TORSION POINTS OF ABELIAN VARIETIES AND F-ISOCRYSTALS

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Today I will talk about a joint work with Emiliano Ambrosi. We found some new properties of the category of F-isocrystals. Our goal was to prove a certain extension of the theorem of Lang–Néron for abelian varieties. The starting point was my previous work on the monodromy groups of overconvergent F-isocrystals, [D'Ad17]. The plan is to start with the statement of our result on abelian varieties. Subsequently, I will explain the translation of the problem in terms of morphisms of p-divisible groups. Finally, I will explain how to solve it using F-isocrystals.

Lang–Néron theorem. The theorem of Lang–Néron is a positive characteristic analogue of the theorem of Mordell–Weil. In contrast with Mordell–Weil, in this case one has to remove the isotrivial cases. Let $\mathbb{F}_q$ be a finite field, $\mathbb{F}$ an algebraic closure of $\mathbb{F}_q$ and $k/\mathbb{F}$ a finitely generated field extension (e.g. $k = \mathbb{F}(t)$) and let A be an abelian variety over k.

Definition 1. We say that A is *traceless* if for every abelian variety $B/\mathbb{F}$, we have $\mathrm{Hom}(B_k, A) = 0$.

Theorem 2 (Lang–Néron)**.** *If A is traceless, then the group $A(k)$ is finitely generated.*

Remark 3. Notice that the condition is also necessary because the Mordell-Weil group of a non-zero abelian variety over $\mathbb{F}$ is an infinite torsion group.

Consequence. We have a chain of finite groups

$$A(k)_{\mathrm{tors}} \subseteq A^{(p)}(k)_{\mathrm{tors}} \subseteq \dots,$$

where the inclusions are induced by the relative Frobenius.

Question 4 (Esnault)**.** *Is the chain eventually stationary?*

We gave a positive answer to her question. We use an equivalent formulation. Let k^{perf} be a perfect closure of k.

Theorem 5. *If A is traceless, the group $A(k^{\mathrm{perf}})_{\mathrm{tors}}$ is finite.*

First observations.

- When $\ell \neq p$, the group scheme $A[\ell^n]$ is étale. Therefore, $A(k^{\mathrm{perf}})[\ell^\infty] = A(k)[\ell^\infty]$ is controlled by Lang–Néron.
- The group schemes $A[p^n]$ are not étale. We have an exact sequence of p-divisible groups

$$0 \to A[p^\infty]^\circ \to A[p^\infty] \to A[p^\infty]^{\text{ét}} \to 0 \quad (*)$$

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which splits canonically over k^{perf}. This implies that $A[p^\infty](k^{\text{perf}}) = A[p^\infty]^{\text{ét}}(k^{\text{perf}}) = A[p^\infty]^{\text{ét}}(k)$.

– By Lang–Néron the subgroup $A[p^\infty](k) \hookrightarrow A[p^\infty]^{\text{ét}}(k)$ is finite. To prove Theorem 5 one has to bound the index of the subgroup.

Theorem 6. *The natural morphism*

$$\mathrm{Hom}(\mathbb{Q}_p/\mathbb{Z}_p, A[p^\infty]) \to \mathrm{Hom}(\mathbb{Q}_p/\mathbb{Z}_p, A[p^\infty]^{\text{ét}})$$

is an isomorphism.

Theorem 6 $\Rightarrow$ Theorem 5. Assume by contradiction $|A[p^\infty]^{\text{ét}}(k)| = \infty$. We can pile-up the points forming an infinite non-zero tower $(P_i)_{i\in\mathbb{N}} \in \varprojlim_i A[p^i]^{\text{ét}}(k)$. This defines a map $\varphi : \mathbb{Q}_p/\mathbb{Z}_p \hookrightarrow A[p^\infty]^{\text{ét}}$ by sending $1/p^i \mapsto P_i$. By Theorem 6, φ lifts to $\mathbb{Q}_p/\mathbb{Z}_p \hookrightarrow A[p^\infty]$. This contradicts Lang–Néron. □

Remark 7. Theorem 6 holds trivially when (*) splits over k. To prove Theorem 6 we prove a weak splitting property for (*). To do so we use the category of F-isocrystals.

F-isocrystals.

Over a point. Let κ be a perfect field, $K := \mathrm{Frac}(W(\kappa))$ and σ the lift of the Frobenius of κ. An F-isocrystal over κ is a finite-dimensional K-vector space E endowed with a σ-linear automorphism. By Dieudonné–Manin classification, the F-isocrystals over $\overline{\kappa}$ form a semi-simple abelian category such that the irreducible F-isocrystals are parameterized by rational numbers. For each $s/r \in \mathbb{Q}$ we have an irreducible F-isocrystal $E_{s/r}$ with slopes $(s/r, ..., s/r)$ where s/r is repeated r times (it is a multiset). For a general F-isocrystal over $\overline{\kappa}$ the slopes are the union of the multisets of slopes of the irreducible summands. The notion of slopes over κ is defined after base-change to $\overline{\kappa}$.

Example 8. When $A_0/\mathbb{F}_q$ is an abelian variety $H^1_{\text{crys}}(A_0/K)$ is an F-isocrystal over $\mathbb{F}_q$ with the F-structure induced by $F := A_0 \to A_0$ the (p-th power) absolute Frobenius. The slopes of an ordinary abelian variety are $(0, 0, .., 1, 1)$ with g zeros and g ones. When A_0 is supersingular the slopes are the multiset $(1/2, 1/2, ..., 1/2)$ with $1/2$ repeated g times.

General base. Let X be a smooth geometrically connected variety over $\mathbb{F}$ such that $k = \mathbb{F}(X)$[1]. We have a category of crystals

$$\mathbf{Crys}(X) := \left\{ \begin{array}{c} \text{Crystals of coherent} \\ \mathcal{O}_{X,\text{crys}}\text{-modules} \end{array} \right\}.$$

Even if the crystalline site is not functorial, its associated topos is functorial. Hence F^* acts on $\mathbf{Crys}(X)$. We define the category of F-isocrystals as

$$\mathbf{F\text{-}Isoc}(X) := \left\{ (\mathcal{E}, \Phi) \middle| \begin{array}{c} \mathcal{E} \in \mathbf{Crys}(X)_{\mathbb{Q}} \\ \Phi : F^*\mathcal{E} \xrightarrow{\sim} \mathcal{E} \end{array} \right\}.$$

[1]It will become clearer later why we prefer to work over a finite field rather than over $\mathbb{F}$, which would be the most natural choice.

The category $\mathbf{F}\text{-}\mathbf{Isoc}(X)$ has different incarnations. For example, it is equivalent to the category of *convergent* F-isocrystals in the sense of Ogus. The convergence condition is guaranteed by the existence of a Frobenius structure.

Theorem 9 (Berthelot–Breen–Messing)**.** *There exists a contravariant functor*

$$\mathbb{D}_{\mathbb{Q}} : \{ \text{ p-div gps/ } X \} \otimes \mathbb{Q} \to \mathbf{F}\text{-}\mathbf{Isoc}(X)$$

which is exact and fully faithful. The functor sends $\mathbb{Q}_p/\mathbb{Z}_p \mapsto (\mathcal{O}_X, \mathrm{id})$[2].

Definition 10. A *unit-root* F-isocrystal is an F-isocrystal which admits uniquely slope 0 at closed points (with some multiplicity).

Lemma 11. *The functor* $\mathbb{D}_{\mathbb{Q}}$ *induces a bijection between étale p-divisible group and unit-root* F*-isocrystals.*

Reduction to F-isocrystals. After the warm up with F-isocrystals we come back to the proof of Theorem 6. Let $\mathfrak{A}/X$ be an abelian scheme with constant slopes such that $\mathfrak{A} \otimes_{\mathbb{F}_q} k \simeq A$. We define

$$\mathcal{H} := \mathbb{D}_{\mathbb{Q}}(\mathfrak{A}[p^\infty])$$
$$\mathcal{H}^{\mathrm{ur}} := \mathbb{D}_{\mathbb{Q}}(\mathfrak{A}[p^\infty]^{\text{ét}}).$$

We have $\mathcal{H} \simeq R^1 f_{\mathrm{crys}*}\mathcal{O}_{\mathfrak{A}}$ and $\mathcal{H}^{\mathrm{ur}} \subseteq \mathcal{H}$ is the maximal unit-root subobject of $\mathcal{H}$. We reformulate Theorem 6 in terms of F-isocrystals.

Theorem 12. *For* $\mathcal{T} = (\mathcal{O}_X^{\oplus r}, \Phi) \in \mathbf{F}\text{-}\mathbf{Isoc}(X)$*, the morphism*

$$\mathrm{Hom}(\mathcal{H}, \mathcal{T}) \to \mathrm{Hom}(\mathcal{H}^{ur}, \mathcal{T}) \ (**)$$

is surjective.

Remark 13. When we pass from the statement of Theorem 6 to Theorem 12 we switch from a geometric situation to an arithmetic one. This is the reason why we need to introduce the constant F-isocrystal $\mathcal{T}$ and we cannot assume that Φ is the identity.

Overconvergence. In order to prove the statement we need to introduce the category of *overconvergent F-isocrystals*, denoted by $\mathbf{F}\text{-}\mathbf{Isoc}^\dagger(X)$. This is defined Zariski-locally using the rigid generic fiber of X. I will not define them, but I will recall some properties we will use.

- There is a canonical functor $\epsilon : \mathbf{F}\text{-}\mathbf{Isoc}^\dagger(X) \to \mathbf{F}\text{-}\mathbf{Isoc}(X)$ which is an equivalence when X is proper (e.g. over a point).
- (Kedlaya) ϵ is fully faithful.
- (Étesse) There exists $\mathcal{H}^\dagger \in \mathbf{F}\text{-}\mathbf{Isoc}^\dagger(X)$ such that $\epsilon(\mathcal{H}^\dagger) = \mathcal{H}$ ("$\mathcal{H}$ is overconvergent").

The category of overconvergent F-isocrystals has some similarities with the category of lisse sheaves. For example, the rigid cohomology groups with coefficients in overconvergent F-isocrystals are finite dimensional. Moreover, over finite fields, we have a *theory of weights* for overconvergent F-isocrystals.

[2] We drop the subscript "crys" from $\mathcal{O}_{X,\mathrm{crys}}$

Theorem 14 (Kedlaya)**.** *For $\mathcal{E} \in \mathbf{F}\text{-}\mathbf{Isoc}^\dagger(X)$ ι-pure of ι-weight w and every i, $H^i_{\mathrm{rig}}(X,\mathcal{E})$ is ι-mixed of weights $\geq w+i$.*

Corollary 15. *If $\mathcal{E}$ is ι-pure, then $\mathcal{E}$ is semi-simple as an overconvergent isocrystal.*

Remark 16. Notice that here the semi-simplicity is without Frobenius structure. This is the analogue of the semi-simplicity of ι-pure lisse sheaves when base-changed to $\mathbb{F}$. Our $\mathcal{H}^\dagger$ is pure of weight 1 and it is actually semi-simple even when we consider its F-structure. This stronger result is a consequence of the positivity of the Rosati involution, as noticed by Weil.

Warning. $\mathcal{H}^{\mathrm{ur}} \hookrightarrow \mathcal{H}$ is not "overconvergent" and in general the injection has no retractions. Therefore, we have a functor ϵ which is fully faithful but which does not preserve semi-simplicity. After we apply ϵ we obtain new subquotients. We need to control this phenomenon. A natural way to do this is using the *monodromy groups*.

Monodromy groups. In order to define the monodromy groups we need to extend the field of coefficients of the categories we have introduced. For a finite extension $K/\mathbb{Q}_p$ we define

$$\mathbf{F}\text{-}\mathbf{Isoc}^{(\dagger)}(X) \otimes K := \left\{ (\mathcal{E},\lambda) \;\middle|\; \begin{array}{c} \mathcal{E} \in \mathbf{F}\text{-}\mathbf{Isoc}^{(\dagger)}(X) \\ \lambda : K \to \mathrm{End}(\mathcal{E}) \ \mathbb{Q}_p\text{-linear morphism} \end{array} \right\}.$$

Taking a 2-colimit when K varies in $\overline{\mathbb{Q}}_p$, we get

$$\mathbf{F}\text{-}\mathbf{Isoc}^{(\dagger)}(X) \otimes \overline{\mathbb{Q}}_p.$$

These categories are *$\overline{\mathbb{Q}}_p$-linear neutral Tannakian categories*, namely abelian $\overline{\mathbb{Q}}_p$-linear $\otimes$-categories, with duals and with an exact and faithful functor to the category of (finite-dimensional) $\overline{\mathbb{Q}}_p$-vector spaces.

The concept of neutral Tannakian category has been introduced by Grothendieck and then further developed by Saavendra–Rivano and Deligne. These categories constitute a linear analogue of Galois categories. Thanks to the Tannaka duality, one can associate to a neutral Tannakian category $\mathbf{C}$ an affine group scheme which completely determines the category. More precisely, $\mathbf{C}$ is equivalent (in a non-canonical way) to the category of finite dimensional representations of the group scheme.

For $\mathcal{E}^\dagger \in \mathbf{F}\text{-}\mathbf{Isoc}^\dagger(X) \otimes \overline{\mathbb{Q}}_p$, $\langle \mathcal{E}^\dagger \rangle^{\otimes} \subseteq \mathbf{F}\text{-}\mathbf{Isoc}^\dagger(X)$ is defined as the smallest category containing $\mathcal{E}^\dagger$ and closed under $\oplus$, $\otimes$, duals and the operation of taking subquotients. The associated affine group scheme is denoted by $G(\mathcal{E}^\dagger)$. Let $\mathcal{E}$ be $\epsilon(\mathcal{E}^\dagger)$. We define as before, the category $\langle \mathcal{E} \rangle^{\otimes} \subseteq \mathbf{F}\text{-}\mathbf{Isoc}(X)$ and the affine group scheme $G(\mathcal{E})$.

In this case, the affine group schemes $G(\mathcal{E}^\dagger)$ and $G(\mathcal{E})$ are actually of finite type. We call them the *monodromy groups* of $\mathcal{E}^\dagger$ and $\mathcal{E}$ respectively. These were initially defined by Crew. We have the following closed embedding

$$??? \to G(\mathcal{H}) \subseteq G(\mathcal{H}^\dagger) \leftarrow \text{reductive}.$$

In the case of lisse sheaves the monodromy groups have been intensively studied by many people, notably Serre, Larsen–Pink, Chin. In [D'Ad17], I studied the right group exploiting the analogy between lisse sheaves and overconvergent F-isocrysals and using the Langlands correspondence.

Example 17 (Crew)**.** When A/k is a non-isotrivial ordinary elliptic curve $G(\mathcal{H}^\dagger) = \mathrm{GL}_2$. Moreover, there exists an exact sequence

$$0 \to \mathcal{L} \to \mathcal{H} \to \mathcal{L}^\vee(-1) \to 0$$

where $\mathcal{L}$ has rank 1 and $G(\mathcal{H})$ is a Borel subgroup of GL_2.

Over a point. If $x := \mathrm{Spec}(\mathbb{F}_{q^n})$ is a closed point of X and $K_n := \mathrm{Frac}(W(\mathbb{F}_{q^n}))$ we have a functor

$$\mathbf{F}\text{-}\mathbf{Isoc}(x) \otimes \overline{\mathbb{Q}}_p = \sigma\text{-}\mathbf{Vec}(K_n \otimes_K \overline{\mathbb{Q}}_p) \simeq \mathbf{Rep}_{\overline{\mathbb{Q}}_p}(\mathbb{Z}) \xrightarrow{\text{forg}} \mathbf{Vec}(\overline{\mathbb{Q}}_p)$$

where the second category is the category of locally-free modules over the algebra $K_n \otimes_K \overline{\mathbb{Q}}_p$, endowed with a σ-linear Frobenius structure. The last equivalence depends on the choice of the embedding $K_n \hookrightarrow \overline{\mathbb{Q}}_p$. This functor to $\mathbf{Vec}(\overline{\mathbb{Q}}_p)$ induces a fibre functor for $\mathbf{F}\text{-}\mathbf{Isoc}^{(\dagger)}(X) \otimes \overline{\mathbb{Q}}_p$ by pre-composing with the inverse image functor from X to x.

For $\alpha \in \overline{\mathbb{Q}}_p^\times$, we define a rank 1 constant $\overline{\mathbb{Q}}_p$-linear F-isocrystal over $\mathrm{Spec}(\mathbb{F}_q)$ as $(\overline{\mathbb{Q}}_p, \overline{\mathbb{Q}}_p \xrightarrow{\cdot\alpha} \overline{\mathbb{Q}}_p)$, denoted by $\overline{\mathbb{Q}}_p^\alpha$. We denote in the same way the inverse image of $\overline{\mathbb{Q}}_p^{(\alpha)}$ to X via the structural morphism.

Rank 1 F-isocrystals. If $\mathcal{L} \in \mathbf{F}\text{-}\mathbf{Isoc}(X)_{\overline{\mathbb{Q}}_p}$ is of rank 1, there exists $\alpha \in \overline{\mathbb{Q}}_p^\times$ such that $\mathcal{L} \otimes \overline{\mathbb{Q}}_p^{(\alpha)}$ comes from a p-adic character of $\pi_1^{\text{ét}}(X)$ (α is necessary to kill the slope of $\mathcal{L}$). If $\mathcal{L}$ is overconvergent, by a theorem of Tsuzuki, the character is potentially unramified, hence by class field theory it has finite order. Therefore, for $\mathcal{E}^\dagger \in \mathbf{F}\text{-}\mathbf{Isoc}^\dagger(X) \otimes \overline{\mathbb{Q}}_p$, there exists $\alpha \in \overline{\mathbb{Q}}_p$ such that $(\mathcal{E}^\dagger)^{(\alpha)}$ has finite order determinant.

Theorem 18 (The global monodromy theorem)**.** *If $\mathcal{E}^\dagger \in \mathbf{F}\text{-}\mathbf{Isoc}^\dagger(X) \otimes \overline{\mathbb{Q}}_p$ is irreducible with finite order determinant, $G(\mathcal{E}^\dagger)^\circ$ is semi-simple.*

Remark 19. By the theorem, if $\overline{\mathbb{Q}}_p^{(\alpha)} \in \langle \mathcal{E}^\dagger \rangle^\otimes$ then α is a root of unity. Indeed, $G(\overline{\mathbb{Q}}_p^{(\alpha)})$ is finite when α is a root of unity and $\mathbb{G}_m$ otherwise.

Frobenius tori. We reach now the crucial part of the proof. For $\mathcal{E}^\dagger \in \langle \mathcal{H}^\dagger \rangle^\otimes$ and $i : x \hookrightarrow X$ a closed point, we have that $G(i^*\mathcal{E}^\dagger) = G(i^*\mathcal{E})$, because over a point every F-isocrystal is overconvergent. The two groups embed in $G(\mathcal{E}^\dagger)$ and $G(\mathcal{E})$ respectively. We already mentioned that by the positivity of the Rosati involution, the Frobenius of $\mathcal{H}^\dagger$ at closed points is semi-simple. This implies that, the group $G(i^*\mathcal{E}^\dagger) = G(i^*\mathcal{E})$ is of multiplicative type. We define $T_x(\mathcal{E}) := G(i^*\mathcal{E})^\circ$, which is called the *Frobenius torus* of $\mathcal{E}$ at x.

Theorem 20 ([D'Ad17, Theorem 4.2.10])**.** *For $\mathcal{E}^\dagger \in \langle \mathcal{H}^\dagger \rangle^\otimes$, the set of closed points $x \in |X|$ such that $T_x(\mathcal{E})$ is a maximal torus of $G(\mathcal{E}^\dagger)$ is Zariski-dense.*

The analogous result for lisse sheaves was proven by Serre, I have extended it to overconvergent F-isocrystal using independence techniques. As $T_x(\mathcal{E})$ is also contained in $G(\mathcal{E})$, we get the following.

Corollary 21. *$G(\mathcal{E})$ contains a maximal torus of $G(\mathcal{E}^\dagger)$.*

We use this corollary to prove the next proposition.

Proposition 22. *For $\mathcal{E}^\dagger \in \mathbf{F}\text{-}\mathbf{Isoc}^\dagger(X) \otimes \overline{\mathbb{Q}}_p$ irreducible with finite order determinant, if $\overline{\mathbb{Q}}_p^{(\alpha)} \in \langle \mathcal{E} \rangle^{\otimes}$ then α is a root of unity.*

Remark 23. Even if the global monodromy theorem is false for F-isocrystals, Proposition 22 is telling us that the consequence discussed in Remark 19 remains true. This will be the key to prove Theorem 12.

We prove Proposition 22 by comparing the two monodromy groups we have introduced with two other ones which are the *geometric* counterparts. The proposition ultimately follows from a count of the dimensions of the maximal tori of the groups involved.

End of the proof of Theorem 12. We can decompose $\mathcal{H}^\dagger = \bigoplus \mathcal{H}_i^\dagger$ with $\mathcal{H}_i^\dagger \in \mathbf{F}\text{-}\mathbf{Isoc}^\dagger(X) \otimes \overline{\mathbb{Q}}_p$ irreducible. This induces a decomposition $\mathcal{H}^{\mathrm{ur}} = \bigoplus \mathcal{H}_i^{\mathrm{ur}}$.

Claim. If there exists a surjective morphism $\mathcal{H}_i^{\mathrm{ur}} \twoheadrightarrow \overline{\mathbb{Q}}_p^{(\alpha)}$ for some $\alpha \in \overline{\mathbb{Q}}_p^{\times}$, then $\mathcal{H}_i^{\mathrm{ur}} = \mathcal{H}_i \simeq \overline{\mathbb{Q}}_p^{(\alpha)}$.

Claim $\Rightarrow$ Theorem 12. We may assume $\mathcal{T} = \bigoplus \overline{\mathbb{Q}}_p^{(\alpha_i)}$, then the LHS and the RHS of the statement of Theorem 12 decompose and it suffices to show that the restriction morphism $\mathrm{Hom}(\mathcal{H}_i, \overline{\mathbb{Q}}_p^{(\alpha_i)}) \to \mathrm{Hom}(\mathcal{H}_i^{\mathrm{ur}}, \overline{\mathbb{Q}}_p^{(\alpha_i)})$ is surjective. This follows easily from the claim.

Proof of the claim.

- If $\mathcal{H}_i = \mathcal{H}_i^{\mathrm{ur}}$, then $\mathcal{H}_i^\dagger \simeq \overline{\mathbb{Q}}_p^{(\alpha)}$ because $\mathcal{H}_i^\dagger$ is irreducible.
- Suppose $\mathcal{H}_i \neq \mathcal{H}_i^{\mathrm{ur}}$, there exists $\beta_i \in \overline{\mathbb{Q}}_p$ with $v_p(\beta_i) < 0$ such that $\det(\mathcal{H}_i \otimes \overline{\mathbb{Q}}_p^{(\beta_i)})$ has finite order.
  After twist, we have $\mathcal{H}_i^{\mathrm{ur}} \otimes \overline{\mathbb{Q}}_p^{(\beta_i)} \twoheadrightarrow \overline{\mathbb{Q}}_p^{(\alpha\beta_i)}$. Thus, by definition, $\overline{\mathbb{Q}}_p^{(\alpha\beta_i)} \in \langle \mathcal{H}_i \otimes \overline{\mathbb{Q}}_p^{(\beta_i)} \rangle^{\otimes}$. By Proposition 22, $\alpha\beta_i$ is a root of unity. On the other hand, $v_p(\alpha) = 0$ and $v_p(\beta_i) < 0$. Contradiction.

Weak (weak) semi-simplicity. We end the talk giving another example of a structural property deduced from the fact that $G(\mathcal{H})$ is a maximal rank subgroup.

Theorem 24. *The group $\mathrm{Ext}^1_{\langle \mathcal{H} \rangle^{\otimes}}(\overline{\mathbb{Q}}_p, \overline{\mathbb{Q}}_p)$ vanishes.*

This follows from the following abstract lemma on algebraic groups.

Lemma 25. *Let $\mathbb{K}$ be a characteristic 0 field, G a reductive group over $\mathbb{K}$, $H \subseteq G$ a subgroup of maximal rank (i.e. that contains a maximal torus of G), then $\mathrm{Ext}^1_H(\mathbb{K}, \mathbb{K}) = 0$.*

Proof. Suppose by contradiction the existence of a non-trivial extension of $\mathbb{K}$ by itself. This implies that there exists a surjective morphism $\varphi : H \twoheadrightarrow \mathbb{G}_a$. The subgroup $K := \mathrm{Ker}(\varphi)$ has maximal rank in G because the same is true for H. Looking at the Lie algebras, one can prove that $K^\circ = N_G(K)^\circ$. On the other hand $H^\circ \subseteq N_G(K)^\circ$, contradiction. □

References

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