TD02: RIEMANNIAN METRICS

M1 - DIFFERENTIAL GEOMETRY, 2019-2020

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Exercise 1 (Standard metrics)**.** Let g_0 denote the Euclidean Riemannian metric on $\mathbf{R}^n$.

(1) Are the translations isometries of $(\mathbf{R}^n, g_0)$? Define a natural Riemannian metric on the flat torus $T^n = \mathbf{R}^n/\mathbf{Z}^n$.
(2) Consider $g = i^*(g_0)$ where $i : \mathbf{S}^n \to \mathbf{R}^{n+1}$ is the canonical inclusion. Do elements of the orthogonal group $O_{n+1}(\mathbf{R})$ induce isometries of $(\mathbf{S}^n, g)$? Define a natural Riemannian metric on $\mathbf{RP}^n$.

Exercise 2 (Metrics on curves)**.**

(1) Let M be a smooth curve without boundary. Show that any two Riemannian metrics g_1, g_2 on M are conformal to each other.
(2) Does the above result remain true for $dim(M) \geq 2$?
(3) What are smooth curves endowed with an intrinsic metric and without boundaries up to isometries?