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The role of complex conjugation in transcendental number theory

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Abstract.
In his two well known 1968 papers "Contributions to the theory of transcendental numbers", K. Ramachandra proved several results showing that, in certain explicit sets $\{x_1, \ldots, x_n\}$ of complex numbers, one element at least is transcendental. In specific cases the number n of elements in the set was 2 and the two numbers x_1, x_2 were both real. He then noticed that the conclusion is equivalent to say that the complex number $x_1 + ix_2$ is transcendental.

In his 2004 paper published in the Journal de Théorie des Nombres de Bordeaux, G. Diaz investigates how complex conjugation can be used for the transcendence study of the values of the exponential function. For instance, if $\log \alpha_1$ and $\log \alpha_2$ are two nonzero logarithms of algebraic numbers, one of them being either real of purely imaginary, and not the other, then the product $(\log \alpha_1)(\log \alpha_2)$ is transcendental.

We survey Diaz results and produce further similar ones.

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