"WHY MATHEMATICS?" YOU MIGHT ASK

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…it seems to me that they have a poor opinion of our religion if they think it needs the protection of philosophy.
Lorenzo Valla, *Dialogue on Free Will*

André Weil, speaking at the 1978 International Congress of Mathematicians at Helsinki, concluded his address entitled "History of Mathematics: Why and How?" with these words: "Thus my original question 'Why mathematical history?' finally reduces itself to the question 'Why mathematics?', which fortunately I do not feel called upon to answer."[1] I heard Weil's address, and the applause that followed, and remember imagining circumstances in which that final question could not be so easily evaded. The House Committee on Science, Space, and Technology, for instance, in 1991 called upon the AMS to answer a very similar question: "What are the main goals in the mathematical sciences?" "No truth without money", wrote French philosopher Jean-François Lyotard, reading the final section of Descartes' *Discourse on Method* as a kind of research grant application.[2] Weil knew his audience, and the committee of twelve mathematicians responding to the government body responsible for research budgets knew theirs.

> "The most important long-term goals for the mathematical sciences are: provision of fundamental tools for science and technology, improvement of mathematics education, discovery of new mathematics, facilitation of technology transfer, and support of efficient computation."[3]

"Meaning is what makes things sell," wrote Roland Barthes,[4] and the AMS adopted the posture of Fourier who, according to a celebrated comment of Jacobi,

> "…had the opinion that the principal aim of mathematics was public utility and explanation of natural phenomena; but a philosopher like him should have known that the sole end of science is the honor of the human mind, and that under this title a question about numbers is worth as much as a question about the system of the world."[5]

Though the AMS seems to have left "honor" a niche in the third goal, the fine print again directs the reader to "unexpected" applications of pure mathematics.

Few pure mathematicians are as indifferent to practical applications as G. H. Hardy, who in *A Mathematician's Apology* famously claimed that "Judged by all practical standards, the value of my mathematical life is nil…." But it's fair to assume that, when addressing

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1 Proceedings of the ICM, Helsinki, 1978, pp. 227-236, quotation p. 236.
2 Lyotard, *La Condition Postmoderne*, Minuit (1979), pp. 73-74.
3 From *Pilot Assessment of the Mathematical Sciences*, prepared for the House Committee on Science, Space, and Technology. Notices of the AMS, vol 39 (1992), 101-110).
4 *Système de la Mode*, Paris (1967), p. 10.
5 C.G.J. Jacobi, letter to Legendre, July 2, 1830, in *Gesammelte Werke*, Vol. I, Berlin (1881), p. 454.

one another, rather than government committees, most pure mathematicians, including those who represented the AMS in 1991, would choose a quite different list of "most important long-term goals."

In this they have long been able to count on the protection of philosophy. It has been a commonplace since Plato ("Let no one ignorant of geometry enter" his Academy) to grant mathematics intrinsic value on metaphysical grounds.[6] The topos of mathematics as a source of certain knowledge was already well established by the second century, when Ptolemy wrote[7]

> "Only mathematics, if one attacks it critically, provides for those who practice it sure and unswerving knowledge, since the demonstration comes about through incontrovertible means, by arithmetic and geometry."

The "crisis of foundations" of the early twentieth century, culminating in Gödel's incompleteness theorems, was largely motivated by the hope to make mathematical certainty safe from dependence on human frailty. As Bertrand Russell wrote in *Reflections on my Eightieth Birthday*:

> "I wanted certainty in the kind of way in which people want religious faith. I thought that certainty is more likely to be found in mathematics than elsewhere.… Mathematics is, I believe, the chief source of the belief in eternal and exact truth.…"[8]

Russell's hope to ground certainty in logic is largely a thing of the past — as Marvin Minsky wrote in another context, "without an intimate connection between our knowledge and our intentions, logic leads to madness, not intelligence" [9] — but his words continue to echo. After being named first recipient of the Abel Prize, Jean-Pierre Serre, was quoted in *Liberation* to the effect that mathematics is the only producer of "totally reliable and verifiable" truths[10]. And Landon T. Clay III, announcing the creation of the $7,000,000 Millennium Prize Fund, linked his decision to devote much of his personal fortune to the support of pure mathematics to "the decline in religious certitude … the pursuit of proof continues to be a strong motivating force in human actions."[11]

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[6] The present essay is mainly concerned with metaphysical certainty. Descartes wrote in *Principles of Philosophy*, CCVI, of "certainty …founded on the metaphysical ground that, as God is supremely good and the source of all truth, the faculty of distinguishing truth from error which he gave us, cannot be fallacious so long as we use it aright, and distinctly perceive anything by it," and cites "the demonstrations of mathematics" as his first example. Plato saw mathematics rather as a source of "knowledge of that which exists forever" (and, not incidentally, useful in war as well). *Republic*, VII, 522 ff. Certainty and its cognates are some — but only some — of the apparent blessings of mathematics that so impressed certain philosophers as to "infect" the whole of their work, as Ian Hacking argues in *What Mathematics Has Done to Some and Only Some Philosopheers*, *Proc. British Academy*, **103**, (2000), 83-138.

[7] Ptolemy, *Syntaxis*, I, ch. 1 16.17-21, cited in G.E.R. Lloyd, *The Ambitions of Curiosity*, Cambridge University Press (2002), p 137, note 13.

[8] In *Portraits from Memory*, quoted in R. Hersh, *What is Mathematics, Really?*, p. 151.

[9] Minsky, *The Society of Mind*, Simon and Schuster (1985, 1986), 18.1. Compare René Thom's comment in connection with his criticism of attempts to reduce mathematics to set theory: "In attempting to attach meaning to all the phrases constructed in ordinary languages, according to Boolean rules, the logician proceeds to a phantasmic, delirious reconstruction of the universe." Reprinted in T. Tymoczko, ed., *New Directions in the Philosophy of Mathematics*, Princeton (1998), pp. 67-78.

[10] *Liberation*, May 23, 2003.

[11] Transcript of interview by Francois Tisseyre conducted on the occasion of the Paris Millennium Meeting, May 24, 2000, graciously provided by the Clay Mathematics Institute.

The mind saves its honor, but only through indenture to a higher power. I would like to express my opinion that the bargain, placing mathematicians on the front lines in defense of metaphysical certainty or any other normative concern of philosophers, is an unnecessary burden that fails to do justice to what is uniquely valuable about mathematics. It also fails to protect pure mathematics from real existential dangers, of which budget cuts are only the most obvious expression. Mathematics is not in danger of collapse for lack of a coherent account of its certainty; but it may well collapse for lack of an account of its value.

One danger that should not worry mathematicians is that of *postmodernism*, about which many thousands of pages have been written, although it is not clear whether or not such a thing exists. I will nevertheless add a few pages of my own, because the term has come to be used as shorthand for a radical relativism that is imagined to call into question not only certainty but rationality in all its forms[12]. One thus finds mathematicians, skeptical of certainty in Russell's sense, who nonetheless express hostility to something they call "postmodernism" as a defense of reason and the value of mathematics as a rational activity.

Applied to architecture, postmodernism designates a reasonably precise tendency. As a trend defining the spirit of the times, it has been called "the cultural logic of late capitalism", differing from modernism in its emphasis on space rather than time, multiplicity of perspective and fragmentation rather than unity of meaning and totality, pastiche (sampling)[13] rather than progress, and much more along the same lines. As a movement in philosophy it is most typically (if abusively) associated with Michel Foucault, Jacques Derrida, Gilles Deleuze, Barthes, Lyotard, and so-called "French theory" of the 60s and 70s more generally. Postmodern prose is eclectic, ironic, self-referential, and hostile to linear narrative. The variant known as posthumanism celebrates the fading of conceptual and material boundaries between human beings and machines.

We are all postmodernists insofar as we have experienced the degradation of public discourse under the influence of advertising slogans, and are therefore likely in spite of ourselves to read Jacobi's invocation of "the honor of the human mind" as a precursor of that genre. Mathematicians can even claim to be the first postmodernists: compare an art critic's definition of postmodernism — "meaning is suspended in favor of a game involving free-floating signs" — with the definition of mathematics, attributed to Hilbert, as "a game played according to certain simple rules with meaningless marks on paper."[14] Mathematics could nevertheless (or for that very reason) safely ignore postmodernism, were it not that the latter is supposed to have no room for certainty, metaphysical or

[12] For example, George Lakoff and Rafael E. Núñez write of a "radical form of postmodernism which claims that mathematics is purely historically and culturally contingent and fundamentally subjective:" *Where Mathematics Comes From*, p. 363. No examples are given of texts espousing this point of view.
[13] "Because his … artistry comes from combining other people's art… the DJ is the epitome of a postmodern artist." http://www.jahsonic.com/PostModernism.html.
[14] Otto Karnik, in *Attraction and Repulsion*, article in *Kai KeinRespekt*, Exhibition Catalogue of the Institute of Contemporary Art, Boston, Bridge House Publishing (2004), p. 48; the Hilbert quotation is easy to find but is probably apocryphal, which doesn't make it any less significant. *Mathematics and the Roots of Postmodern Thought*, by Vladimir Tasi'c, is an extended speculation on postmodernism's mathematical antecedents; see my review in *Notices of the AMS*, August 2003.

otherwise.[15] So it is not surprising that authors considered postmodernists have had some perplexing run-ins with science and mathematics. Others can connect the dots between Venturi et al.'s invitation to *Learn*[ ] *from Las Vegas* and David Mumford's call[16] for "the dawning of the age of stochasticity;" it is these conflicts that are relevant to the present essay.

The conflicts that have attracted the most attention have hinged on misunderstandings. Geographer David Harvey's Marxist analysis sees postmodernism as a cultural reflection of change in the world economy, the "regime of accumulation," which has dramatically transformed our experience of space and time. Questioning the relevance for social sciences of the physicists' geometric model, Harvey "think[s] it important to challenge the idea of a single and objective sense of time or space, against which we can measure the diversity of human conceptions and perceptions."[17]

More controversial accounts of postmodernism sound like this:

> "Science and philosophy must jettison their grandiose metaphysical claims and view themselves more modestly as just another set of narratives" (Terry Eagleton, quoted in Harvey, *op. cit.*, p. 9)

As far as mathematics is concerned, relativism of this kind has more to do with English-language postmodernism than with the French original. One might have thought that mathematical progress from axioms to theorems and from lesser to greater abstraction or generality constituted a prime example of the sort of "master narrative" French postmodernists regarded with suspicion, and a particularly tempting target, given the special role Enlightenment thinking reserved for mathematical explanation. That seems not to have been the case. The most prominent French philosophers identified as

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[15] For example, "[Derrida's] thought is based on his disapproval of the search for an ultimate metaphysical certainty or source of meaning that has characterized most of Western philosophy." From the *Encyclopedia Brittanica Online*.

[16] Mumford suggests a modification of the Zermelo-Fraenkel axioms of set theory, inspired in part by his decades of research on computer vision: V. Arnold et al., *Mathematics: Frontiers and Perspectives*, AMS 2000, 197-218. In keeping with his quasi-empiricist outlook, Gregory Chaitin has argued in a number of books and articles that axioms for set theory should be adapted to their intended applications.

[17] David Harvey, *The Condition of Postmodernity*, Oxford: Basil Blackwell (1989), pp. 201-205; see also p. 253. I suspect that most of the bizarre references to mathematics and physics by the likes of Deleuze-Guattari and Virilio that so exercised Sokal and Bricmont in *Fashionable Nonsense* (New York: Picador, 1999) were attempts to find an appropriate language to grasp the same cultural transformations. Not particularly clear or successful attempts, but that's another matter. Regarding the controversial role of rhetoric in French theory, Perry Anderson wrote "What is clear is that the hyperbolic fusion of imaginative and discursive forms of writing, with all its attendant vices, was also inseparable from everything that made this body of work most original and radical." in *London Review of Books*, Vol. 26 No. 17, dated 2 September 2004.

I strongly recommend Alain Badiou's short article "Philosophy's French Adventure," in *New Left Review*, Vol 35, Sept/Oct 2005, pp. 67-77. for an account of the ambitions of French philosophy in the second half of the twentieth century — in other words, the period beginning with Sartre, continuing through existentialism, structuralism, deconstruction, and postmodernism, and ending with … Badiou — including its ambivalent relations to mathematics and science, political engagement, and stylistic experimentation. Though Badiou is a perpetrator as well as an observer, and may therefore be suspected of partiality, his article is remarkably clear, concise, and coherent, and perhaps for those reasons convincing. Note especially his remark that "French philosophers sought to wrest science from the exclusive domain of the philosophy of knowledge…" seeing science (and by extension mathematics) as a "practice of creative thought, comparable to artistic activity, rather than as the organization of revealed phenomena," an "operation" that "finds its extreme expression in Deleuze" (pp 70-71).

postmodernists, though metaphysical skeptics in other regards, had no quarrel with mathematics' metaphysical pretensions per se; but they did question their relevance to the human sciences. For Derrida, thinking of Leibniz in particular, "[mathematics] was always the exemplary model of scientificity" and Foucault worried, uncontroversially, that

> Mathematics has certainly served as a model for most scientific discourse in their efforts to attain formal rigour and demonstrativity; but for the historian who questions the actual development of the sciences, it is … an example … from which one cannot generalize."[18]

Conversely, mathematics coexists harmoniously with a methodology strongly influenced by Foucault in Ian Hacking's studies of the "historical ontology" of the concepts of probability and statistics[19].

At least one of postmodernism's canonical French texts does take on the issue of certainty in science and mathematics directly. Alluding to the trilogy of Gödel's theorems, uncertainty in quantum mechanics, and fractals,[20] Jean-François Lyotard saw in contemporary mathematics

> "a current that calls into question precise measurement and prediction of the behavior of objects at the human scale … postmodern science…produces not the known, but the unknown."[21]

Various authors have reminded readers that Gödel's theorems and the Uncertainty Principle (and chaos) are statements about formal systems in mathematics and particle physics (and non-linear differential equations), respectively, and as such have no bearing on metaphysics.[22] The arguments are often eloquent but altogether beside the point, and of little comfort to seekers of certainty like Russell. Metaphysical certainty, whatever it may be, can't be any less binding than a mathematical proof.[23] Gödel's theorem on the impossibility of proving consistency of a formal system within the system can reasonably be taken to mean that metaphysical certainty cannot be guaranteed by mathematical

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[18] Derrida, *Of Grammatology*, p. 27; Foucault, *The Archeology of Knowledge*, pp. 188-89. "Why don't you ask a physicist or a mathematician about difficulty?" was Derrida's response to a 1998 *New York Times* question about deconstruction; see *Jacques Derrida, Abstruse Theorist, dies at 74*, *NY Times*, October 10, 2004. Appeals to the presumed value of even the most abstruse mathematics, in order to legitimate obscurity elsewhere, are common. I first encountered such an argument in an article by composer (and former mathematician) Milton Babbitt, entitled "Who cares if you listen?" (*High Fidelity*, February 1958): "Why should the layman be other than bored and puzzled by what he is unable to understand, music or anything else?" With this sort of talk, the justification of pure mathematics on aesthetic grounds is turned upside down. That's why I only address this possible answer to the title's question — by far the most popular among my colleagues — in a footnote.

[19] *The Emergence of Probability* and *The Taming of Chance*, Cambridge University Press (1975, 1990). The postmodern "brand" was attached to Foucault retroactively but it has stuck, whether or not it is appropriate.

[20] A cliché for the succeeding generation of literary critics: for a sample emphasizing chaos rather than Gödel, see N. Katherine Hayles, ed., *Chaos and Order*, University of Chicago Press, 1991.

[21] Lyotard, *op. cit.*, pp.94, 97.

[22] Much of *Prodiges et vertiges de l'analogie* by Jacques Bouveresse, Raisons d'Agir (1999), is devoted to just this sort of reminder.

[23] For example, Husserl, for whom "the mathematical disciplines" alone "at the present time …could effectively represent the idea of a scientific eidetic," and who remains a primary reference for French philosophy of mathematics, wrote, pre-Gödel, that "*In a mathematically definite manifold the concepts 'true' and 'formal implication of the axioms' are equivalent*": *Ideas*, §§71-72, italics in original. The word "manifold" refers here to a kind of formal system,; "eidetic" has to do with essences and here distinguishes mathematics from the empirical sciences.

means alone.[24] But Serre in his comments to *Liberation* surely meant something more than the tautology that mathematical truth is totally reliable and verifiable *by the standards of mathematics*… The struggle to pin down this "something more," what one might call mathematics' "essence," is what keeps philosophy of mathematics revisiting the scenes of its many past defeats.

Lyotard doesn't do it very well, but a case can be made for the existence of a "postmodern" sensibility in recent science, encompassing everything from Stephen Jay Gould's insistence on the contingent nature of evolution to complexity theory to the study of consciousness as an "emergent" phenomenon. What these developments have in common is a rejection of reductionism and related top-down "master narratives" not because they are wrong but because they are irrelevant and useless. I wouldn't want to insist that this amounts to a new Kuhnian paradigm (the notion is in any case widely criticized as oversimplified), but the science I have in mind doesn't feel like the disciplines that inspired analytic philosophy of science. Although Jürgen Jost has written a book entitled *Postmodern Analysis* and some specialists now claim to be working in "postmodern algebra," I don't see any real sign of this sensibility in pure mathematics itself, [25] where I'm not sure it even makes sense to draw the line between modern and postmodern. Hilbert's definition of mathematics as a game really does sound like something from Derrida, but if Hilbert's foundational program ("wir müssen wissen, wir werden wissen") isn't a prime example of high modernism, I don't know what is. On the other hand, the abandonment of all forms of foundationalism in Tymoczko's anthology *New Directions in the Philosophy of Mathematics* is a rejection of "master narratives" within philosophy of mathematics, and indeed the blurb calls the anthology "postmodern."[26]

While Weil is supposed to have discounted Gödel's metaphysical menace by making it into a joke — "God exists since mathematics is consistent, and the Devil exists since we cannot prove it" — his fellow Bourbakist Dieudonné attempted a counterattack:

> Just as physicists and biologists believe in the permanence of the laws of nature, solely because they have observed this up to now, … the mathematicians called — wrongly — 'formalists' (…at present the near totality of mathematical researchers) are convinced that no contradiction will appear in set theory, none having manifested themselves for 80 years. [27]

This is either an inductive (empirical), sociological or pragmatist argument. All these trends are indeed present in postmodernism, more typically in English sociology of science than in French philosophy:

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[24] Predictably, religion steps in to fill the gap: see http://www.asa3.org/ASA/topics/Astronomy-Cosmology/PSCF9-89Hedman.html#16. John D. Barrow takes the implications of Gödel's theorems for physics seriously, while denying they necessarily limit scientific objectivity; see for instance *Domande senza risposta*, in *Matematica e cultura 2002*, M. Emmer, ed., Springer (2002), pp. 13-24.

[25] It might be argued that experimental mathematics, and specifically the treatment of results of computer experiments as empirical science, is sufficiently novel to require a separate philosophical treatment; but I don't see this as postmodern in the sense I described above.

[26] Tymoczko, *op. cit.*. The anti-foundationalism of this anthology is largely inspired by Gödel's theorems,

[27] Weil's joke is quoted in at least 85 sites found via Google; no primary source is given. Dieudonné's comment is naturally from *Pour l'honneur de l'esprit humain*, Hachette (1987), pp. 244-245. Borel's remarks on the "self-correcting power of mathematics", in his contribution to the discussion of the article "Theoretical mathematics: toward a cultural synthesis of mathematics and theoretical physics" by A. Jaffe and F. Quinn, express a more modest form of pragmatism: *Bull. Am. Math. Soc.* **30**, (1994), p. 180.

> The compelling force of mathematical procedures does not derive from their being transcendent, but from their being accepted and used by a group of people. The procedures are not used because they are correct, or correspond to an ideal; they are deemed correct because they are accepted. (David Bloor, 1983)[28]

There is ample reason to believe[29] that Lyotard's real target was not science and mathematics so much as the notion of political progress (or progressive politics). "Science plays its own game, it cannot legitimate other language games. For example, that of prescription…" [30] (i.e., of guiding political transformation in practice). Mathematics' role here is solely through its applications, as science of prediction in the most general sense; metaphysical certainty is relevant insofar as it guarantees the correctness of predictions.[31] The *Sociology of Scientific Knowledge* (SSK) movement, of which David Bloor was a founder, also has a political agenda, shared with some other branches of the broader trend in sociology and history of science known as *Science Studies*. The agenda is diffuse but revolves around the notion that science is too important in modern societies to allow scientists the last word regarding its meaning as well as its use. [32] Jon Agar, criticizing what he perceived as the defensiveness of Bloor and his coauthors of the 1998 book *Scientific Knowledge*, expressed this agenda particularly crudely:

> "their stance of disinterestedness makes the authors very shy of suggesting that their arguments could be used to criticize science.… The idea that theory is neutral is reminiscent of the American gun lobby slogan: 'guns don't kill people, people do.'"[33]

The combative stance aside, SSK is firmly rooted in postwar philosophy of science in the analytic tradition. The later Wittgenstein's discussion of mathematics and knowledge more generally in terms of "language-games," "forms of life," and learning to follow rules, emphasizes social factors, and SSK is enthusiastically Wittgensteinian. Of course, Wittgenstein's work is notoriously unsystematic and lends itself to a variety of interpretations. I find it wrong to see the Wittgenstein who wrote "Grounds for *doubt* are lacking!"[34] as a skeptic. My reading of Wittgenstein left the impression that, beyond the social factors to which he drew explicit attention, he perceived "something more" specifically in mathematics ("the hardness of the logical must"), to which our language and philosophy are not able to do justice.

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[28] In *Wittgenstein. A Social Theory of Knowledge*. London, Macmillan Press.

[29] See for example John Sanbonmatsu, *The Postmodern Prince*, Monthly Review Press (2004), esp. p. 122 ff,; "L'inquiétude de l'actualité", interview of Foucault by Roger-Pol Droit, in a supplement to *Le Monde*, 19-20 Sept. 2004, p. VIII.

[30] Lyotard, *op. cit*., p. 66. Lyotard and other postmodernists did not see this as a bad thing at all: "The decline, perhaps the ruin, of the universal idea can free thought and life from totalizing obsessions." Quoted in Sanbonmatsu *op. cit*., p. 92.

[31] Eric Hobsbawm defends history against "relativists and postmodernists" as a "rational investigation of the course of human transformations" in terms similar to the Sokal-Bricmont defense of scientific rationality: "Manifeste pour l'histoire," in *Le Monde Diplomatique*, December 2004. In all these debates mathematics is treated, if at all, exclusively as a source of applications, This is where postmodernism becomes relevant to Congressional committees.

[32] One occasionally finds this sentiment in the writings of Michel Callon and Bruno Latour, leading French practitioners of Science Studies. Otherwise SSK has little or nothing to do with French philosophy, and as far as I know Bloor has never called himself a postmodernist. Nor have most (or any?) of the other authors mentioned here.

[33] *Social Studes of Science*, **28/4** (August 1998), p. 651.

[34] *On Certainty*, Basil Blackwell (1969), par. 4; *Philosophical Investigations I*, Blackwell (1958) par. 437.

Can sociology succeed where philosophy failed? Bloor's militant "naturalist" response to the question "whether sociology can touch the very heart of mathematical knowledge"[35] is less an exercise in debunking metaphysics than an attempt to seize the metaphysical high ground for sociology. An otherwise subtle ethnographic study by Claude Rosental of the resolution of a conflict among logicians betrays a similar sensibility, as does his suggestion that training in mathematics and logic might have constituted a "serious handicap" to carrying out his project.[36] The classic declaration of Science Studies' epistemic independence from the science observed is due to Bruno Latour and Stephen Woolgar:

> "…we do *not* regard prior cognition… as a necessary prerequisite for understanding scientists' work. This is similar to an anthropologist's refusal to bow before the knowledge of a primitive sorcerer. There are, as far as we know, no a priori reasons for supposing that scientists' practice is more rational than that of outsiders."[37]

But one can also imagine sociologists paying serious attention to mathematicians' accounts of their experience, addressing in the process the question that Weil did not. In her fieldwork at the Max-Planck-Institut in Bonn, for example, billed as the first study of mathematics from the perspective of constructivist sociology of science, Bettina Heintz also worries about "going native" and "overidentifying with the dominant culture." But her subject is the eminently sociological one of determining how mathematicians reach consensus, and her methodology, far from treating practicing mathematicians as "primitive sorcerers", records their epistemic perspectives sympathetically and at length. One has the impression that, in spite of the limitations of her methodology, Heintz is more interested in accounting for the "real mathematics" to which we return below, whereas Bloor and Rosental are preoccupied with marshalling evidence to counter the metaphysical preoccupations of philosophers.

Under siege from Gödel's theorem, Popper's attack on verificationism, Kuhn's theory of Scientific Revolutions, Lakatos' dialectical approach to the contents of knowledge in *Proofs and Refutations*, as well as Wittgenstein, certainty in Russell's sense has largely been scrapped[38]. As for the needs — social, philosophical, spiritual — the notion of metaphysical certainty was designed to address, they remain. Thus, on the one hand, the tendencies I have described as postmodernist continue to express skepticism regarding certainty, seemingly unaware that their target is now little more than an advertising slogan having little to do with mathematicians' concerns; on the other hand, analytic philosophy has sought to substitute more flexible notions. The term "warrant," for example, is used

[35] Bloor, *Knowledge and Social Imagery*, University of Chicago Press (1976), p. 74.

[36] C. Rosental, *La trame de l'évidence*, Presses Universitaires de France (2003), p. 14.

[37] Latour and Woolgar, *Laboratory Life*, Princeton University Press (1986), pp. 29-30.

[38] Lakatos' posthumous "A Renaissance of Empiricism in the Recent Philosophy of Mathematics," presents a long series of quotations by mathematicians and a few philosophers - including Russell in 1924! – acknowledging that mathematics is uncertain, after all. Naturally, most of those cited refer directly or indirectly to Gödel's theorem. The article was reprinted in T. Tymoczko, ed., *op. cit.*, pp. 29-48. "Only dogma or theory has made people say that mathematics as a whole has a peculiar certainty," writes Hacking in *What Mathematics Has Done…*, *op. cit.*, p. 116. Certainty persists, however, in titles of philosophy books, e.g. Marcus Giaquinto's optimistic *The Search for Certainty: a Philosophical Account of Foundations of Mathematics*.

in Philip Kitcher's influential attempt to develop a consistent account of mathematics on empirical rather than aprioristic grounds. Kitcher recalls Frege's frustration with the mathematicians of his time, observing that "When Frege emphasizes the possibility of complete clarity and certainty in mathematical knowledge, he is advancing a picture of mathematics that is almost irrelevant to the working mathematician."[39] However, Kitcher as well as SSK remain obsessed by the problem of "how our mathematical knowledge [is] acquired,"[40] where knowledge is taken to be true and justified belief. Compare this with the formulation by self-identified Social Constructivist Paul Ernest:

> "The fundamental problem of the philosophy of mathematics is that of the status and foundation of mathematical knowledge. What is the basis of mathematical knowledge? What gives it its seeming certainty, and is this certainty justified?" [http://www.ex.ac.uk/~PErnest/soccon.htm]

Reading Heintz, one learns that now, as in Frege's day, mathematicians themselves widely consider these problems outdated or beside the point.[41] The most controversial aspect of SSK's "Strong programme," formulated by Bloor and Barry Barnes, is the "thesis of symmetry," the insistence that truth or falsity not be taken into account when investigating how a scientific claim comes to be accepted as knowledge. Heintz' fieldwork suggests this is compatible with the view prevailing among mathematicians regarding acceptance of a mathematical proof, a "kind of consensus theory of truth."[42]

A striking instance of "how a mathematical proof comes to be accepted as knowledge" is playing out even as I am writing these lines. Grigori Perelman's announced proof of the Poincaré Conjecture is undergoing unprecedented scrutiny in a small number of specialized centers, with the hope of determining the truth or falsity of Perelman's claim. This is going on quite beyond the spotlight of sociology, as far as I know, and with no guidance from philosophy, even though the $1,000,000 prize offered by the Clay Mathematical Institute is in no sense platonic, and the rules for awarding the prize presuppose the fallibility of the mathematical community, in terms very similar to those Heintz's informants expressed spontaneously.[43] The case is exceptional, however; "certifying knowledge," in Rosental's sense, is as such relatively unimportant to mathematicians, and I suspect Perelman's close readers would describe what they are doing as attempting to *understand* his proof rather than "certifying" it as knowledge (for the sake of the community, or a generous benefactor, or philosophers or sociologists).[44]

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[39] Kitcher, *The Nature of Mathematical Knowledge*, Oxford University Press (1984), p. 269.

[40] Kitcher, *op. cit.* p. 4.

[41] B. Heintz, *Die Innenwelt der Mathematik*, Springer (2000); cf. pp. 137-139.

[42] Heintz, *op. cit.*, p. 178. Heintz quotes Yu. I. Manin – "A proof only becomes a proof after the social act of 'accepting it as a proof'" – as well as René Thom's "community"-theory of truth. One can of course always ask whether Heintz selectively quoted mathematicians whose positions support her thesis. This question can be asked of any sociological study, and it's best to let the sociologists work out their methodological issues. An important remark, however: though Heintz' original goal was to account for the formation of consensus among mathematicians within a science studies framework — with questionable success, but that's another matter — she does not defend a particular school within philosophy of mathematics, In this she differs from Bloor, for instance, who identifies himself explicitly as an empiricist.

[43] See http://www.claymath.org/millennium/Rules_etc/, third and subsequent paragraphs.

[44] "Having shown how the production of certified knowledge in logic could constitute an object of a sociological investigation and analysis, a vast field of research takes shape." Rosental, *op. cit.*, p. 350. I suspect that identifying and accounting for the priorities expressed by mathematicians themselves would constitute a much richer field of research. See also note 62.

"By far the larger part of activity in what goes by the name *philosophy of mathematics* is dead to what mathematicians think and have thought, aside from an unbalanced interest in the 'foundational' ideas of the 1880-1930 period, yielding too often a distorted picture of that time," announced David Corfield, presenting his efforts to develop a "Philosophy of Real Mathematics."[45] Corfield contrasted the traditional apriorist's concerns:

> How should we talk about mathematical truth? Do mathematical terms or statements refer? If so, what are the referents and how do we have access to them?" Corfield, pp. 10-11.

with the list of questions Aspray and Kitcher consider typical of the "Maverick Tradition" in philosophy of mathematics:

> "How does mathematical knowledge grow? What is mathematical progress? What makes some mathematical ideas (or theories) better than others? What is mathematical explanation?" quoted by Corfield, p. 18.

The Mavericks, well represented in Tymoczko's anthology, have moved a welcome step away from certainty. Nevertheless, the philosophers and philosophically-minded sociologists I've mentioned — with the partial exception of Corfield, to be explored below — still often write as though mathematicians were creating Truth or Knowledge, almost as a favor to philosophy or sociology, to show how such a feat is possible[46]. Or just to show it *is* possible. Even Ian Hacking, who could write that "the most striking single feature of [twentieth century philosophy of mathematics] is that it is very largely banal," named mathematics' "astonishing power to establish truths about the world independently of experience"[47] his central concern in commenting on the "mathematical style of reasoning" in the sense of A.C. Crombie. We mathematicians, on the other hand, are quite convinced we are creating *mathematics*, and it is the "why" of that activity, without the ennobling assimilation to the generic objects of interest to epistemology, that, as Weil understood, required no explanation in Helsinki.

"Whoever undertakes to set himself up as a judge in the field of Truth and Knowledge is shipwrecked by the laughter of the gods," wrote Einstein. Mathematicians tend to

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My anecdotal evidence, for what it's worth, suggests mathematicians are generally reluctant to "certify" anything. Some of my colleagues are in the habit of expressing uncertainty in sentences beginning, "If you held a gun to my head…." No one ever completes the sentence "... I would unhesitatingly assert that the proof is correct."

[45] D. Corfield, *Towards a Philosophy of Real Mathematics*, Oxford (2003). "Real mathematics" for Corfield, who is remarkably well-informed about trends in the most diverse branches of mathematics, is "real" in the same way as "real ale. " I readily agree that skepticism to this sort of realism is self-defeating.

[46] Many of the authors in Tymoczko's anthology, (note 36) originally published in 1986 also look to the (real) practice of mathematics for philosophical insight, but Truth and Knowledge keep creeping in. Arriving in France in 1994, I was astonished to discover that the concerns of twentieth century French philosophers of mathematics are entirely different. Following Husserl, the French concentrate largely on the phenomenological experience of the individual mathematical subject. It is only a slight exaggeration to say that the French-language and English-language traditions in philosophy of mathematics have become mutually incomprehensible. Fortunately, mathematicians writing in French and in English have no trouble citing each others' works.

[47] I. Hacking, *Historical Ontology*, Harvard University Press (2002), p. 212, p. 183. I hasten to add that my isolation of this quotation is misleading. More representative of the article is the following sentence: "The truth of a sentence (of a kind introduced by a style of reasoning) is what we find out by reasoning using that style," *op. cit.*, p. 191. For Hacking's philosophy of mathematics, see the article *What Mathematics Has Done…* cited in note 7.

respond with dismay rather than laughter, and then only to blunders so egregious as to be universally recognized as such.[48] Although those who find fault with philosophical speculation regarding the nature of mathematics seem to be under an implicit obligation to propose a speculative alternative, experience suggests that the practice of mathematics renders one unfit to do so. This, more than the fear of ridicule, is the main reason I would not venture my own speculative philosophy of mathematics. If it's hard "for those who are used to thought processes stemming from geometry and algebra" to "develop the sort of intuition common among physicists,"[49] bridging the gap between mathematicians and metaphysicians is probably hopeless. There are superficial parallels, to be sure: a metaphysical abstraction like "essence", like a mathematical abstraction like "set", designates nothing in itself, but in each case refers to a canonical body of specialized texts in which the term in question plays a central role. I would like to argue that the nothing designated by "set" is somehow different, and more fruitful, than the nothing designated by "essence." But the only means at my disposal for making such an argument — available to the "form of life" my training and experience have made me, the "essence," if you like, of my intellectual practice — take the form of mathematical reasoning, application of which to the point I want to make will lead, at best, into a vicious circle.[50] More bluntly, and for reasons akin to those Serre invoked in his *Liberation* interview, I cannot be satisfied with an answer less certain than what mathematics provides; for a mathematician, a merely pragmatist answer to Weil's question is an admission of defeat. And yet I am aware that (metaphysically certain) grounds for distinguishing mathematical certainty from pragmatic certainty are lacking!

Another reason to steer clear of speculation, possibly more profound, is that, whereas philosophy presents itself as a dialogue extending over millennia, each new contribution to which would ideally require attention to all previous contributions, mathematics is in principle supposed to be derivable by pure reason from a small number of axioms. A philosophical proposition, in other words, remains attached to its origins and context; a mathematical proposition floats free. That this principle, an important constituent of the aura of metaphysical certainty surrounding mathematics, bears no resemblance to mathematics as actually practiced — "one of humankind's longest conversations", as Barry Mazur puts it[51] — does not change the fact that what little I know of the philosophical tradition is completely unreliable and that the list of footnotes is primarily the fruit of a random walk (or random surf, or remix) among scraps of the literature I have happened to encounter.

If I am nevertheless writing about philosophy, it is in large part because of a question I was posed in 1995, during a presentation of Wiles' proof of Fermat's Last Theorem to an audience of scientists. An October 1993 article in *Scientific American* entitled *The Death*

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[48] As Serre put it, "Si vous ne voulez pas que les choses soient parfaites, ne faites pas de maths." *Libération*, *op. cit*. Heintz' book is an inquiry into the roots of this apparent universal tendency to consensus, and finds it in the institution of the proof; Rosental's treats a (highly unusual) case in which universal consensus apparently failed. The Einstein quotation is in Morris Kline, *Mathematics The Loss of Certainty*, Oxford (1980), p. 325.

[49] R. MacPherson, quoted in *Quantum Fields and Strings: A Course for Mathematicians*, Vol. 1, p. 2.

[50] "Truth is always the possibility of its proper destruction," according to the (non-postmodern) French philosopher, Alain Badiou, taking Gödel's theorem as an exemple: http://www.egs.edu/faculty/badiou/badiou-truth-process-2002.html

[51] B. Mazur, *Imagining Numbers (particularly the square root of minus fifteen)*, Farrar Straus Giroux (2003), p. 225.

of Proof had called Wiles' proof a "splendid anachronism," citing Laszlo Babai and his collaborators, among others, in support of the thesis that, in the future, deductive proof in mathematics will be largely supplanted by computer-assisted proofs and probabilistic arguments. That same month the *Notices of the AMS* published Doron Zeilberger's manifesto *Theorems for a Price*, predicting a rapid transition from rigorous proofs to an "age of *semi-rigorous* mathematics, in which identities (and perhaps other kinds of theorems) will carry price tags" measured in computer time and proportional to the degree of certainty desired, to be followed in turn by "abandoning the task of keeping track of price altogether, and … the metamorphosis to non-rigorous mathematics."[52]

Feeling called upon to answer the question Weil avoided, I argued that the basic unit of mathematics is the concept rather than the theorem, that the purpose of a proof is to illuminate a concept rather than merely confirm a theorem, and that the replacement of deductive proofs by probabilistic or mechanical proofs should be compared, not to the introduction of a new technology for producing shoes, say, but rather to the attempt to replace shoes by the sales receipts, or perhaps the cash profits, of the shoe factory. The audience had its own question: was I talking about certainty? Of course not. That option has been philosophically discredited, as I have tried to explain; other normative prescriptions fall victim no less easily to the laughter of philosophers. On the other hand, I see no pragmatic reason why probabilistic or mechanical proofs wouldn't suit the five goals on the AMS committee's list as well as deductive proofs, nor any sociological reason why they shouldn't be as effective in commanding consensus in the event of a paradigm shift. So what was I talking about?

Such a question, at this point in the essay, practically begs to be answered by an advertising slogan. For example:

> The practice of making what one writes "reliable and verifiable" fosters critical thinking in general.

This is a popular argument for teaching proofs, and probably even true, but how would one go about verifying such a claim?[53] I am very much tempted to say that the concepts that serve as material for"one of humankind's longest conversations," deserve to be

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[52] John Horgan, *Scientific American*, October 1993, 92-102; D. Zeilberger, *Notices of the AMS*, **40** (1993) 978-981. In the pop posthumanist scenarios promoted by Hans Moravec, Ray Kurzweil, and the like, computers acquire all human capabilities, including generation and proof of theorems — for some reason this is always considered a landmark — by the middle of the 21st century. The distinction between humans and computers subsequently fades away rather rapidly, making Zeilberger's prediction moot.

A more recent, and much more nuanced discussion of prospects for automatic theorem proving has been posted on the internet by Marco Maggesi and Carlos Simpson: *Information Technology Implications for Mathematics, a view from the French Riviera*, at http://math1.unice.fr/maggesi/itmath/ (undated, but apparently not posted before 2004). For the related topic of automated proof checking, see the 1994 *QED Manifesto* (http://www.cs.ru.nl/~freek/qed/qed.html), whose motivations include combating "the degenerative effects of cultural relativism" and "preserv[ing] mathematics from corruption." Interestingly, though certifying correctness of a proof is a primary goal of the authors of the *Manifesto* — a goal shared, for obvious reasons, by Thomas Hales, whose Flyspeck Project has set itself the goal of formal verification of his proof of the Kepler conjecture (http://www.math.pitt.edu/~thales/flyspeck/) — the *Manifesto* explicitly cites aesthetics rather than metaphysics as its "foremost motivation."

[53] Anecdotes abound. Arriving in Ohio at the beginning of the invasion of Iraq, and having been briefly exposed to what passed for television journalism in the U.S. in those days, I was more than relieved to discover that not one of my American mathematical colleagues gave the slightest credence to the official reasons for the war.

appreciated on their own terms. Note that nothing is more "emergent" than a conversation. But that would be unfaithful to the spirit of Mazur's book, one of whose strengths is its refusal to conform to a linear narrative. Anyway, I am aware that a similar argument from tradition can be made in favor of religious faith.

Rather than hazard an answer to Weil's (non-)question here, I will take a cue from Corfield and suggest that one can best account for the value of pure mathematics by attending to what mathematicians write and say. A handful of commonplace words appear consistently, invested with unexpected power, when mathematicians attempt to account, formally as well as informally, for their value judgments that collectively constitute an answer to the question Weil left in suspension.

Hermann Weyl's book "The Idea of a Riemann Surface"[54] refers in his preface to Plato, just as Plato could have written a book entitled "The *eidos* [form] of a square," referring to himself. The word "concept" which was central in my reply to the audience is closer to this use of the term "Idea" as used by any number of philosophers, including most of those mentioned in this essay. A square, or a Riemannian manifold, would be a concept or "Idea" in this sense, and the usage is current among mathematicians, who generally reserve the word "idea" to designate something else. In Plato's *Meno*, the proof of the doubling of the square — draw diagonals and fit the resulting triangles together — which the slave "remembers" under Socrates' coaching, is taken by Plato to be contained in the Idea of the square. For a mathematician, drawing the diagonals and moving the triangles *are* the ideas.

That a contrast can be drawn as I did in 1995 between "illuminating concepts" and "confirming theorems" is something of a truism among mathematicians and even some philosophers. Already in 1950 Popper had argued that "A calculator… will not distinguish ingenious proofs and interesting theorems from dull and uninteresting ones."[55] Corfield wisely takes it for granted that "What mathematicians are largely looking for from each other's proofs are new concepts, techniques, and interpretations" rather than merely "establishing the truth or correctness of propositions" [p. 56]. No less wisely, though he devotes a chapter to the "extremely complex subject" of "mathematical conceptualization," he does not dwell on concepts (or Ideas) as such; nor will I. It's next to impossible to talk in general terms about mathematical concepts without getting caught up in the debate over their reality (and provoking the laughter of the philosophers). Those who write about mathematics[56] have an irritating tendency to claim that most mathematicians are Platonists, whether or not they have committed themselves explicitly to a philosophical position. Maybe it can be (and has been?) argued that mathematics is Platonist in the intentionality expressed in the syntax of mathematical statements; maybe this is what Weil meant by his claim, quoted by Bourguignon, that most mathematicians "spend a good portion of their professional time behaving as if they were" Platonists.[57] In

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[54] Hermann Weyl used the word *Idee* in his title but applied the term *Begriff* (concept) elsewhere in the text. Both terms arrived in English as "concept."

[55] Quoted in Heintz, *op. cit*., p. 176.

[56] Mathematicians included: see R. Hersh, *What is Mathematics, Really?*, *passim*.

[57] J.-P. Bourguignon, "A basis for a new relationship between mathematics and society", in B. Engquist and W. Schmid, eds., *Mathematics Unlimited — 2001 and Beyond*, Springer (2001), p. 176. Plato saw things quite the other way around: "Their language [speaking of mathematicians] is most ludicrous, *though they cannot help it*, for they speak as if they were doing something and as if all all their words were directed toward action." *Republic* VII.527a, my emphasis.

practice I would guess most mathematicians are pragmatists, in the spirit of the remarks of Dieudonné quoted above.

On the other hand, there is no doubt whatsoever that the "ideas" that matter to mathematicians are real. A mathematician, according to a joke attributed to Weil,[58] can be defined as someone who has had two ideas (mathematical, of course). But then, Weil worried, so-and-so would be a mathematician. The climactic event in Poincaré's celebrated account of the role of the unconscious in mathematical discovery was the coming of an idea ("the idea came to me") as he placed his foot on the steps of the omnibus.[59]

More to the point, consider Ian Hacking's justification of his commitment himself to a realist ontology of electrons: "*So far as I'm concerned, if you can spray them then they are real*."[60] By the same token, if you can steal ideas then they are real. Every mathematician knows ideas can be and often are stolen. Polemics then ensue, considerably juicier than the epistemic controversy studied by Rosental.[61]

Nothing in the life of mathematics has more of the attributes of materiality than (lower-case) ideas. They have "features" (Gowers), they can be "tried out" (Singer), they can be "passed from hand to hand" (Corfield), they sometimes "originate in the real world" (Atiyah) or are promoted from the status of calculations by becoming "an integral part of the theory" (Godement).[62] At some point they come into being: it is generally understood, for example, that "new ideas" will be needed to solve the Clay Millennium Problems. (Lower-case) ideas can also be counted. I once heard Serre introduce the proof of a famous conjecture by saying that it contained two or three real ideas. where "real" was intended as high praise. The ambiguity did not concern the number of ideas — there were three, which Serre enumerated — but whether all three were original with the author. Ideas are public: necessarily so, in order to be stolen, or to be presentable as Serre did in his lecture. Poincaré's idea was a sentence ("the transformations of which I had made use to define Fuchsian functions were identical to those of non-Euclidean geometry"); the slave's idea in *Meno* was a line in the sand.

Early in his unpublished memoirs *Récoltes et Semailles*, Grothendieck wrote that "ideas, even dreams" were, in Allyn Jackson's terminology, the "essence and power" of his

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[58] I heard this joke reported by several people who claimed to have heard it from Shimura, and I believe but am not certain I first heard it from Shimura himself.

[59] ""l'idée me vint…" Poincaré, *Science et méthode*, Ch. III, Editions Kimé (1999), p. 49.

[60] I. Hacking, *Representing and Intervening*, Cambridge University Press (1983), p. 23.

[61] This is a value judgment and it deserves elaboration. Rosental's book is largely concerned with accounting for the evolutiion of a controversy regarding the correctness of a purported proof by an outsider of a theorem in fuzzy logic. This is fascinating but to my mind remains within the optic of "certifying knowledge" which, on Rosental's reading, turns out to be more complicated than some philosophers (but not most mathematicians) might imagine. The polemics concerning theft of mathematical ideas, on the other hand, are in my opinion not only "juicier" as material for gossip but also for the questions they raise regarding the nature of what is being stolen.

[62] T. Gowers, *Mathematics. A Very Short Introduction* Oxford (2002), p. p. 135; I. M. Singer, quoted at http://www.abelprisen.no/en/prisvinnere/2004/interview_2004_7.html; Corfield, p. 199; Atiyah, preface to V. Arnold et al., *Mathematics: Frontiers and Perspectives*, AMS (2000); Godement, preface to *Analyse Mathématique I*, Springer (2001).

mathematical work. [63] An idea is typically symptomatic of "insight," and the capacity for insight is generally called "intuition." Mathematicians have borrowed all of these terms from philosophy but use them to completely different ends.[64] Philosophers tend to follow Kant in attributing intuitions — the ones that without concepts are blind — to transcendental subjects or their more down-to-earth offspring. Intuition in this sense is a poor substitute for certainty, even for the Mavericks. "Intuition … is frequently a *prelude* to mathematical knowledge," wrote Kitcher. "By itself it does not warrant belief…" Poincaré called intuition "the tool of invention," a "je ne sais quoi" that holds a proof together, but contrasted it to logic, "the tool of demonstration," that "alone can provide certainty". Saunders MacLane expressed himself in much the same terms nearly a century later. David Ruelle considered reliance on (visual) intuition a characteristic feature of human (as opposed to extraterrestrial) mathematics. [65]

In each case intuition belongs to the *private* sphere, and is relegated to the "context of discovery," as opposed to the "context of justification" deemed worthy of philosophy's full attention. When mathematicians refer to "intuition" in the sense I have in mind, it is crucially *public*,[66] As in the quotation from MacPherson a few paragraphs back, it can be transmitted from teacher to student or through a successful lecture, or developed collectively by running a seminar and writing a book on the proceedings. It has something in common with a "style of reasoning," but on a smaller scale. Grothendieck resorted to perceptual metaphor when describing Serre's ability to communicate something akin to intuition:

> The essential thing was that Serre each time strongly sensed the rich meaning behind a statement that, on the page, would doubtless have left me neither hot nor cold-and that he could "transmit" this perception of a rich, tangible, and mysterious substance-this perception that is at the same time the *desire* to understand this substance, to penetrate it. –*Récoltes et Semailles*, p. 556

"Even those who try to articulate, to classify, the fruits of the imagination, and who are committed to the existence of an inner experience concomitant with it, admit to dark difficulty in describing it," wrote Mazur, elaborating an unusual array of literary and rhetorical strategies to chip away at the difficulty[67]. This much is certain: this inner experience of imagination, or of understanding, is what drives people to become mathematicians, and it is why Weil could count on his audience's silent assent. Heintz recorded some of her informants' attempts to describe this inner experience:

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[63] A. Jackson, "Comme Appelé du Néant--As If Summoned from the Void: The Life of Alexandre Grothendieck," *Notices of the AMS*, October 2004, p. 1052.

[64] A consequence of the relative indifference of mathematics to the philosophical tradition is that mathematicians really are in the enviable position of Humpty Dumpty when it comes to talking about what they do in general terms. Not about the specifics, however.

[65] Kitcher, *op. cit.*, p. 61; Poincaré, *La valeur de la science*, Flammarion (1970), pp. 36-37; MacLane, in Atiyah et al., Responses to: A. Jaffe and F. Quinn, "Theoretical mathematics: toward a cultural synthesis of mathematics and theoretical physics" *Bull. AMS*, **30** (1994), no. 2, 178-207; D. Ruelle, "Conversations on mathematics with a visitor from outer space," in V. Arnold et al. *op. cit*, 251-260. Catherine Goldstein reminds me that the meaning of the word "intuition" has evolved along with its cultural background — in theoretical psychology, for example. The word translates easily from one language to another, the cultural background much less easily. All the more remarkable, then, that the word is used with such (apparent) consistency by contemporary mathematicians.

[66] This is also true of the normative program of intuitionism associated with Brouwer, but that is definitely not what I have in mind.

[67] Mazur, *op. cit.*, p. 43.

[in mathematics] you have concrete objects before you and you interact with them, talk with them. And sometimes they answer you.

She even talks about the "idea" that helps put the pieces together. " And suddenly you see the picture," she was told. Yet all this raw ethnographic data is presented in a chapter whose title — "Beauty and Experiment: Discovery of Truth in Mathematics" — betrays her relentlessly epistemological preoccupations.[68]

"The specific ways that mathematical truths move from person to person, and how they are transformed in the process, are as difficult to capture as the truths themselves," wrote Mazur, in what could have been a comment on Grothendieck's remarks on Serre.[69] The central notion in Mazur's book is that of "imagination." I've chosen the terms "idea" and "intuition" not for their intrinsic importance, though I believe each of the terms points to ways of talking about the famous "flash in the middle of a long night" that ends Poincaré's *The Value of Science*: "But this flash is everything." What strikes me about these terms is how their pervasiveness in mathematicians' conversations — the sense that they, more than the definitive theorems, are "everything" — contrasts so starkly with their near exclusion from philosophical consideration, even though the words themselves can be seen on practically every page of philosophy of mathematics. Maybe their very banality makes them appear philosophically trivial. Or maybe the problem is that the same words serve so many distinct purposes. Corfield uses the same word to designate what I am calling "ideas" ("the ideas in Hopf's 1942 paper", p. 200) as well as "Ideas" ("the idea of groups", p. 212) and something halfway between the two (the "idea" of decomposing representations into their irreducible components for a variety of purposes, p. 206). Elsewhere the word crops up in connection with what mathematicians often call "philosophy," as in the "Langlands philosophy" ("Kronecker's ideas" about divisibility, p. 202), and in many completely unrelated conections as well. Corfield proposes to resolve what he sees as an anomaly in Lakatos' "methodology of scientific research programmes" as applied to mathematics by

> a shift of perspective from seeing a mathematical theory as a collection of statements making truth claims, to seeing it as the clarification and elaboration of certain central ideas… (p. 181)

He sees "a kind of creative vagueness to the central idea" in each of the four examples he offers to represent this shift of perspective; but on my count the ideas he chooses include two "philosophies," one "Idea," and one which is neither of these.

Other value-laden terms are no less important. In the wake of Bourbaki, quite a few philosophers (Cavaillès, Lautman, Piaget, and more recently Tiles) have made serious attempts to make sense of "structure" in mathematics. I've read a number of philosophical attempts to account for mathematical aesthetics, though none has left much of an impression. The practically universal use of dynamical or spatiotemporal metaphors ("the space X is fibered over Y", etc.) , and the pronounced tendency to present proofs as series of actions playing out in time ("now choose an orbit passing arbitrarily close to the point x") have attracted little attention from philosophers.[70] These

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[68] Heintz, *op. cit*, pp. 144-153, The extracts are from a long quotation on pp. 152-153, and on p. 150.

[69] Mazur, *op. cit*., p. 159.

[70] R. Nuñez' article "Do *real* numbers really move?" makes interesting points regarding mathematicians' use of metaphors of motion, though he limits his analysis to examples specifically related to the mathematics of motion: in R. Hersh, ed., *18 Unconventional Essays on the Nature of Mathematics* , Springer (2006), 160-181. Plato's specifically disapproved of mathematicians' use of action metaphors, cf. Note 57.

phenomena may be linked to the curious preference of many mathematicians for blackboards over contemporary audiovisual technology, which in turn draws attention to the neglected (and emergent) aspect of mathematical communication as *performance*, a word that manages to be typically post-modern and pre-modern at the same time.

For his part, Corfield doesn't talk much about "intuition" and is ambiguous about what he means by "ideas," but his discussions of "natural" and "importance," in the context of an analysis of the debate on the relative merits of groups and groupoids, are philosophically insightful while remaining faithful to the use of the terms by "real" mathematicians. His treatment of "postmodern algebra", where "diagrams are not just there to illustrate, they are used to calculate and to prove results rigorously" (p. 254), also has street credibility. It's true that much of his book remains concerned with "Maverick" questions, like accounting for plausible reasoning. I also have mixed feelings about his two chapters devoted to automated theorem proving and conjecture formation, less for the reasons mentioned above — his goal is "useful concept formation" rather than mere truth verification — than for his scanting of the "emergent" approach to artificial intelligence, exemplified by the remarks of robot designer and AI theorist Rodney Brooks:

> To me it seemed that these sorts of intelligence capabilities [chess, calculus, and problem solving]… are all based on a substrate of the ability to see, walk, navigate, and judge…they arise from the interaction of perception and action.[71]

But there is no question that Corfield likes mathematics, and for the right reasons; his book, unlike the normative treatise in philosophy of mathematics, is definitely part of the "conversation."

Morris Kline called the "loss of certainty" entailed by Gödel's theorems an "intellectual traged[y]"[72] and actually counseled "prudence" in designing bridges "using theory involving infinite sets or the axiom of choice." The word "tragedy" seems misplaced but the pathos is real, as it was for Russell. Pathos and its twin, resolute optimism, have found an unlikely home in the philosophy of mathematics:

> If this conception of mathematics [as "human knowledge of structures gained by employing reason beyond the bounds of logic"] can be sustained, mathematics could once again serve as a source of an image of reason liberated from formal imprisonment, freed to confront apocalyptic post-modern visions.[73]

Whether or not it carries weight with congressional committees, I find this goal appealing, but it's a goal for philosophers, not for mathematicians. I'm willing to apply the "principle of charity" to philosophers if they will do the same for me. Corfield wrote:

> Human mathematicians pride themselves on producing beautiful, clear, explanatory proofs, and devote much of their effort to reworking results in conceptually illuminating ways. Philosophers must not evade their duty to treat these value judgments in mathematics. (p. 39).

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[71] R. Brooks, *Flesh and Machines*, Pantheon (2002), p. 37. Not surprisingly, AI has had to come to terms with Poincaré's "je ne sais quoi" in its efforts to model "real mathematics" electronically. I have no objection in principle to robot mathematicians, as long as they are disposed to join humans in the "conversation." Some specific difficulties of conversation with expert systems are reviewed in Maggesi and Simpson, *op. cit.*

[72] Kline, *op. cit.*, pp. 3, 351.

[73] Mary Tiles, *Mathematics and the Image of Reason,* p. 4.

They also have a duty, it seems to me, to account for terms like “idea” and “intution” — and “conceptual” for that matter — used by human mathematicians (at least) to express their value judgments. An answer to the question “Why Philosophy?” might well begin there.[74]

Postscript: In December 2004 my university joined a number of other institutions in France and elsewhere in hosting a traveling UNESCO-sponsored exhibition entitled “Pourquoi les mathématiques?” Hoping to learn the answer before my submission deadline, I spent a few hours at the exhibition, which was clever and engaging, presenting a variety of — pure — mathematical ideas with a sprinkling of practical applications, but in no way addressed the “Pourquoi?” of the title. An organizer was on hand, and when I turned to her for guidance she explained that the French title was a solution to a problem of translation. The English title, which came first, was “Experiencing Mathematics.” This, she assured me, has no adequate French translation, so “Pourquoi les mathématiques?” was chosen as the best substitute.

Maybe the solution to the problem of my title is simply to accept the translation in the opposite direction. Even the most ruthless funding agency is not yet so post-human as to require an answer to “Why experience?”[75]

I thank: Cathérine Goldstein and Norbert Schappacher for pointing me in the directions of the Rosental and Heintz books, among other source material, and for vigorously criticizing my project as well as its execution; Mireille Chaleyat-Maurel for explaining the title of the UNESCO exhibition; Ian Hacking for critically reading an earlier version of the manuscript with tolerance and rigor; David Corfield, for several helpful clarifications; and especially Barry Mazur, for many suggestions, much encouragement, for help with the title, and most of all for showing in his Imagining Numbers… *that there is at least one way out of the fly-bottle.*

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[74] I should at least acknowledge, at Ian Hacking’s suggestion, that if I want to argue that mathematical ideas are more fundamental than theorems, then I need to explain just how mathematical ideas differ from other kinds of ideas — the idea that “ideas are more fundamental than theorems,” for example, or the idea of making such a comparison. Idem for “Ideas.” After giving some thought to the question, and after having considered and rejected as personally unsatisfying the characterization of mathematical ideas on purely historical or purely sociological grounds, I find the question seems to lead inevitably to characterizing Mathematics as such, given that the fact of being centrally concerned with ideas hardly suffices to distinguish Mathematics from other disciplines. It’s probably most prudent to confess to being out of my depth and to fall back on the transparent attempt to pass the buck which ends the paragraph to which this footnote is attached.

[75] Or, as Hermann Weyl put it, “with [mathematics] we stand precisely at the point of intersection of restraint and freedom that makes up the essence of man itself.” Note the word “essence.” From 'The Current Epistemogical Situation in Mathematics' in Paolo Mancosu (ed.) From Brouwer to Hilbert. The Debate on the Foundations of Mathematics in the 1920s, Oxford University Press, 1998, pp. 123-142. I thank David Corfield for this quotation.