A Tribute to Henri Cartan

This collection of articles paying tribute to the mathematician Henri Cartan was assembled and edited by Pierre Cartier, IHÉS, and Luc Illusie, Université Paris-Sud 11, in consultation with Jean-Pierre Serre, Collège de France. The collection begins with the present introductory article, which provides an overview of Cartan's work and a short contribution by Michael Atiyah. This overview is followed by three additional articles, each of which focuses on a particular aspect of Cartan's rich life.

—Steven G. Krantz

Jean-Pierre Serre

Henri Cartan
8 July 1904–13 August 2008

Henri Cartan was, for many of the younger generation, the symbol of the resurgence of French mathematics after World War II. He died in 2008 at the age of 104 years.

Personal Life

Henri was the eldest son of the mathematician Élie Cartan (1869-1951), born in Dolomieu (Isère), and of his wife Marie-Louise Bianconi, of Corsican origin.

Born in Nancy in 1904, he entered the École Normale Supérieure (ENS, 45 rue d'Ulm) in 1923. It was there that he forged the friendships with mathematicians who were to play a major role in his life, beginning with André Weil, who had entered the ENS a year before; others included Jean Dieudonné, Jean Delsarte, René de Possel, and Charles Ehresmann. He left the ENS in 1926, supported by a grant until the completion of his thesis in 1928, and briefly became a teacher at the Lycée Malherbe de Caen. He was then appointed to positions at the University of Lille and subsequently the University of Strasbourg, where he taught from 1931 to 1939. The year 1935 was a particular high point of both his professional and his personal life: with his friends Weil, Dieudonné, de Possel, and others, he founded the Bourbaki group, which he left only at the statutory age of fifty years; and he married the young and charming Nicole Weiss, daughter of one of his physics colleagues at Strasbourg University.

This happy marriage, which lasted until his death (followed, a few months later, by that of his wife), produced five children: Jean, Françoise, Étienne, Mireille, and Suzanne.

In September 1939, at the beginning of the war, he moved to Clermont-Ferrand, where the University of Strasbourg had been evacuated. A year later he got a chair at the Sorbonne, where he was given the task of teaching the students of the ENS. This was a providential choice that allowed the "normaliens" (and many others) to benefit for more than twenty-five years (1940-1965) from his courses and seminars. In fact there was a two-year interruption when he returned to Strasbourg from 1945 to 1947—alas for me, because I was then a student at the ENS and could not make his acquaintance until my final year.

He left the ENS in 1965 and, a few years later, to escape the internal disputes between the component parts (Paris VI and Paris VII) of the former Sorbonne, he accepted a chair at Orsay, where he taught until his retirement in 1975. A lecture theatre in the mathematics building has recently been named after him.

Further details on the life of Henri Cartan can be found in two interviews (Schmidt 1990, Jackson 1999).

Mathematical Work

Henri Cartan worked on many subjects but there was one to which he was particularly attached, and that was the theory of functions of several complex variables (which later became the theory of complex varieties and also "analytic geometry"). I will begin with this topic.

His thesis (Joel, no. 3) dealt with analytic functions of one variable, one of the most popular topics of the period in France. Cartan continued the work of André Bloch and Rolf Nevanlinna,

References in this form refer to the bibliography at the end of the text.

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endowed with a sheaf of rings. For Cartan this sheaf was a sub-sheaf of the sheaf of continuous functions; Grauert-Remmert and Grothendieck showed a little later that it was better not to make such a hypothesis so as to allow nilpotent elements.

In subsequent years Cartan never lost interest in functions of several complex variables. He took great pleasure in expounding in Bourbaki seminars the works in this area of other mathematicians, notably those of Hirzebruch (exposé 84), of Grauert (exposé 115), of Douady (exposé 296) and of Ramis (exposé 354).

Let us now change the subject slightly and turn to topology. I have already mentioned the expository talks, clarifying sheaf theory, in the seminars of 1948/1949 and 1950/1951. He had done something similar for fiber spaces in the seminar of 1949/1950. Other results: the spectral sequence giving the cohomology of a Galois covering (with J. Leray), the method of “killing homotopy groups” (with me), and the study of the real cohomology of principal fiber bundles of Lie groups (with Chevalley, Koszul, and Weil). However, his most original contribution to topology was without doubt the long series of lectures in the 1954/1955 seminar (reproduced in [Oe], no. 93), where he determined the homology of the Eilenberg–Mac Lane complexes (“which required great efforts”, as he said in an interview in 1982—I can readily believe it). This work is now classified not as part of topology but as part of what is called “homological algebra”, a terminology introduced by Cartan and Eilenberg in their book with that title ([CE], completed in 1953 but only published in 1956). A “fundamental” book in the precise sense of that term, it collected scattered results and organized them in a systematic way, transforming them into an instrument of great power. Cartan also worked on other subjects, which I will simply mention:

- Classes of infinitely differentiable real functions (with S. Mandelbrojt) ([Oe], nos. 63–68);
- General topology: introduction of the notion of a filter ([Oe], nos. 61 and 82) and construction of the Haar measure ([Oe], no. 69);
- Potential theory ([Oe], nos. 70–75 and 84); see the report by J. Deny (1975);
- Harmonic analysis (with R. Godement) ([Oe], no. 80);
- Real analytic spaces (with F. Bruhat) ([Oe], p. XVI; [Oe], nos. 45–46).

**Cartan’s Influence**

One cannot reduce the influence of Cartan to a mere list of the theorems he proved. He did much more than that. As I said at the outset, Cartan represented (both in France and abroad) the revival of mathematics in France after World War II. How did this come about? It is difficult to answer precisely. There were several factors, among which were the following:

- The large numbers of students whom he trained (in chronological order: Deny, Koszul, Godement, Thom, myself, Cerf, Douady, Karoubi, and several others); he did not give them a research topic (believing, no doubt, that a mathematician who does not ask himself questions is not a real mathematician), but once they had started he helped them to prove their results, to clarify them, and to write them up properly. This took him on occasion much time (I am thinking in particular of a certain thesis in topology on which he—and I—spent many hours). But the pupil learned much.
- Another reason for his influence: the Cartan Seminars. I have mentioned several above. There were sixteen of them (from 1948 to 1964), and all except one (that of 1952/1953) have been written up; a summary can be found in Serre (1975). What made these seminars original and interesting was that they started from scratch and gave essentially complete proofs; despite this, at the end of the year (and after some twenty lectures) they culminated in interesting and occasionally novel results. Many mathematicians, French and foreign, learned their topology or their functions of several complex variables from these seminars.3

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3 The Cartan Seminars had a predecessor: the "Séminaire Julia", organized between 1935 and 1938 by Weil, Chevalley, Cartan and others. Here also there was an annual theme (such as class field theory, Hilbert spaces, the work of Élie Cartan, ...) and the lectures were written up. And there was a successor: the impressive "Séminaire de Géométrie Algébrique" of Grothendieck at the Institut des Hautes Études Scientifiques (1960–1969), where the proofs were even more complete—if I may say so—and the results even more novel. Since 1970, mathematical seminars have multiplied, in France as elsewhere, but none, to my knowledge, has tried to follow the difficult model of Julia-Cartan-Grothendieck: one settles for inviting, week after week, a lecturer who presents (usually without proofs) his latest results, and then discusses them with specialists. This is not the same thing.
studying in particular the properties of analytic curves in complex projective spaces of any dimension (for example, curves not meeting a given family of hyperplanes). This sort of topic was highly fashionable at the time, but it became less so in later years (despite the work of Lars Ahlfors and H. and J. Weyl). It finally came back into the limelight thanks to the work of Shoshichi Kobayashi on hyperbolic manifolds (1970-1980) (see Demaillie 1997) and also to that of Paul Vojta (around 1980), who created an astonishing dictionary relating Nevanlinna invariants to the heights of rational points on algebraic varieties.

Shortly after writing his thesis, his eyes were opened, by Weil, to the charms of functions of several complex variables. Cartan was definitely seduced by this new field. Between 1930 and 1940 he published many articles in collaboration with the German school (Heinrich Behnke and Peter Thullen), with whom he made great bonds of friendship that withstood World War II. A summary can be found in [An], sections 2-5. In particular, we can note the following:

- the introduction in ([Oe], no. 23), with Thullen, of the notion of “convexity” relative to a family of holomorphic functions.

- the following result ([Oe], no. 32), related to the work of Élie Cartan: the group of automorphisms of a bounded domain in $\mathbb{C}^n$ is a real Lie group, and the subgroup that fixes a point is compact and embeds into $\text{GL}(n, \mathbb{C})$.

Starting in 1940 it was the “Cousin problems” that attracted him most ([An], section 6). This involves the construction of functions whose local singularities (additive or multiplicative) are given. Is this possible, and if not what are the conditions that need to be met? The problem is reasonable only if one works in a domain of holomorphy, which is what Cartan assumes. He gets very close to his aim, thanks to a theorem on invertible holomorphic matrices ([Oe], no. 35), but he lacked two auxiliary results (which he later interpreted as statements of “coherence”). It was the Japanese mathematician K. Oka who proved the first of these two results. He published the proof and sent it to Cartan, who immediately saw how the same methods led at once to the second result ([Oe], nos. 36 and 38). The first Cousin problem was thereby solved, at least for domains of holomorphy.

The second Cousin problem, in contrast, does not always have a solution. There are obstructions of a topological nature: the problem should have continuous solutions (a minimal requirement if one is searching for holomorphic solutions). How can one concretely exhibit these obstructions and, moreover, show that there are no others? I suppose (I never thought of asking him) that this was one of the reasons that led Cartan to become interested in algebraic topology around 1945–1950. There were some striking analogies—for those who could see them—between certain concepts introduced by Oka (the “ideals of indeterminate domains”) and the theory of sheaves, which was being created by Jean Leray. In his first seminars at the ENS (1948–1951), Cartan took up Leray’s theory in a slightly modified form that was easier to use. In a subsequent seminar (1951/1952) he reaped the fruits of his labors. He began by clarifying the notion of “coherence”, implicit in Oka’s work, defined “coherent analytic sheaves”, and proved a vast generalization of the Cousin-type theorems: the famous “Theorems A and B”.

The stronger statement is “Theorem B”, which says that the higher cohomology groups of a coherent analytic sheaf are zero; in other words that every reasonable problem (of additive type) has a solution (provided the underlying manifold is a “Stein manifold”, the natural generalization of a domain of holomorphy).

Theorems A and B are very powerful tools. Cartan and I described several applications of them in a colloquium in Brussels in 1952; apparently these theorems made a strong impression on the participants because one of them (a German) said to his neighbor, “The French have tanks (Panzer); we only have bows and arrows” (see Remmert 1995). Indeed the idea of applying the (algebraic-topological) theory of sheaves to objects relevant to analysis (holomorphic functions) was a new idea; it was used later in many other situations (for example, solutions of partial differential equations) and has now become standard.

Another original idea of Cartan (now equally standard) was that, developed in the 1953/1954 seminar, of defining a complex analytic space (possibly with singularities) as a topological space

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Another reason may have been the translation by Weil of the Cousin problems in terms of holomorphic fiber bundles with additive structure group (for the first problem) and multiplicative structure group (for the second problem)—see ([Oe], no. 39, section 5).
Going beyond his own mathematics, I should mention the efforts that Cartan made to improve relations between French and German mathematicians after World War II. He was also active, with L. Schwartz and M. Broué, in the "Comité des Mathématiciens" that came to the aid of mathematicians imprisoned for political reasons in various countries (notably in the USSR), for example L. Pliouchtch, A. Chitcharansi, A. Chikhanovitch, and L. Massera.

**Distinctions**

Henri Cartan was a member of the Académie des Sciences de Paris and of several Academies in Germany, Belgium, Denmark, Spain, Finland, Italy, Japan, Poland, Russia, Sweden, and the United States. He was also an honorary member of the London Mathematical Society and a Foreign Member of the Royal Society of London.

He had honorary degrees from ETH (Zürich), Athens, Cambridge, Münster, Oslo, Oxford, Saragossa, Stockholm, and Sussex.

In France he had received the Gold Medal of the Centre National de la Recherche Scientifique in 1976 and he was Commandeur des Palmes Académiques, Grand Officier de l'Ordre National du Mérite, and Commandeur de la Légion d'Honneur.

He received the Wolf Prize in 1980 and the Heinz R. Pagels Human Rights of Scientists Award in 1989.

He had been president of the French Mathematical Society (1950) and of the International Mathematical Union (1966-1970). He had also been president (and subsequently honorary president) of the Mouvement Fédéraliste Européen (1974-1985).

**Michael Atiyah**

I got to know Cartan mainly through serving with him on the Executive Committee of the International Mathematical Union. This gave me a chance to see him operate on the international scene. By then he was an elder statesman, and he looked the part, always impeccably dressed in a style that one associates with earlier periods. But this formality hid a charming and friendly personality, and the twenty-five years that separated us were no barrier to our friendship.

I first met Cartan at a conference in Mexico in 1956, and, although he was then over fifty, I was struck by his restless intellectual energy. During the lectures his eyes were alarmingly alert; he seemed to be on the verge of springing from his seat with impatience at the slow pace of the lecturer. But he was never aggressive or rude, just interested and enthusiastic. I can only imagine what a live wire he would have been twenty years earlier.

I was very pleased when in 1973 Oxford gave him an honorary degree. The public orator in his (Latin) speech referred (as Serre has done) to the important role played by Cartan in maintaining links with German mathematicians after 1945. He also mentioned the fact that a younger brother of Henri, a talented composer who died young, had one of his compositions played in a prewar concert in the same theater (the Sheldonian) where Henri received his honorary degree.

My last memory of Cartan is of his attending a lecture of mine in Paris when he was at a very advanced age and seriously infirm. It was a touching symbol, both of his friendship and of his dedication to mathematics.

**References**

**Books**


Séminaires de l'École Normale Supérieure
1948/49 Topologie algébrique
1949/50 Espaces fibrés et homotopie
1950/1951 Cohomologie des groupes, suites spectrales, faisceaux
1951/1952 Fonctions analytiques de plusieurs variables complexes
1952/1953 Groupes d'homotopie (unwritten)
1953/1954 Fonctions automorphes et espaces analytiques
1954/1955 Algèbres d'Eilenberg-Mac Lane et homotopie
1955/1956 (with C. Chevalley) Géométrie algébrique
1956/1957 Quelques questions de topologie
1957/1958 (with R. Godement and I. Satake) Fonctions automorphes
1958/1959 Invariant de Hopf et opérations cohomologiques secondaires
1959/1960 (with J. C. Moore) Périodicité des groupes d'homotopie stables des groupes classiques, d'après Bott
1960/1961 (with A. Grothendieck) Familles d'espaces complexes et fondements de la géométrie analytique
1961/1962 Topologie différentielle
1962/1963 Topologie différentielle
1963/64 (with L. Schwartz) Théorème d'Atiyah-Singer sur l'indice d'un opérateur différentiel elliptique

Other texts


Cartan as a Teacher

Pierre Cartier

"We Are All Your Students, Mr. Cartan"

At the end of June 1965, at the (then) new mathematics library of the École Normale Supérieure in Paris (ENS), a party was held in honor of Henri Cartan. He was leaving his position of "directeur des études mathématiques" at the ENS, after twenty-five years of service. Many mathematicians gathered on the occasion of this farewell. Cartan, in one of his customary understatements, commented: "I asked to extend the invitation to my former pupils, and I see so many people...". To which Vladimir Arnol'd, visiting France for the first time, and with his standard wit, answered: "But... Mr Cartan... in Moscow as well as in Paris, we are all your students." Cartan was extremely pleased.

Of the two great creations of the revolutionary Convention (1793), if École Polytechnique is, in the words of Arthur Wightman, a "peculiar combination of West Point and M.I.T.", École Normale is properly the highest in a network of teacher's colleges.

When I was admitted at ENS in 1950, the mathematics department consisted of a rich and spacious library, with two offices at the entrance gate, shared by the professor, his teaching assistant (a "caiman" in our student's slang), and the librarian. The professor was Henri Cartan; his caiman was Jean Fresnel, a veteran from WWII; the librarian was the wife of the head librarian of the school, named Madame Martin. There were two more classrooms, called E and F, with the standard joke that every event was announced to take place in Room E or F. Everything took place in these two small rooms: the Bourbaki Seminar, the standard lectures for the students, the entrance examination, and also the Cartan seminar (see later).

Mathematics students were allowed only three years for the standard curriculum, whereas our friends in the other departments were given four years. Obtaining a fellowship for a supplementary year (to begin a doctoral thesis) was quite hazardous.

Cartan was everywhere. He himself taught the three years. In the first year, he supplemented—and modernized—the Sorbonne course on calculus. In this first year, we were supposed to follow at the Sorbonne the main courses: calcul différentiel et intégral (calculus), general physics, and classical mechanics. These courses were quite old-fashioned.

In the second year, instead of writing a master's thesis, the mathematics students were required to follow an advanced course: algebra (or analysis, or "géométrie supérieure"). But the main feat was for us Cartan's course called "cours aux carrés."

Having married during the summer of 1951, to the great dismay of Cartan, rather conservative in social habits, I was less faithful a student, but I remember memorizing hastily Erdős-Selberg's proof of the prime number theorem as part of "algèbre supérieure". Cartan impressed me very much with a course on potential theory, using the new tool of distributions and a simplified version of Sobolev spaces known as Dirichlet space.

For many years, the main academic obligation for the students of ENS was to submit to a national competition called "agrégation". Officially, this is a qualifying examination for the profession of high school teacher, in an extended sense, from junior

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1 alligator
2 And a Jew who left France in 1942 to join de Gaulle's Free French Forces.
3 Second-year students
4 A so-called "elementary proof", a tricky and uninformative extension of Čebyšev reasoning!
high school to the elite undergraduate program known as "classes préparatoires".

In our third year, still under the guidance of Cartan, we prepared for the "agrégation". He read very carefully our memoirs, twice a month, and trained us for the oral examination. I hated the completely old-fashioned curriculum and was impatient to return to more serious—and modern—matters. But preparing for the oral examination—a course in which one teaches how to teach—is a fruitful exercise, and I was later happy to teach such a course.

I remember a somewhat embarrassing episode. I was supposed to explain the standard result that a function, whose derivative exists everywhere and is identically zero, is a constant. I prepared carefully during the Christmas vacation, using a proof in Bourbaki allowing some exceptional points. When it was my turn, I explained to the class that I intended to "improve" Bourbaki's proof by allowing an exceptional set of measure 0. I remember Cartan commenting: "Now, listen carefully, Cartier is going to prove his first theorem!" I understood, by his tone, that something was wrong. I often taught Lebesgue's integral and never slipped again!

But the real event was Cartan's seminar. I vividly remember my first class at ENS, the second Monday of November 1950. According to what became the weekly routine, I attended in the morning Cartan's class for the "conscrits" (freshmen), then at 2:30 p.m. his seminar, and in the late afternoon, we were supposed to learn from the craftsmen the art of cutting and assembling metal, wood, glass, for a possible career in experimental physics. I was not very good at that and learned just enough to help my friends prepare fantastic illuminations for the school's night in the next spring.

Cartan had an indomitable curiosity and openness. He was also very tenacious, and his friends often called him "mosquito" for his insistence on biting. Though not a Huguenot, he was as rigorous. I recently learned that, during the summer, he was the organ player for a small Protestant community in Die, where the mourning took place after his death. He could be quite formal, but in a very British way, compatible with wit. I remember my first visit to his just reunited family (in their summer home). His daughter was wearing trousers for the first time, and in a very formal tone he asked his wife: "My dear friend, have you noticed the dress of your daughter?" Despite this formality, he had an open mind: always well dressed, he never blamed me for my sloppiness, intended to provoke my elders!

A glance at the table of contents of his seminar shows an unusual diversity of topics: algebraic topology, sheaf theory, several complex variables, automorphic functions, algebraic geometry, index theorem... He himself gave many of the lectures, the other speakers spent long hours with him for preparation, and he wrote (and typed) a large part of the proceedings himself.

After a few weeks, during which I understood nothing, I had acquired an elder brother, Jean-Pierre Serre, and a benevolent uncle, Samuel Eilenberg, a Protestant and a Polish Jew, well in line with my roots in Mittel Europa. I made an acquaintance with all—or almost all—French mathematicians. At that time, everything in mathematics was in Paris, around Cartan, with two extensions in Nancy—where Dieudonné, Delsarte, and Schwartz maintained Bourbaki's spirit—and in the University of Strasbourg with Ehresmann (for a while a Bourbaki accomplice). The glorious generation—Hadamard, Borel, Fréchet, Paul Lévy, Élie Cartan—was very old or already gone, and the ambitious youth gathered around Cartan. By his central position at the ENS, Cartan was more or less the thesis advisor of everyone (at least formally).

From Eilenberg, in the academic year 1950-51, we heard a long series of lectures in which he developed an axiomatic theory of group homology as a model for his well-known axiomatization of algebraic topology in his book with Steenrod. This series was to be followed by a similar attempt toward the cohomology of sheaves—a much more difficult subject, whose completion was one of the first major achievements by Grothendieck.

Three times a year, the same group of people attended the Bourbaki seminar in its ascending

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5 The other arm of Bourbaki was in Chicago; hence the famous series of Nancago mathematics books.

6 Father of Henri Cartan!
phase. There we learned about Weil's proof of the Riemann hypothesis in the case of function fields, Zariski's work (as reported by P. Samuel), Koszul's thesis about the homology of Lie algebras, the work of Petrovsky in partial differential equations and of Gelfand in the theory of group representations...all the hot subjects in mathematics. Moreover, the supposedly secret drafts of forthcoming Bourbaki volumes were freely circulated by Serre, Cartan, and others. Among the closest friends of Cartan were André Weil, who visited France every summer and gave a series of lectures at the ENS in the winter of 1951 (about adeles and ideles), Armand Borel, who introduced Cartan to Leray's work on sheaves, and Claude Chevalley, who became a professor at La Sorbonne after 1954 (and one year in Japan).

French mathematics was at a turning point. The undergraduate curriculum (even in its enhanced form for the "classes préparatoires") was a mix of coordinate geometry, synthetic geometry (based on the "theorem" that every one-to-one correspondence between the points of a projective line is given by a Möbius transformation), differential calculus with applications to geometry and kinematics. The foundations were sometimes shaky, there was hardly any hint of groups of transformations (in geometry), and the use of matrices was ignored or not advised. The good teachers dared to give the foundations of the real number system, but in the absence of set-theoretical terminology, the exposition was quite obscure. In the land of Lebesgue, hardly any mention was made of the Lebesgue integral, and we had to learn Lie groups from the thesis of Élie Cartan or in Pontrjagin!

By a sequence of well-planned steps, Cartan made General Bourbaki win! He managed to hire the ambitious youth at La Sorbonne: Schwartz, Choquet, Dixmier, Godement, and Chevalley. In 1957 the takeover was complete (except for André Weil, who was never forgiven for his refusal to be drafted in 1939, at the beginning of WWII). The curriculum was deeply renovated, and the textbook of textbooks became Bourbaki (whose golden age extends from 1950 to 1975). The forceful gesticulations of Dieudonné, as well as the power of persuasion of Choquet and Lichnerowicz (both not members of Bourbaki) convinced everyone to worship general topology, linear algebra, functional analysis, and group theory. Henri Cartan was always a moderate in this debate and never threw the baby out with the bath water. He made one major mistake—discarding classical mechanics with this comment: "The teaching of classical mechanics (in France) is very poor"—which was true!—"and for the physicist, only quantum mechanics matters." We know better, as no one can understand quantum mechanics without a thorough acquaintance with its Newtonian (or Hamiltonian) version. One of the reasons for this mistake was the almost complete ignorance of the challenges of mathematical physics by the members of the Bourbaki coterie. It is also to be said that the teaching of physics in France was even more backward than in mathematics. One example: the first serious course on quantum mechanics was given in 1964, not in Paris, but in the National Center for Nuclear Physics (at Saclay).
So around 1960, Bourbaki had won, in main part due to the efforts of Cartan, helped by the political know-how of Schwartz. There followed the dubious episode of the so-called "modern math", or the failed attempt to use Bourbaki as a textbook for kindergarten! The fiercest proponents were ultra-zealous disciples of Bourbaki, not of the same mathematical caliber, helped by an attraction toward abstractness, an integral part of the then prevalent fashions (in art and elsewhere).

For about fifteen years French mathematics was ruled by two enlightened despots: Cartan and Schwartz, controlling between them most of the academic world. Namely, every doctoral student in mathematics had one of them as thesis advisor, and they suggested research topics. Cartan's way was more open and deep, but everyone benefited from their valued advice. Committee work was reduced to a couple of days off for our masters, where everything was settled, and in most cases, in an optimal way.

Under this reign, what was the fate of the young French mathematician? Let me take myself as an example. At the end of my first year at ENS, I was at pain to choose my future; I was attracted by philosophy and (experimental) physics as well as by mathematics. The advice of Althusser, the professor of philosophy, was warm support, with a fatherly caution about the degrees to be earned. Yves Rocard, head of the physics department, whose son Michel was later a prime minister of France, wanted to recruit me to build him the French atom bomb, to be exploded ten years later. Cartan, following the advice of Ellenberg, invited me to one of those secret meetings of Bourbaki.

My fate was decided.

A few years later, Cartan turned me down for the Princeton fellowship (awarded later on to Douday) on the excuse that this didn't fit a married man\(^{10}\) (he was not especially happy about my early marriage\!). Then he turned me down for the position of "calman" at the ENS, explaining to me later that I would have sacrificed my research work because of my involvement in teaching (he was perfectly right!).

A deeper interference came later in 1961. After completing my thesis, two years of postdoc at the Institute for Advanced Study in Princeton, and almost three years of military service at the time of the war of independence of Algeria, it was time for me to apply for a professorship (I was almost thirty!). For each position I applied to, I got the same answer: "Why should you come here, since Cartan says that you have been appointed at Strasbourg?" (where I didn't apply!). There was no way out. I tried to rebel and told Cartan I would stay only two years in Strasbourg—no more. He laughed, and I willingly spent ten years there—in the opinion of my wife, our best years! He later gave me an explanation. He was enormously interested in bringing back together French and German mathematicians, as witnessed by his long-lasting friendship with Behnke and Hirzebruch. In Cartan's opinion, I was the best fit in the young generation to work in this direction. He was right, and I immensely enjoyed working for the Versöhnung\(^{11}\) with the help of my German friends Dold and Puppe. I was from that time on as convinced as he was of the necessity of a Federal Europe (still in the making), well in line with the thoughts of my mother, who explained to me, when I was five, that I should see the day of the United States of Europe.

Some years later, in 1974, I didn't yield to his pressure to join Orsay University, but it was already another time, in the aftermath of the 1968 student revolution... Grotheendieck was gone already...

Thank you, Henri Cartan, for your fatherly influence on me! It made me grow up!

**Jacques Dixmier**

**Our Teacher**

I was a student at the École Normale, from October 1942 to September 1945. This was a time of war, and Cartan was unable to return to his alma mater in Strasbourg, which was then under Nazi rule. In this way, we could benefit from an outstanding teacher.

In my first year in the school, I had to attend the calculus course at La Sorbonne, Cartan's course being officially a kind of tutorial to help us master the curriculum in calculus. But this curriculum was rather outdated, and the aim of Cartan was to "modernize" the subject matter. The main point was to introduce us to so-called modern algebra (groups, rings, vector spaces...). Therefore, Cartan's lectures were combining a rather standard course on calculus but in the spirit of the not yet published volume of Bourbaki, entitled *Functions of a Real Variable (Elementary Theory)*. So, for instance, the $\mathbb{F}$-function was described as in the famous booklet by Emil Artin. In the more "modern" or "advanced" part, we were told about integers mod $p$, Grassmann calculus, Fourier transforms, and fixed-point theorems.

Cartan knew how to involve us in the class, and we would have been more impressed if we had known his research work. We were seventeen in the class, to be reduced to nine during the second year, because of the hardships of the war and also

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\(^{10}\)But not much later, I was invited for two years at the Institute for Advanced Study, where my wife was welcomed!

\(^{11}\)Mutual forgiveness, in German!

Jacques Dixmier is professor of mathematics, retired from the Université Paris VI.
because in the second year he taught only the students who had chosen mathematics as a major. After consulting the class, he decided to teach a course on Lie groups. He developed all the important notions from a rather general point of view but always illustrated by elementary examples. His goal was to lead us up to Lie's third theorem (reconstructing a Lie group from its Lie algebra). Because we were lacking the relevant background in topology, he concentrated on the local theory. I would not claim it was an easy course, and he admitted it frankly.

In our third year, our curriculum was centered around the so-called "agrégation", a national competition for prospective teachers. The main emphasis of the course was—and still is—to prepare the students to deliver a lecture—in a sense teaching how to teach.

So, for each of many years, he offered to the second-year students a new basic course. These courses were enormously influential, and he offered them besides his many commitments: research work, doctoral students, academic infighting, family tragedies... He was a real master, and we cannot be too grateful to him.

Adrien Douady

Memories of the Cartan Seminar

On June 28, 2004, a celebration was held at the École Normale Supérieure in Paris in honor of Henri Cartan on the occasion of his 100th birthday. Among the speakers was Adrien Douady.12 Douady gave recollections of the Cartan seminars. Here are excerpts of his talk,13 prepared by Régine Douady, his widow, from his handwritten notes, translated into English and slightly edited by P. Cartier and L. Illusie.

Nowadays there are two kinds of seminars: the colloquia, in which a prestigious visitor is invited to talk on a topic of his or her choice, and groupes de travail (work groups) in which a small team decides to study some question, to fully understand an important article. The Cartan seminar was of this second type. Cartan would choose a theme or a recent result that he wanted to understand in detail. He would assemble around him a team of speakers and assign the talks to be given, reserving a good number of them for himself. The seminar took place in Room U of the ENS. In the audience you would sometimes find Serre, Well, Dieudonné, Godement, Chevalley (who ran his own seminar right afterward), and, of course, Cartan. He did not tolerate the slightest inaccuracy, the slightest imprecision, and he criticized the speaker to the point of totally destabilizing him. You had to be well prepared. One month earlier Cartan would hand you a paper to read and digest. He would sometimes explain the plan. You were allowed, and even encouraged, to reconstruct the whole thing. I was of course happy to do that. I've never been able to read a paper (I go to sleep after page 3).

I remember my talks, and those of Bernard Morin as well. I wrote on the blackboard for him. Blind, he had an acute geometric vision. That was his revenge: seeing what others don't.

But what really mattered for Cartan were the notes of the expositions. Again he would tolerate no imprecision. It was out of the question to say that two groups were isomorphic without specifying an isomorphism between them, or to say that a diagram commuted up to sign: the sign had to be given. Above all he wanted the text to be perfectly clear. For this he asked you to revise your text as many times as necessary. Vingt fois sur le métier remettez votre ouvrage (Redo your work twenty times), said Boileau. With Cartan it was rather thirty times than twenty. His point was that a text should have at least thirty readers, otherwise there's no need to write it. Therefore, if you spend half an hour to spare the reader a minute of perplexity, then it's well worth it. He would return your manuscript covered with annotations made with a red pen in his small, curly handwriting. Then you would revise it and give him back a new version. There was no word processor at the time. I typed. I had a green typewriter, given to me by my father, which I still have. I photocopied my text, cut out shreds of 2 or 3 lines, pasted, photocopied again. I was allergic to the droplets falling from the ribbon of the typewriter when you type. You worked hard. A good reason for this is that when your text was eventually "Cartan acceptable", another person was to work on it: Denise Larteaux, the secretary of the jih (Institut Henri Poincaré). She used stencils, whose smell impregnated her office and her blouse. You could make corrections—with a correcting red ink having an even worse smell—but only by replacing one letter by another one, or a word by another one of the same length. All that made you confident that your text was really in final form.

Through this hard training all Cartan's students acquired a solid mathematical style. Serre's style reached perfection: it's a model of clarity, saying all that has to be said without an unnecessary word. Unfortunately Serre had very few Ph.D. students.

We try to pass the torch to our students. They toil and sweat, but they are grateful to us. One of them recalled that after I had said to her, "This is not clear", she had replied, "But it's clear in my
head”. Then I said: “What do you want to give to the reader: your text or your head?”

A few words, now, about the contents of the Cartan seminars. There were two main themes: algebraic topology and complex analytic geometry. In the 1940s one discovered the extraordinary power of the cohomological methods created for algebraic topology. These methods were later applied to algebraic geometry, complex analytic geometry, number theory, etc. Their development in each of these fields, as well as in their foundations (homological algebra), occupied a large (too large?) part of the activity of mathematicians up to the 1980s. Weil, Zariski, Cartan, Serre, Grothendieck, and Dieudonné were leaders in this vast movement, with the Cartan seminar as a prominent place for it.

Just after I had passed the "agrégation", I spent 1957-58 in Princeton. I attended the course of John Moore on algebraic topology and of Spencer in complex analytic geometry (I had tried to attend Kodaira's, but in vain, as he did not speak loudly, and I was already a little deaf). I had benefited from many conversations with Cartan and, above all, Andreotti. When I came back, in October 1958, Cartan immediately enrolled me in his seminar, starting with two expositions on spectral sequences. The goal was a construction of Adams to prove that the only spheres admitting a continuous composition law with neutral element are $S^0, S^1, S^3, S^7$ and are parallelizable. At the time $K$-theory didn't exist, and you used "secondary cohomological operations". They were horrible, ill-defined machines. A few years later, thanks to $K$-theory, a much simpler proof was found, and it was observed that all "secondary cohomological operations" are actually differentials in the Atiyah-Hirzebruch spectral sequence relating cohomology and $K$-theory.
mathematicians. It existed for sixteen years, up to 1964, and it was not replaced afterward. Each year, Cartan devoted his seminar to the proof of a recent result. He started from scratch so that a student with a general knowledge in mathematics was able to understand. He developed the necessary tools, gave complete proofs, and, at the end of the year, he obtained a proof of the chosen result, very often better than the original one. Each year, the seminar was written down, and it remained a classical reference.

I had the opportunity to attend this seminar for six years. In the first one, I was very young and ignorant. The subject was automorphic functions, and I was astonished by such things, completely new for me. In the second and third year I learned some algebraic topology. The seminar was dedicated to Adams's works on Hopf spaces, with the theory of secondary cohomological operations, then to the periodicity of stable homotopy groups. In the next year, Cartan returned to functions of several complex variables and analytical spaces.

The topic was suggested by Kodaira and Spencer's works on families of complex analytical manifolds and by the new results of Ahlfors and Bers on the moduli of compact Riemann surfaces. The seminar began with talks by Douady on Kodaira and Spencer's works. Then Cartan asked me to explain Ahlfors and Bers's work. At this time, Grothendieck, who attended the seminar, said that he had a way of constructing the moduli space of compact Riemann surfaces of given genus. His method was connected with his work in algebraic geometry. He proposed to Cartan to explain it, and his explanation occupied eleven sessions of the seminar: Cartan's seminar became Grothendieck's seminar! Cartan generously welcomed the new state of affairs. At the end of the year, Grothendieck asked me to explain local properties of analytical spaces. In his talks, he had used some of these properties without proof, and this complement was necessary. For me, this was hard work, and I still remember the care with which Cartan read my text.

For people of my generation, Henri Cartan was a master and a model. We were impressed by the power of his mathematical work but also by his moral qualities and by his sense of humor. He was engaged in the defense of persecuted mathematicians in dictatorial countries and in the construction of a political Europe. He was a good pianist and a fine musician. Through the reading of his correspondence with André Weil and Jean Dieudonné just after the war, I got a better knowledge of him. I hope I can publish these letters someday.

Jean-Pierre Kahane

Cartan at Orsay

Henri Cartan spent the last years of his academic career, the years from 1969 to 1975, at Orsay. Although it was a short period of time, Cartan played a decisive role in the development of Orsay, and he remains a glorious figure in the brief history of Université Paris-Sud. There are no archives of this interesting time, and I didn't try to collect testimonials; this article relies on my personal recollections.

Before 1968

The scientific center of Orsay was created around 1955, when the laboratories of nuclear physics moved from Paris to Orsay, under the supervision of Irène Joliot and Frédéric Joliot-Curie. Shortly afterward other important physics laboratories also moved from Paris or were created directly at Orsay. Until 1965 all these laboratories were part of the Faculté des Sciences de l'Université de Paris. This Faculté des Sciences, together with the Faculté des Lettres, were both located at the Sorbonne, but the building of the Sorbonne was too small to contain everything. A new location in Paris was necessary; it was built at the end of the 1960s on the former site of the wine market, La Halle aux Vins, and became the Jussieu campus. In the meantime, the mathematicians gathered at Institut Henri Poincaré, where the mathematics department of the Faculté des Sciences was located.

In 1958 it was decided to organize part of the teaching in all scientific subjects at Orsay. The first professors of mathematics who had their teaching duties at Orsay were successively Delange, Deny, Lesieur, Malgrange, myself, Malliavin, Néron, Cerf, and Poitou. We were professors at the Faculté des Sciences of Paris, and Orsay was merely a part of this Faculté, "annexe de la Faculté des sciences".

That situation changed in 1965, when Orsay became the second Faculté des Sciences de l'Université de Paris, with a dean having the same status as the Parisian dean. We then became professors at the Faculté des Sciences d'Orsay. However, we kept only one mathematics department, common to the two Facultés. During those years the president of the department was Henri Cartan (except in 1967 when he spent one year in Princeton and asked me to replace him). In this position he became a key actor in the development of mathematics at Orsay, treating his younger colleagues as equal members of the department (for example, though it was not so convenient at the time, he decided that one or two plenary meetings of the department should take place at Orsay instead of Paris). We organized

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together the advanced courses and the thesis defenses; the *commission des thèses* in mathematics had no formal power, but the simple procedure of registering and discussing every planned defense proved very efficient in ensuring a good standard for the theses accepted by the department. Scientific life was developing at Orsay, with advanced courses and seminars, and a mathematical library that soon became quite good.

**1968-1969**

In 1968 the Faculté des Sciences de Paris moved to its new location, the quadrangle of Jussieu. Meanwhile Orsay developed all branches of science. The cooperation-competition was supervised by the two deans, Marc Zamansky and Georges Poitou, both mathematicians.

Occasionally there were common meetings of all professors of both Facultés. The joint mathematics department met regularly at Institut Henri Poincaré.

The student uprising of May 1968 was an explosion in Paris and a happening at Orsay. As a sign of solidarity with the students, Laurent Schwartz and Henri Cartan initiated a dangerous action, namely individual resignations from their academic positions. This act was emulated by a number of professors and became a kind of dramatic collective resignation during a joint Paris-Orsay meeting chaired by the dean, Zamansky. Zamansky disapproved and left the meeting; Poitou took the chair and collected all papers and signatures and did what had to be done, that is, nothing. The action had taken place, and the danger, having the resignations accepted, was over.

Poitou had a special style as dean and grand views about the future of universities. The same can be said about Zamansky, except that the style and the views were mutually opposed. It was agreed by everybody that the old universities, as empty shells, were dead. The dead body of the enormous University of Paris had to be replaced by new and living universities. For Zamansky there was no doubt: the Faculté des Sciences de Paris had to become the Scientific University of Paris. For Poitou all new universities should be multidisciplinary, and he acted in order to include Orsay in a much larger thematic and geographic domain that became the Université de Paris-Sud (now Université Paris-Sud). Zamansky failed, and the Faculté des Sciences de Paris split into two parts, which became the scientific parts of Université Paris 6 (now Université Pierre et Marie Curie) and Université Paris 7 (now Université Denis Diderot). The splitting involved different factors, including political and personal antagonisms. Mathematicians suffered from strong and partly artificial oppositions (now forgotten). Chevalley chose a new university, Paris 8-Vincennes, and Cartan chose Orsay.

Of course Cartan was more than welcome at Orsay and also at the new Université de Paris-Sud. A constitutive council of the university was created, elected by the different sections of the new university; that is, medicine at Kremlin-Bicêtre and other important hospitals and research laboratories; pharmacology at Châtenay-Malabry, with the greatest number of pharmacology students in France; law in Sceaux; institutes of technology in different branches at different places: Sceaux, Cachan and Orsay, with Orsay becoming the scientific center of the new university. Cartan was elected a member of this constitutive council. Then the first meeting of the council elected him as president. Though he didn't keep this position very long, his academic experience and the unique way he had of chairing a meeting were highly appreciated: he was always exquisitely polite and perfectly clear. Among the members of the council was the director of the Institute of Technology of Sceaux, Mrs. Alice Saulnier-Seité, a trade-unionist at the time, who became minister of the universities a few years later. There were strong personalities, opposing views, bright speakers, and challengers, and Cartan proved a perfect helmsman of this moving boat.

**1969-1975**

As soon as he joined Orsay, Cartan took part in all aspects of the life of the mathematics department. By the way, the math building at Orsay, bâtiment 425, is named Mathématique, without an "s", as in Bourbaki "Éléments de mathématique". Cartan could feel at home.

The scientific environment was, and still is, quite favorable: Institut des Hautes Études Scientifiques (IHÉS) at Bures-sur-Yvette, a large campus of CNRS (National Center of Scientific Research) in Gif-sur-Yvette, several engineering schools in the immediate neighborhood (École Supérieure d'Electricité, École Supérieure d'Optique, and, in the mid-1970s, École Polytechnique). The IHÉS is within walking distance of the math department, and Cartan kept a strong relationship with permanent and visiting members of this institute.

We had several foreign members in the math department. They were invited on a temporary basis, and many of them wanted to settle in France and become French citizens. The same was true for some of the research people working at IHÉS. This was never easy and sometimes very difficult. Cartan got involved in the more difficult or urgent cases; he made the necessary steps and proved efficient in a sometimes incredible way.

The scientific life in the math department was organized around the library. From the very beginning we had decided to use all the funds we had for buying books and collections. We had obtained substantial help from the CNRS and a large endowment from the State through an occasional
resource called 5e Plan (the French policy included at that time some large programs, called Plans).

We had used this fund very carefully, but it came to its end as Cartan arrived at Orsay. We had an ordinary allowance for mathematics from the university, but we needed more. Cartan was a regular user of the library, and he appreciated the policy we had in this matter. Together with Gustave Choquet, who later moved from Paris to Orsay, he collected the information about the holdings and needs of the library, which he presented to the newly elected president of the University, Bernard Picinbono. Picinbono was convinced, all relevant committees and councils were equally convinced, and the library was saved.

Actually the needs of the library were also the

first reason to organize the scientific life in a new way. We had a strong but informal link with CNRS, and the policy of CNRS (a very good policy) was to favor formal associations between CNRS and university laboratories or research teams. From the university point of view the math department was a laboratory; for purely financial reasons it appeared that it would be advantageous to have associated research teams. We formed research teams in harmonic analysis, topology, number theory, numerical analysis, and probability. Cartan was a member of the topology team, directed by Jean Cerf. Not only did he play an important role in the recruitment of topologists, but he perfectly understood the need to favor the fledgling teams in applied mathematics, numerical analysis, and probability and statistics. This enlargement of the mathematical field proved essential to maintaining and improving the scientific level of the department.

Cartan was a teacher. He had been the teacher of generations of mathematicians before moving to Orsay. In Orsay he had ordinary university students, and he taught classical subjects. The ordinary students of this time were actually quite decent students, interested in mathematics, interested in their studies, and very demanding of their teachers. Cartan impressed them and made them work. He, too, was very demanding of students and collaborators, checking everything with his personal style, blending rigor and irony. On the other hand he was extremely kind and thoughtful toward students and collaborators when they had personal difficulties. As a mathematician his figure was severe, but he was sensitive to other aspects of life and more than friendly as a human being.

His course was called "Algebra and Geometry". It included hyperbolic geometry and some important theorems in number theory (the Dirichlet theorem on prime numbers, for example). There was a mimeographed version of the course, but it was never published. His style of exposition was not what could be expected from a founder of Bourbaki. He wanted things to be understood in

the most intuitive way. Examples (recollected by one of his assistants): an affine space is nothing but a linear space that lost its origin; or, a homographic transformation in the plane is simply a dilation/rotation (or a translation), if you place the fixed points at a right place.

When Cartan moved to Orsay, he was president of IMU, the International Mathematical Union. He chaired the Fields Medals Committee for the Fields Medals awards of 1970 (Baker, Hironaka, Novikov, Thomson). Just before retiring he was elected a full member of the French Academy of Sciences (he had been a corresponding member for nine years).

After the Retirement

Henri Cartan retired on October 1, 1975.

A symposium in his honor, on analysis and topology, was organized in the main amphitheater of Orsay on June 18, 19, and 20, 1975. I had been elected president of the university one month earlier, and it was the first scientific event that I attended in this capacity. A special booklet was issued by the university with the general nontechnical speeches, and the scientific contributions were published by Astérisque.

Cartan stayed on as emeritus in the department; he only had to share his office, and his name remained on the door until he died. He continued to use the library. For some time he was the only mathematician at Orsay who was a member of the Academy of Sciences. He was very active as an academician. The Academy of Sciences still applies an unwritten rule, la règle de Cartan, for electing new members: the number of people to be considered should be strictly larger than twice the number of available positions. He took part in a most important decision of the Academy: to decrease the average age of the members of the Academy, a number of new members are required to be younger than fifty-five at the time of
Cartan with a granddaughter, Dolomieu, August 1977.

He took part in every meeting as far as he could, and even after he was 100 he never failed to send a proxy when important votes were going to occur. I had the privilege of being called to the telephone on these occasions and hearing his firm, clear voice.

He was awarded the Médaile d'Or du CNRS in 1976 and the Wolf Prize in 1980, as well as being a member of a number of foreign academies. A celebration of his 100th anniversary took place in Paris. The most recent celebration was organized at Orsay, when the main amphitheater of the university was named Amphithéâtre Henri Cartan.

Of course, Cartan didn't need this in order to be remembered in Orsay. He had been one of the founders of the university and a kind of father of the mathematics department. We are proud to be part of his heritage.

Max Karoubi

Some Souvenirs of Henri Cartan

The first time I met Henri Cartan was at the École Normale Supérieure in Paris, when I was admitted in 1959. Despite excellent results in previous mathematics studies, the contact with the École Normale was a shock to me, as for most students of my generation. Actually, according to French tradition, we spent a lot of time at the École Normale, trying to understand Bourbaki's books, although we had no research experience...

Personally, I was not very successful at this exercise and largely preferred to attend Cartan's lectures, either at the École Normale or at the University of Paris. Indeed, Cartan was the best professor I ever knew. He had a unique way of captivating the audience from his first words, by the depth of his knowledge, of course, but also by his eloquence and perfect French. Nothing was left in the dark, no proof was omitted. It was clear that everything was thought out, not only in the details, never boring, but more importantly in the research on the most elegant way to prove theorems. At the end of his lectures, we all felt more clever!

Cartan was also supervising carefully the math studies of everyone, and we often had the opportunity to meet him during our four years at the École Normale. This was particularly important at the end of this period, when we started a research project. Then I naturally applied to a position at the CNRS (Centre National de la Recherche Scientifique), the analog of a "predoc" position today. Unfortunately, the application was rejected, probably because my research records were too thin... Cartan was surprised by this bureaucratic decision, not only for me, but also for other rejections he had heard about. He had the conviction that young students should not miss a chance to start scientific research if they were willing to.

What happened then gives an idea of his prestige and influence: the French administration shifted gears over the following days, after some vigorous phone calls from Cartan to high rank officials in the government!

After this event, I had no other choice than starting to work seriously on a Ph.D. As a matter of fact, Cartan did not suggest any research project; his point of view (shared by some university professors) was simple. A student really interested in math should be able to raise problems according to his own taste, solve them, and, if sufficiently clever, write his Ph.D. after their solution... The role of Cartan was to check the interest of the questions involved and the correctness of the proofs.

This situation was at the same time stressful and challenging. Therefore, I started to look at many books; among them was Marston Morse's *Calculus of Variations in the Large*, a basic reference for the subject I was interested in. Reading this book was for me like swimming across the Atlantic Ocean, since I was not mature enough to understand the deep connections between differential geometry and algebraic topology.

Being Cartan's student saved me again. Indeed, I attended the 1963/1964 Cartan/Schwartz
seminar about the Atiyah-Singer index theorem [2]. Its purpose was not only to understand the proof of this remarkable achievement but also to offer an opportunity for the students to approach exciting new research. The seminar was organized so that each student had the responsibility of some "exposés" which Cartan wanted written the "right" way. For instance, I was in charge of the exposés 4 and 5, and this was all right. However, Cartan was not satisfied with the exposé 16 I was in charge of, too. He decided to rewrite it completely (under my name). Actually, Cartan himself wrote many exposés during the years his famous seminar was running.

This Cartan/Schwartz seminar was the starting point of my research. After this experience, I did exactly what Cartan asked us to do: raised my own questions and tried to solve them. The new field of $K$-theory was a fascinating one (and still is), after the fundamental work of Grothendieck, Bott, Atiyah, Singer, Hirzebruch, and others. Many questions were left over, like the relation between Clifford algebras and Bott periodicity, the possibility of deriving the $K$-functor in order to get cohomology theories on a wide class of objects... During this period (1964–65), I had the chance to meet Grothendieck at the IHES. Although I was not formally his student, he helped me a lot, together with Cartan, to get my Ph.D. finally written. This was not finished, however: Cartan was asking for a "second thesis", which meant a different subject I should be able to master! Serre was kind enough to recommend to me a book about modular forms, a course he was giving at the Collège de France.

The positions at the CNRS were not supposed to be permanent (this has now changed). Therefore, after my thesis in 1967, I applied for a position at the University of Strasbourg, where Cartan was professor before the war and a little bit after. At that time, and also in 1972, when I applied for another position at the University of Paris, Cartan wrote a letter of recommendation. Of course, I don't know the contents of these letters, but from the decisions of the committees, I believe that the recommendations were positive in both cases. This is leading me to the special filial relationship between Cartan and his former students. He was ready to help when he had an opportunity, such as giving lectures about our work, so that we could get some recognition. For instance, when in Princeton during the years 1966–67, Cartan gave a lecture about my thesis before I visited the United States myself. He also gave a talk at Bourbaki's seminar about my work, as he did previously for others: Koszul, Douady, and so on. After this period, he was always inquiring about my research advances, reading my notes in the Comptes Rendus, asking about my own students, and this continued until the last years of his life.

Cartan influenced my mathematics directly much later than my thesis, not just in the field of $K$-theory but also in another aspect of my research linked with classical algebraic topology. The story started from a lecture Grothendieck gave at the IHES about a possible generalization to the integral case of the Quillen-Sullivan theory of rational homotopy types. Cartan wrote a short paper in Inventiones (in honor of Serre) about this subject [1]. In particular he wrote this sentence: "L'auteur expose ici ce qu'il croit avoir compris lors d'une conférence de Grothendieck à l'IHES le 12 décembre 1975."

Despite this modesty, I found Cartan's paper very inspiring and going beyond what Grothendieck explained in his lecture. Although the problem of algebraically determining integral homotopy types is still open and difficult, the way Cartan presented it was the origin of many of the papers I wrote the following years, for instance, the recent reference [4].

It is important to underline the deep European convictions of Cartan, after the two wars that were a devastation in Europe and caused so many deaths (including that of Cartan's own brother). I had the opportunity to work with him in more favorable circumstances when the first European Congress of Mathematics was launched in Paris during the year 1992. It had a difficult birth, due to the skepticism of many influential French mathematicians. I deeply think that, without Cartan's support, this Congress, and the ones which followed it, would have had no chance to take place.

As a conclusion, I would like to add some words about the affability of Cartan and his wife, unfortunately deceased six months after him. When invited a few times to their homes, in Paris or Dolomieu, and also on many other occasions, I was greeted so warmly that I had the feeling of being part of the family circle.

We have lost a great mathematician and a man with a great sense of human values.

References

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14 The author explains here what he thought he has understood from a lecture of Grothendieck at the IHES on December 12, 1975.
15 In Budapest, Barcelona, Stockholm, Amsterdam...
Jean-Pierre Bourguignon

Remembering Henri Cartan, a Highly Influential Mathematician, a Passionate Advocate for Europe and Human Rights

Henri Cartan died on August 13, 2008, at the age of 104. His professional life had been extremely full, with many commitments, some strictly mathematical and others addressing more general societal issues.

His scientific achievements, and in particular his involvement in the birth and development of the Nicolas Bourbaki group, will be presented and discussed elsewhere.

His role as a teacher at the Université de Strasbourg, both in Strasbourg and in Clermont-Ferrand, where the university moved during the war, then at the Université de Paris, and most notably at the École Normale Supérieure (ENS), left a long-lasting impression on the many mathematicians who attended his lectures. The seminar that he organized and forcefully led at ENS in the 1950s has become legendary. His role in shaping a new generation of mathematicians cannot be underestimated, as he both attracted exceptional people and offered them the most advanced teaching, while orienting them towards worthwhile and challenging problems.

I personally followed his famous "Introduction à la topologie algébrique" course in 1967-68 but had to do it at a distance, as I was at that time a student at École Polytechnique with a busy schedule there.

Henri Cartan spent the last years of his career in the early 1970s as professor in Orsay at the mathematics department of the newly founded Université Paris-Sud.

He kept informed about what was happening in the mathematical community at large up to the very end of his life. Getting a few words from Henri Cartan on one of his personal cards was always moving and a delight because of the care taken in the wording.

A first manifestation of Henri Cartan's public concern for the free circulation of scientists occurred in connection with the International Congress of Mathematicians held in Boston in 1950. The visa application Laurent Schwartz had made to attend the ICM, where he was to receive the Fields Medal, had been set aside by the U.S. Embassy in Paris. In order to exert maximum pressure, Henri Cartan collected the passports of all the French ICM participants and threatened that there would be no French participation if Schwartz was not allowed to enter the United States. Schwartz received his visa at the very last minute, but still in time for the French delegation, led by Henri Cartan, to take the boat to Le Havre to cross the Atlantic.

Later, in 1974, he, Schwartz,1 and a few concerned mathematicians engaged in the defense of a number of mathematicians prosecuted by their governments, such as Leonid Pliouchtch, Andrei Chikhanovitch, and Anatoli Chitcharanski in the Soviet Union, José Luis Massera in Uruguay, and Sion Assidon in Morocco. All kinds of pressures were exerted, and in the end the action of the Comité des Mathématiciens proved remarkably successful.

Later in this article, two eminent German mathematicians discuss Henri Cartan’s remarkable contributions to the German-French cooperation in difficult times, and in particular his (communicative) determination to restore the flow of exchanges right after the Second World War. His scope was broader than German-French relations and embraced Europe as a whole. He in particular tried and set the practical foundations of an academic Europe by ensuring that students would be able to move from one institution to another while progressing in their studies.

He made a very public political stand for Europe through his engagement in the “Mouvement Fédéraliste Européen”. This led him to become a candidate for the European Parliament.


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Human Rights

Jean-Zélie Bounanfong

[Text continues on the page]
Reinhold Remmert

Henri Cartan 1904–2008

In December 1949 Henri Cartan came to Münster for the first time after World War II. I was a freshman. Heinrich Behnke encouraged me to attend Cartan’s lecture. I went out of curiosity. The speaker discussed his forthcoming paper on ideals and modules of holomorphic functions, Oeuvres II, p. 618. I understood nothing. However, I felt as if I were in good company. After the talk there was a reception. Cartan concluded his short address (in German) with the toast “À l’Europe!” I must have looked like a doubting Thomas.

In 1952 Cartan became Doctor Honoris Causa of the University of Münster. This was his first honorary degree. In his words of thanks he pleaded strongly for the reconciliation of scientists on both sides of the Rhine.

In 1953 Karl Stein attended a conference on several complex variables in Brussels. Cartan and Serre presented their Theorem A and Theorem B for Stein manifolds to a dumbfounded audience. Back in Münster, Stein said to me: "The French have tanks. We only have bows and arrows." ("Die Franzosen haben Panzer, wir nur Pfeil und Bogen.")

Complex manifolds with many holomorphic functions were baptized “variétés de Stein” by Cartan. In the late 1950s Cartan teased Stein at a conference in Oberwolfach: "Cher ami, avez-vous aujourd’hui une variété de vous dans votre poche? (Dear friend, do you have one of your varieties in your pocket today?)" Stein looked embarrassed and said: "I never use that expression." Cartan advised him to circumvent the notation by using a variation of a well-known phrase of Montel: "... les variétés dont j’ai l’honneur de porter le nom (... the varieties whose name I have the honor of bearing)."

Henri Cartan was on very friendly terms with Heinz Götze, the wizard of Springer Verlag, Heidelberg. Both men were extremely pleased when, in 1979, the Oeuvres of the French mathematician Cartan were published by the German publishing house. During the ceremony at La Tour d’Argent, where the leather-bound volumes were presented, numerous jubilant toasts à l’Europe were given.

In 1981 Götze suggested having the famous ten papers by Kiyoshi Oka edited. I asked Cartan for advice. He immediately agreed to write commentaries. Later he told me that he enjoyed doing this, however completely underestimated the work involved.

The last time I met Cartan was in 1997 in Paris at the “Journée en l’honneur d’Henri Cartan”. We talked about bygone years and his friendship with Behnke and Stein.


Friedrich Hirzebruch

Henri Cartan 1904–2008

I met Henri Cartan for the first time in Oberwolfach in 1951. We met for the last time during the celebration of his 100th birthday in Paris 2004. I gave a lecture with the title "Henri Cartan: A great friend, mathematician, and European". I shall use the part of this talk that does not overlap Remmert's report.

On the occasion of Behnke’s eightieth birthday on October 8, 1978, celebrated in Münster, Henri Cartan gave a beautiful dinner speech. We were all sad that Heinrich Behnke unexpectedly could not attend the dinner because of illness. He died

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one year later. Cartan's dinner speech was printed by Springer-Verlag under the title "Quelques souvenirs par Henri Cartan". In his speech Cartan recalled his first visit to Münster in 1931. Behnke, a young professor, then thirty-two years old, had decided to make Münster an active and interesting center for the young people around him. For this purpose he had invited a young French mathematician of twenty-six years having related interests who gave four lectures in German and one in French during his one-week visit. Cartan met Peter Thullen, and this was the beginning of a scientific cooperation and long-lasting friendship. Cartan reported also about his second visit to Münster in 1938. In the meantime the famous Ergebnisse-Bericht (Springer-Verlag) by Behnke and Thullen had appeared in 1934. Thullen had left Germany. The political atmosphere was depressing. There were not many students. But still mathematics went on. Behnke's assistant was Karl Stein, who had received his Ph.D. degree in 1936.

During the war, the friendship between Cartan and Behnke was not interrupted; Behnke, for example, received a mathematical letter from Oka in December 1940 and informed Cartan about it. In 1943 Cartan's brother Louis was deported to Germany. About this tragedy, Cartan says in "Quelques souvenirs" addressed to Behnke:

"Je ne puis pas non plus oublier toutes les démarches que vous avez faites durant les années 1943 et 1944 (en vain, hélas) pour tenter de retrouver la trace de mon frère Louis, déporté en Allemagne au mois de février 1943, et qui ne devait jamais revenir. (I cannot forget, too, all the efforts you made during the years 1943 and 1944 (in vain, alas) to try to find any trace of my brother Louis, who was deported to Germany in the month of February 1943, and who never returned.)"

Already in 1946 Cartan came to Oberwolfach, where he met Behnke again after eight years. The Oberwolfach guest book records that Cartan participated in a concert (Haydn, Bach, Beethoven) on November 1, 1946, and lectured on Galois theory for noncommutative fields on November 4, 1946. In this way Cartan began his efforts to reconcile the mathematicians on both sides of the Rhine (cf. Remmert's contribution).

Cartan was always interested in the work of Behnke and his students, in particular Stein, Grauert, Remmert, and myself.

For Stein's sixtieth birthday (1973) Cartan lectured at a conference in Munich and wrote an article "Sur les travaux de Karl Stein". He reported in particular about Stein's Habilitationsschrift (1940), which concerns Cousin's second problem. The title (translated into English) is: "Topological conditions for the existence of holomorphic functions with a given zero divisor". This is related to the famous Theorem B of Cartan and Jean-Pierre Serre.

Cartan reported about my thesis (written under Behnke and Hopf) in the Bourbaki seminar of December 1953. In the thesis I had introduced complex spaces of dimension 2 and described the resolution of their singularities.

In his Habilitationsschrift, Hans Grauert proved that, for a Stein manifold $X$ and a complex Lie group $L$, the classification of topological principal fiber bundles over $X$ with structural group $L$ coincides with the classification of analytic principal fiber bundles over $X$ with structural group $L$. This includes the solution of Cousin I and II ($L = C$ or $C^*$, respectively). Grauert published his work in three parts in Mathematische Annalen in 1957 and 1958 and thanked Cartan for advice. Cartan lectured on
Grauert’s results in the “Symposium Internacional de Topologia Algebraica, Mexico 1956”.

In his contribution Remmert shows that Henri Cartan was a real European. I want to emphasize this by the following remarks:

The first European Congress of Mathematics took place in Paris from July 6 to July 10, 1992. In his opening speech, Cartan calls the congress an event of great importance showing that the mathematicians know the solidarity of the countries of Europe, which are different in so many ways but have a rich common heritage and a common future. Cartan was especially glad that this first European Congress reunited the mathematicians from the two parts of Europe that were separated for such a long time. Cartan’s eighty-eighth birthday was celebrated during the first European Congress at the residence of the German Ambassador in Palais Beauharnais.

The Association Européenne des Enseignants (European Association of Teachers) was founded in Paris in 1956. Cartan was president of the French section. As such he took the initiative to invite participants from eight European countries to a meeting in Paris in October 1960. Emil Artin, Heinrich Behnke, and I were the German members. The second meeting of this committee was in Düsseldorf in March 1962. As a result, the Livret Européen de l’Etudiant (European Student’s Record) was published and distributed by the European Association of Teachers. The booklet contained a description of minimal requirements for basic courses. It was supposed to increase the mobility of students from one country to another. The professor of one university would mark in the booklet the contents of courses attended by the student. The professor at the next university would then be able to advise the student in which courses to enroll. The booklet was not used very much. For me it was often useful when reforms of the contents of courses were discussed.

The efforts of Cartan to harmonize mathematical studies in Europe date back more than forty-five years. Now we are implementing the Bologna process. In all European countries bachelor’s and master’s degrees are to be introduced. Is this the harmonization we wanted?

Cartan, the European, was also active at the international level. He was president of the International Mathematical Union for the four years 1967 to 1970. He addressed the International Congress of Mathematicians in Nice in 1970 during its opening ceremony and announced the names of the Fields Medal winners. Sergei Novikov, one of the four winners, was unable to attend, indicating the political difficulties of the time.

The mathematicians of my generation, from Germany and everywhere else, learned from Henri Cartan. His papers, books, and seminars were a source of inspiration. He showed us the right way of developing international cooperation. He and his wife were charming hosts for many visitors to Paris. He has left us, but we will always remember him with gratitude.