

MR0488029 (58 #7605) [55B15](#) ([55F50](#))**Karoubi, Max**★ *K*-theory.

An introduction.

Grundlehren der Mathematischen Wissenschaften, Band 226.

Springer-Verlag, Berlin-New York, 1978. xviii+308 pp. \$39.00. ISBN 3-540-08090-2

This book represents only the second comprehensive text on topological *K*-theory. It extends the foundational text of M. F. Atiyah *K*-theory, Benjamin, New York, 1967; [MR0224083 \(36 #7130\)](#)], while at the same time bringing to the material which is common to both the benefit of innovations which have come to light in the meantime.

The cohomological properties, basic vector bundle constructions and Bott periodicity (complex and real), are established. The proof of periodicity in the complex case follows R. Wood's generalisation to arbitrary Banach algebras of an early, elementary proof due to Atiyah and R. Bott. Real periodicity is proved by introducing $K^{p,q}$ -groups which are defined in terms of $C^{p,q}$ -vector bundles ($C^{p,q}$ is the usual Clifford algebra). This approach forces eight-fold periodicity from the similar periodicity in Clifford algebras. This proof was developed by the author [Ann. Sci. École Norm. Sup. (4) **1** (1968), 161–270; [MR0238927 \(39 #287\)](#)] and independently by Atiyah [Quart. J. Math. Oxford Ser. (2) **17** (1966), 367–386; [MR0206940 \(34 #6756\)](#)]. Clifford algebras are given a brief review. The Thom isomorphism is derived and the *K*-theory of complex Grassmannian-, flag-, and projective-bundles is computed. The computation of the *K*-theory of real projective bundles is given in terms of Clifford-bundle *K*-theories. This material has not appeared in a textbook before.

The final chapter deals with the classical successes of *K*-theory: Hopf invariant one, vector fields on spheres (here an innovation due to Woodward is included), the Riemann-Roch theorem and a little of Adams' work on $J(X)$. The last application is rather limited by the author's desire to keep the book self-contained. It would have been appropriate and very gratifying to see something about the Adams conjecture in the historical note at the end of Chapter Five.

Incidentally, each chapter ends with a historical note, which should be very useful in enabling a nonexpert to pursue the literature. Also the historical note at the end of the chapter on periodicity enables the author to make some amends for not presenting anything about the index construction by referring the reader to articles containing the proofs which use elliptic operators.

Reviewed by *V. P. Snaith*