier from October 1976 to April 1977, wrote among other things²:

The tone of the lecture was very personal and enthusiastic. Nothing was rattled off. [...] In spite of rumors that mathematics was not a priority for him, I never noticed any lack of motivation during the course. [...] I never heard a bad word about G. from the seminar participants. On the contrary, everything I heard about him was filled with evident admiration and sympathy. [...] We had already heard that he had switched to raising sheep; I can't remember if that was supposed to be understood literally. People also said that he only gave courses as a way of earning his living.

¹ This could be an error, as according to Wikipedia the Rubik's Cube became known only in 1979.

² From an email to the author.

And then something that might perhaps be of particular interest to young ladies:

Concerning his appearance I would have described him as good-looking. But the question always bothered me as to how one could manage to have such wrinkly shirts. Also his hairstyle was remarkable. A shaven head, but always with lots of scratches. It was said that he did that himself. Oh well.

In connection with the 1974/75 course, we mentioned Grothendieck's motorbike accident. In the fall of 1974, due to carelessness, Grothendieck crashed head on into a car. One of his legs received multiple fractures and he had to stay in the hospital for several weeks. As Ladegaillerie recounts, Grothendieck only accepted treatment with antibiotics when told that his leg would otherwise have to be amputated. He refused anesthesia for the necessary operations and insisted that acupuncture would have to suffice, which then did not after all turn out to be quite the case. As can be seen from photos, he was still using crutches in 1975.

It is possible that he needed further treatment a year later in Paris. In any case Karoubi relates that as he was already in Paris for a hospital stay, he also gave a lecture at a colloquium in Paris. This talk – his last at any colloquium – was held on 12 December 1975, on the theme “Cohomologie de de Rham à puissances divisées”. Karoubi reports that many people came (about a hundred) and that the lecture was considered a major event on the Paris mathematical scene. It dealt with generalizing the description of the “rational homotopy type” (after Sullivan) to a description of the “full homotopy type.” H. Cartan incorporated parts of this talk into his paper “Théories cohomologiques”. Finding the “right” approach to homotopy theory remained a persistent theme in Grothendieck’s mathematical “meditations” dating from the 80s.

A letter to his German friends confirms the fact that although Grothendieck’s mathematical activity was not very intense, he could not completely get away from it either: “I waste my time on foolish math”. These friends also relate that Grothendieck always did math when he was not doing well emotionally, although it is not clear which was the cause and which the effect. Grothendieck confirms this himself in a letter dated January 18, 1978: “For the last four or five months my life's curve has been once again passing through a ‘relative minimum’ - a characteristic sign of this is that I am doing math quite intensively [...] and my sense of being alive is correspondingly dulled.”

After Grothendieck had given standard courses for some years, he set himself higher goals. He saw his mission as that of motivating the students, encouraging them to a true engagement with mathematics, and leading them to independent thought and original research. And he tried to make it as easy as possible for them.

In the academic year 1977/78 he veered away from the usual curriculum. Presumably he had thought deeply about what he wanted to offer the students. We are quite well informed about this academic year, partly through Grothendieck himself, who spoke of the events of that year in his text *Esquisse d'un Programme*, and partly by two German visiting students, Rudolf B. and Volker Diekert³.

In the winter semester of 1977/78 (when the letter quoted above was written), he gave a course on the geometry of the cube, which was continued in the summer semester with a course on the geometry of the icosahedron. In the summer semester of 1978 he gave two two-hour C4 courses (courses for the fourth year or eighth semester) in the department of pure mathematics. One of them was “Geometry of the icosahedron” and the other, “Arithmetic geometry of regular polygons”. Both sets of lectures took place on Fridays, beginning on March 3 and ending on May 26.

According to the very sketchy lecture notes available, Grothendieck took a combinatorial-group theoretical approach to these geometric objects, working over an arbitrary base field (or even an arbitrary commutative base ring), so that for finite fields with few elements the so-called exceptional isomorphisms between finite linear groups played a role. Grothendieck obviously could not refer to any of the standard literature for these courses, and he developed the material himself. Because it dealt with fairly elementary things, it was without doubt appropriate material to lead students towards independent thinking. Volker Diekert wrote his master’s thesis (for the degree known as “Diplôme des Etudes Supérieures”) with Grothendieck; it was essentially a write-up of the icosahedron course. As mentioned above, Grothendieck spoke of these classes in his memoir *Esquisse d'un Programme*:

It was in 1977 and 1978, parallel to two C4 courses on the geometry of the cube and that of the icosahedron, that I began to get interested in regular polyhedra, which then appeared to me as particularly concrete “geometric realizations” of combinatorial maps;

³ Diekert received his doctorate later under Kay Wingberg and became a professor of computer science in Stuttgart.

the vertices, edges and faces are realized respectively as points, lines and planes in an appropriate affine 3-dimensional space, and respecting incidence relations. This notion of geometric realization of a combinatorial map still makes sense over any base field and even over an arbitrary base ring.

Little more could be learned about Grothendieck's teaching activities: in 1981/82 he gave a seminar for a small number of students, but possibly without any lectures, and in 1983/84 a seminar on Teichmüller groupoids was planned together with Yves Ladegaillerie and Carlos Contou-Carrère. Grothendieck gave a few talks, but a proper seminar did not take place. In a letter to Mebkhout dated June 15, 1983 he mentioned that he intended to give a preparatory course for the "agrégation"⁴. Presumably this course did not take place either, as Grothendieck ended up taking a CNRS research position and did not visit the university after about 1984.

Even though lecturing did not take a central place in Grothendieck's life, the evolution of his teaching activities in Montpellier is, fundamentally, a tragic tale, which reflects on a small scale the tragedy of his life. He wished with all his heart to interest and inspire his math students to "true research", to independent questioning and thinking, and he would have helped them in every way along this path. Presumably he would have accepted it even if, instead of mathematics, they had undertaken anything at all autonomously and of their own accord. He could not understand, however, that they thought differently from him, that they had other goals in mind, that they did not follow his lead, but simply wanted to pass their exams. Ultimately they could not reach an understanding.

Grothendieck must have felt this himself, because at the beginning of the next academic year, 1978/79, he started by presenting the declaration quoted below to his future students. He was planning to give a course called "Introduction to research" for the fourth year students, and in October 1978 he prepared an information sheet for the class entitled *En Guise de Programme* ("In lieu of a programme"). This is an interesting text, whose significance extends far beyond the information that he intended to communicate. For the first time, numerous themes are touched on that later came to play a central role in his meditations. It is difficult, perhaps almost impossible, to adequately translate the poetic diction into German (if only because many words in the text are given special emphasis in that they begin with a capital letter.) Nevertheless an attempt must be made.

⁴ French examination usually taking place after the fourth year of university studies.

When an intense curiosity animates research, we advance as if carried on impatient wings. Are we not then like a venturesome bark, which avidly plows the immeasurable ocean? Yes, we are surrounded on all sides by billowing mists, ceaselessly taking shape, illuminated by our searching gaze, ceaselessly dissolving in order to challenge us all the more to penetrate them! [...]

Ardent curiosity alone is creative, it carries us straight to the heart of the Unknown. Is not This our only true inheritance, placed in each of us before we are born? Imperceptible seed, and yet from it springs the Flower of one thousand petals and the Tree of myriad branches... There is nothing which is not born from It. And if we only let it blossom in us, there is nothing that our Thirst for knowledge cannot bring forth. It alone gives us wings, It alone animates the force which drives us to the essence of things. Where It is not, there is no Creation, no Love.

When this thirst is absent, what meaning is left in our lives? What meaning is there in work where there is neither creation nor love? What is left when there seems to be no trace of the child in us who plays and asks questions? What is the future of a world that lets its unique inheritance perish?

For the last three years I have taught as a blind man would paint. I talked about things as I was discovering them, to people who came to hear me out of some strange sense of obligation. Certainly, the things which were presented and discussed were so simple and tangible that, with me as a playmate, a curious child could have discovered them – and I spoke as if to this child or to myself. And carried away by this imaginary dialogue I remained blind to the fact that I was holding a monologue before pupils busy taking notes on a course which they did not care about. They only cared about the exam. Although the things discussed were child-like and vital – it was as if so many random and inanimate objects piled themselves into a heavy jumble inside inert souls – struck with paralysis. [...]

And now?

What shall we do, we, the new protagonists, at the beginning of this new, alas academic, year, in order to meet the desiderata of an official course without limiting ourselves to a repetition of the immutable scenario of a schoolmaster perorating before his pupils? Every lesson is castrating, every speech in vain, if it is addressed to beings whose curiosity has not been awakened. If curiosity is absent - perhaps eradicated even to the memory of the remote time when it was still alive in us – what can be done in order to bring it to life again? That is our first, our principal question, the one that takes precedence over all others. As long it is suspended, as long as the desire to Play has not been awakened in each and every one – every invitation to a voyage of discovery together will remain completely void of sense.

Our main proposition will thus be to provoke the child to play, the child that slumbers equally in the pupil focused on his baccalaureate and in the teacher. But is it really the role of the schoolmaster to provoke this – is it not rather the role of each of us, to provoke all the others, beginning with oneself? [...] To your places then, “Teacher” and “Pupils” to obediently perform your dance.

It lies with us whether we will be the child absorbed in a fascinating game – or hopping marionettes...

One can conjecture that the fourth year students may have discovered this

“course declaration” with some consternation, not really knowing what to make of it. In any case it is difficult to imagine that a didactic success could really be achieved in this manner, notwithstanding the best of intentions. Quite the contrary! But in fact, the author does not know whether the course actually took place, nor what the intended subject was, nor the final result.

Almost exactly six years later, when writing *Récoltes et Semailles*, Grothendieck referred to this declaration in the following terms:

Six years ago I wrote a two-paged text entitled *En guise de programme* for the C4 course Introduction to Research; the text was an introduction, or more precisely, a declaration of intention on the subject of the spirit of the “course” I meant to teach. After writing this text, which emerged from my pen in the most spontaneous way in the world, I was struck by the abundance of images, one springing from the other, replete with erotic connotations. It was perfectly clear to me that this was neither coincidence nor the result of a simple literary intention – but that it was an unequivocal sign of a profound kinship between the two passions that had dominated my adult life. [R&S, p. 544]

A few pages later he mentions this declaration again in a footnote, referring to certain features of his work that he identifies as being necessarily present in all creative work:

I believe that the first written text in which I develop some of these features is the one from October 1978, *En guise de programme*. After that text, I never took the trouble to explore and elaborate my observations on this subject until this reflection, *Récoltes et Semailles*, from this year. [R&S, p. 597]

In January 1982 Thomas Friedrich from Berlin (in those days East Berlin in the German Democratic Republic) stayed in Montpellier for three weeks. He had discussions with Grothendieck and also went to his “seminar”. This took place in the clubroom of the institute: about five young people attended, each of them bringing quantities of food, most of it obviously from their own cultivations. They conversed animatedly on the most varied subjects, such as for example the groceries they had brought, and their production. After some two hours of copious dining and much wine the meeting was over and a new one was scheduled. Maybe this was just a coincidence, but it is possible that by this time Grothendieck had given up any hope of being able to interest his students in mathematics.

Although Grothendieck did not actively do research and certainly did not participate in conventional academic life, he kept himself informed on the important developments in mathematics through extensive correspon-

dence, and he answered questions addressed to him⁵. He no longer gave lectures, did not go to any conferences, and ceased all publishing activity such as that with Springer-Verlag, and so forth. He received only occasional visits from other mathematicians, such as the visit by Deligne before his move to the USA in 1984. He was still interested, however, in what was happening in mathematics. He was highly impressed by Falting's proof of the Mordell-Tate conjecture, and in a long letter attempted to win Faltings over to his theory of anabelian geometry⁶. In the 80s, however, these ties to the world of mathematicians and mathematics became increasingly weaker and finally ceased almost completely. The end of his relationships with Deligne and Serre, formerly his closest confidants, advisors and discussion partners, seem to be typical: after the last visit by Deligne in 1984, there was no more contact between them, and the correspondence with Serre ground to a halt between 1970 and 1987. A few more letters were exchanged in connection with the sending of *Récoltes et Semailles*, and after that all contact ceased definitively.

In the section entitled *Epilogue outre-tombe – ou la mise à sac*⁷, Grothendieck described in *Récoltes et Semailles* his last contact, in June 1985, with the mathematical institute in Montpellier. In May of that year a secretary informed him that his office on the fourth floor of the institute had been cleared out. Seeing this incident as an egregious example of the general decline of mores, an outraged Grothendieck appealed to M. Lefranc, the director of the institute, to the president of the university, to Mme. Charles, the person directly responsible for issues concerning offices, and, in an open letter, to his colleagues from the mathematical institute. (In another letter written later, he excused himself for having, in his haste, omitted to send the open letter to the staff and the graduate students.)

In her answer, Mme. Charles described the situation from her point of view, and pointed out that Grothendieck lived very far away, was difficult to reach, and had not been seen at the institute for a long time. A meeting of lecturers from the institute (UER 5) took place in the presence of Grothendieck took place, and the director of the institute sent him a collective letter of apology: “Les enseignants de Mathématiques présentent

⁵ His correspondence was only partially made available to the author, and could not be systematically mined for use in this book.

⁶ This letter, together with a translation into English, has been published in *Geometric Galois Actions I* (L. Schneps, P. Lochak, eds.), London Math. Soc. Lecture Notes 242 (1997).

⁷ Epilogue from beyond the grave – or the looting

leurs excuses à Monsieur Grothendieck”.

Grothendieck wrote to his former colleagues for the last time on June 7, 1985, in response to this letter. He observed that only three of them had written to him individually; apparently he expected that each one would excuse himself separately. The letter ends as follows:

As far as possible, and particularly during the period of my appointment to the CNRS, I have decided in the future to renounce my office at the USTL [Université des Sciences et Techniques du Languedoc] and surrender it without a fight to Lapscher, Charles and their ilk. If I can avoid it, I will not undertake any more teaching activities at the USTL. [...] It is likely that this meeting of the UER 5 was the last one in which I will participate, just as this letter is the last one which I will have occasion to write to you. And this time I do not expect an answer.

Alexandre Grothendieck

As was briefly mentioned above, and can be understood from this letter, Grothendieck was applying for a position at the CNRS. This will be discussed further in Chapter 15.