Seminar on Successful Indo–French S&T Cooperation through CEFIPRA

Indo–French cooperation in mathematics

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Two Indo–French achievements

The most important part of cooperation between France and India in mathematics is constituted by the new results proved by the joint works of mathematicians from both countries. We give two such outstanding results.

Waring's Problem

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Problème de Waring pour les bicarrés. I : schéma de la solution, II : résultats auxiliaires pour le théorème asymptotique, C. R. Acad. Sci. Paris, **303**, (1986), 4, 85–88 & 5, 161–163

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Waring's Problem

In 1770, a few months before J.L. Lagrange solved a conjecture of Bachet (1621) and Fermat (1640) by proving that every positive integer is the sum of at most four squares of integers, E. Waring wrote :

"Omnis integer numerus vel est cubus, vel e duobus, tribus, 4, 5, 6, 7, 8, vel novem cubis compositus, est etiam quadrato–quadratus vel e duobus, tribus, &, usque ad novemdecim compositus, & sic deinceps"

"Every integer is a cube or the sum of two, three, ...nine cubes ; every integer is also the square of a square, or the sum of up to nineteen such ; and so forth. Similar laws may be affirmed for the correspondingly defined numbers of quantities of any like degree."

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$$79 = 64 + 15 = 4 \cdot 2^4 + 15 \cdot 1^4 = 2^4 + 2^4 + 2^4 + 2^4 + 1 + \cdots + 1$$

requires $4 + 15 = 19$ terms.

Previous estimates for $g(4)$

$g(4) \leq 53$ (J. Liouville, 1859)

$g(4) \leq 47$ (S. Réalis, 1878)

$g(4) \leq 45$ (É. Lucas, 1878)

$g(4) \leq 41$ (É. Lucas, 1878)

$g(4) \leq 39$ (A. Fleck, 1906)

$g(4) \leq 38$ (E. Landau, 1907)

$g(4) \leq 37$ (A. Wieferich, 1909)

$g(4) \leq 35$ (L.E. Dickson, 1933)

$g(4) \leq 22$ (H.E. Thomas, 1973)

$g(4) \leq 21$ (R. Balasubramanian, 1979)

$g(4) \leq 20$ (R. Balasubramanian, 1985)

On Waring's Problem : $g(6) = 73$

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- The number $703 = 63 + 10 \times 64$ requires $63 + 10 = 73$ terms. Hence $g(6) \geq 73$.

Estimates for $g(6)$

$g(6) \leq 970$ (Kempner, 1912)

$g(6) \leq 478$ (Baer, 1913)

$g(6) \leq 183$ (James, 1934)

$g(6) = 73$ (Pillai, 1940)

Results on Waring's Problem

$g(2) = 4$	J–L. Lagrange	(1770)
$g(3) = 9$	A. Wieferich	(1909)
$g(4) = 19$	R. Balasubramanian, J–M. Deshouillers, F. Dress	(1986)
$g(5) = 37$	Chen Jing Run	(1964)
$g(6) = 73$	S.S. Pillai	(1940)
$g(7) = 143$	L.E. Dickson	(1936)

Sequence of values of $g(k)$

$1,\ 4,\ 9,\ 19,\ 37,\ 73,\ 143,\ 279,\ 548,\ 1079,\ 2132,\ 4223,\ 8384,\ 16673,\ 33203,\ 66190,\ 132055,\ 263619,\ 526502,\ 1051899,\ 2102137,\ 4201783,\ 8399828,\ 16794048,\ 33579681,\ 67146738,\ 134274541,\ 268520676,\ 536998744,\ 1073933573,\ 2147771272\ldots$

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Neil J. A. Sloane's encyclopaedia
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The value of $g(k)$ is known for $3 \leq k \leq 471\ 600\ 000$.

Serre's Modularity Conjecture

Serre's Modularity Conjecture was proved in 2006 in a joint work by Chandrashekhar Khare and Jean–Pierre Wintenberger :

Let

$$\rho : G_{\mathbb{Q}} \to GL_2(F).$$

be an absolutely irreducible, continuous, and odd two–dimensional representation of $G_{\mathbb{Q}}$ *over a finite field* $F = \mathbb{F}_{\ell^r}$ *of characteristic* ℓ*, There exists a normalized modular eigenform*

$$f = q + a_2 q^2 + a_3 q^3 + \cdots$$

of level $N = N(\varrho)$*, weight* $k = k(\varrho)$*, and some Nebentype character* $\chi : \mathbb{Z}/N\mathbb{Z} \to F^*$ *such that for all prime numbers* p*, coprime to* $N\ell$*, we have*

$$\mathrm{Trace}(\rho(\mathrm{Frob}_p)) = a_p \quad \textit{and} \quad \det(\rho(\mathrm{Frob}_p)) = p^{k-1}\chi(p).$$

Pure and Applied Mathematics
List of projects (Start 1989–1994)

- NUMERICAL MODELLING OF THE OCEAN–ATMOSPHERE SYSTEM WITH SPECIAL REFERENCE TO MONSOONS
  One year (April, 1989 to March, 1990)

- A STUDY OF SOME FACTORISATION AND COMPOSITION PROBLEMS IN GRAPHS
  Three years and six months (September, 1992 to February, 1996)

- NONLINEAR HYPERBOLIC EQUATIONS AND APPLICATIONS
  Three years (March, 1992 to February, 1995)

- GEOMETRY AND NUMBER THEORY
  Three years (February, 1992 to January, 1995)

List of projects (Start 1995–1996)

- ASYMPTOTIC ANALYSIS IN PARTIAL DIFFERENTIAL EQUATIONS
  Three years (February, 1995 to February, 1998)

- INTEGRABILITY ASPECTS OF DISCRETE AND CONTINUOUS EQUATIONS
  Three years (August, 1995 to July, 1998)

- CHAOS, TURBULENCE AND COLLECTIVE RELAXATION IN NON–EQUILIBRIUM PLASMAS
  Four years (December, 1995 to November, 1999)

- ARITHMETIC AND AUTOMORPHIC FORMS
  Three years (November, 1996 to October, 1999)

List of projects (Start 1997–1999)

- NONLINER HYPERBOLIC AND ELLIPTICAL EQUATIONS AND APPLICATIONS

  Three years (May, 1997 to April, 2000)

- GEOMETRY

  Three years (May, 1997 to April, 2000)

- RIGOROUS RESULTS ON SCHRODINGER EQUATIONS AND FOUNDATIONS OF QUANTUM THEORY AND APPLICATIONS TO PARTICLE PHYSICS AND ASTROPHYSICS

  Three years and six months (March, 1999 to August, 2002)

- THEORETICAL STUDY OF ELECTRONIC AND MOLECULAR DYNAMIC

  Three years and six months (March, 1999 to August, 2002)

List of projects (Start 1999–2002)

- NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS AND CONTROL
  Three years and six months (July, 1999 to December, 2002)

- NON–CUMULATIVE MARKOV PROCESSES AND OPERATOR SPACES
  Three years (May, 2001 to April, 2004)

- ALGEBRAIC GROUPS IN ARITHMETIC & GEOMETRY
  Three years (September 2001 to August, 2004)

- STUDIES IN GEOMETRY OF BANACH SPACES
  Three years (November 2001 to October, 2004)

- MATHEMATICAL TOPICS IN HYPERBOLIC SYSTEMS OF CONSERVATION LAWS
  Four years (July 2002 to June, 2006)

List of projects (Start 2003–2008)

- ANALYTIC AND COMBINATORIAL NUMBER THEORY
  Three years (October 2003 to September 2006)

- ADVANCED NUMERICAL METHODS IN NONLINEAR FLUID MECHANICS AND ACOUSTICS : NONLINEAR ANALYSIS AND OPTIMISATION
  Three years (March, 2006 to February, 2009)

- CONSERVATION LAWS AND HAMILTON JACOBI EQUATIONS
  Three years (September, 2006 to August, 2009)

- ARITHMETIC OF AUTOMORPHIC FORMS
  Three years (September, 2007 to August, 2010)

- CONTROL OF SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS
  Three years (February, 2008 to January, 2011)

List of projects (Start 2009–2010)

- NUMERICAL TREATMENT OF INTEGRAL OPERATORS WITH NON-SMOOTH KERNELS

  Three years (September 2009 to August 2012)

- KLEINIAN GROUPS : GEOMETRICAL AND ANALYTICAL ASPECTS

  Three years (September, 2010 to August, 2013)

- DISCONTINUOUS GALERKIN METHOD FOR NONLINEAR ACOUSTICS

  Three years (September, 2010 to August, 2013)

Indo–French relations in mathematics

The relations between mathematicians from France and from India are old. The first links were established by A. Weil in 1930, and shortly after that, Father Racine played a major role in the development of mathematical research in India.

André Weil in India

In 1929 Syed Ross Masood, Vice–Chancelor of Aligarh Muslim University, proposed a chair of French civilization to André Weil, who was recommended to him by a specialist of Indology, Sylvain Levi. A few months later this offer was converted into a chair of mathematics. Weil reached India in early 1930 and stayed there for more than two years.

André Weil in India

Weil wrote a report on the situation of the universities in India in 1931 and 1936. In his first report he had suggested actions for the improvement of Indian mathematics. The conclusion of the second report deals with the potential of this country and the possibility for India to soon take one of the leading places in the international mathematical community.

Father Racine

Father Racine (1897–1976) reached India in 1937 as a Jesuit missionary after having taken his Doctorate in Mathematics in 1934 under Élie Cartan. He taught mathematics first at St Joseph's College in Tiruchirappally (Trichy, Tamil Nadu) and from 1939 onwards at Loyola College (Madras). He had connections with many important French mathematicians of that time like J. Hadamard, J. Leray, A. Weil, H. Cartan. His erudition was clear from his lectures, his courses were research oriented in contrast with the traditional way of teaching which aimed only at leading the largest number of students to success in their exams.

Father Racine

Father Racine taught his students to read recent books, like the one of L. Schwartz on distributions.

Father Racine encouraged his best students to join the newly founded Tata Institute of Fundamental Research (TIFR) in Bombay with K. Chandrasekharan and K.G. Ramanathan. This explains why so many mathematicians from that generation who were the leaders in TIFR came from Tamil Nadu.

The Tata Institute

Right after the creation of theTata Institute of Fundamental Research in Bombay, many influential French mathematicians visited the Tata Institute of Bombay and gave courses. In the 50's, L. Schwartz visited it several times, followed by H. Cartan, F. Bruhat, J.L. Koszul, P. Samuel, B. Malgrange, J. Dieudonné, P. Gabriel, M. Demazure, A. Douady and many others, invited by the Director of that time, K. Chandrasekharan. Later, at the end of the 60's, A. Weil and A. Grothendieck visited TIFR.

Indo–French cooperation in mathematics

The influence of French mathematicians on the development of mathematics in India has played a leading role in at least two topics : algebraic geometry in the 1960's and theoretical partial differential equations in the 1970's.
J-L. Verdier was responsible of a PICS Inde (PICS=*Programme International de Coopération Scientifique*) of the CNRS (*Centre International de la Recherche Scientifique*) from 1986 to 1989. A report on this cooperation was published in the *Gazette des Mathématiciens* of the Société Mathématique de France (n$^\circ$ 49, juin 1991, pp. 59–61).
A second report dealing with the activities from 1986 and 1995 was published in the same *Gazette des Mathématiciens* (n$^\circ$ 71, 1997, pp. 62–65).

Indo–French cooperation in applied mathematics

In applied mathematics also the cooperation between mathematicians from France and from India is quite strong. While J.L. Lions was at the head of INRIA (*Institut National de Recherche en Informatique et Automatique*) in Rocquencourt, he developed close relations with several Indian institutions : IISc (*Indian Institute of Science*) in Bangalore, IIT *Indian Institute of Technology* in Delhi, and most of all with the small group of mathematicians working on partial differential equations in the Bangalore section of TIFR. In September 1997 a Master of Scientific Calculus was created at the University of Pondicherry, thanks to a cooperation directed by O. Pironneau.

Indo–French cooperation in applied mathematics

The cooperation on *Scientific Calculus for Mechanics and Engineering* between the laboratory of Numerical Analysis of Paris VI and INRIA Rocquencourt in France and IISc Bangalore, TIFR Bangalore and IIT Delhi in India, started in 1975 and the agreements have been renewed in 1993 ; this program is supported by CEFIPRA, the French Ministry of Foreign Affairs and the *Pôle de recherche commun Dassault-Aviation/Université Paris VI.*

Indo–French Institute of Mathematics

The *Indo–French Institute of Mathematics* (IFIM=*Institut Franco–Indien de Mathématiques*) is a virtual institute which was created in 2003 with the support of NBHM (National Board for Higher Mathematics) and DST (Department of Science and Technology) on the Indian side and MAE (Ministère des Affaires Étrangères) and CNRS (Centre National de la Recherche Scientifique) on the French side. One of the main objectives is to provide financial supports for doctoral, postdoctoral and research positions.

Memorandum of Understanding

There are several MoU between French and Indian Universities. One of them involved the University of Pondicherry in India and the universities of Paris VI and Poitiers in France. During a number of years there were many scientific exchanges under this agreement with a strong support of the French Embassy in Delhi.

Memorandum of Understanding

Thanks to an agreement (MoU) between the CMI (*Chennai Mathematical Institute*) and ENS (*École Normale Supérieure*, rue d'Ulm, Paris), every year since 2000, some three young students from ENS visit CMI for two months and deliver courses to the undergraduate students of CMI, and three students from CMI visit ENS for two months. The French students are accommodated in the guest house of IMSc, which participates in this cooperation.

Memorandum of Understanding

Another MoU has been signed in 2009 between the University of Paris VI Pierre et Marie Curie and the two institutes of Chennai, CMI (*Chennai Mathematical Institute*) and IMSc (*Institute of Mathematical Sciences*). An item in this MoU follows a recommendation of the COPED (*Committee for Developing Countries*) of the French Academy of Sciences : each year, one full time teaching duty of a mathematician from Paris VI will be given in Chennai. In practice, two professors from Paris 6 will go to CMI each year to teach an graduate program for one term each.

Indo–French Conference in Mathematics 2008

A joint *Indo–French Conference in Mathematics* took place from December 15 to 19, 2008, at the Institute of Mathematical Sciences of Chennai. There were some 10 plenary lectures and 30 lectures in parallel sessions, half of them given by Indian mathematicians and the other half by French mathematicians.
Since H. Cartan passed away a few days before this meeting (at the age of 104), two special lectures (by J Oesterlé and C.S. Seshdari) were devoted to him the last day.

CIMPA–SMF–SMAI wiki project

A joint initiative of CIMPA (*Centre International de Mathématiques Pures et Appliquées*), SMF (*Société Mathématique de France*) and SMAI (*Société de Mathématiques Appliquées et Industrielles*) gave rise to a *wiki*– style website concerning mathematics in the world, with an emphasis on cooperations involving French mathematicians and mathematicians from developing countries :

`http://smf4.emath.fr/International/Projet-CIMPA-SMAI-SMF/`

CIMPA : Centre International de Mathématiques Pures et Appliquées

http://www.cimpa-icpam.org/

The CIMPA is a non-profit international organization established in Nice (France) since 1978, whose aim is to promote international cooperation in higher education and research in mathematics and related subjects, particularly computer science, for the benefit of developing countries, organized several research schools in India. Here is the list :

CIMPA Research Schools (1996–2002)

January 1996 : Pondicherry University

Nonlinear Systems
Y. Kosmann-Schwarzbach, B. Grammaticos, K. M. Tamizhmani.

September 2002, TIFR Mumbai (Bombay)

Probability measures on groups : Recent Direction and trends, *Tata Institute of Fundamental Research, Mumbai (Bombay), S. Dani, P. Gratzyck, Y. Guivarc'h.*

December 2002 : ISI Kolkata (Calcutta)

Soft Computing approach to pattern recognition and image processing.
Ashish Ghosh, Sankar K. Pal.

CIMPA Research Schools (2003–2008)

February 2003 : Pondicherry

Discrete Integrable Systems, Pondicherry,
Basil Grammaticos, Yvette Kosmann-Schwarzbach, Thamizharasi Tamizhmani.

January 25 - February 5, 2005 : IISc Bangalore

Security for Computer Systems and Networks.
K. Gopinath, Jean-Jacques Lévy.

January 2-12, 2008 : IIT Bombay (Mumbai)

Commutative algebra
L. L. Avramov, M. Chardin, M. E. Ross, J. K. Verma, T. J. Puthenpurakal.

CIMPA Research Schools in 2013

November 25-December 6, 2013 : University of Delhi

Generalized Nash Equilibrium Problems, Bilevel programming and MPEC
Didier Aussel, C. S. Lalitha.

November 18-30, 2013 : Shillong.

Fourier analysis of groups in combinatorics.
Gautami Bhowmik, Himadri Mukherjee.

July 8-19, 2013 : Indian Institute of Science Bangalore.

Current Trends in Computational Methods for PDEs
Blanca Ayuso de Dios, Thirupathi Gudi.

CIMPA Research Schools in 2014

August 11-22, 2014 : Kerala School of Mathematics, Kozhikode

Mock Modular Forms

Lothar Goettsche, Manickam Murugesan, Kathrin Bringmann, Lothar Goettsche, Ken Ono.

International Research Staff Exchange Scheme (IRSES) Project "Moduli" (submitted)

- moduli spaces and geometric structures
- moduli spaces and physics
- moduli spaces and arithmetic geometry

Joergen Andersen (Aarhus), V. Balaji (CMI, Chennai), Olivier Biquard (UPMC and ENS, Paris), José Ignacio Burgos (CSIC, Madrid), Sinnou David (UPMC, Paris), Oscar García-Prada, Coordinator (CSIC, Madrid), Tomás Gómez (CSIC, Madrid), Nigel Hitchin (Oxford), Gadadhar Misra (IISc, Bangalore), D.S. Nagaraj (IMSc, Chennai), M.S. Narasimhan (IISc, Bangalore), Nitin Nitsure (TIFR, Mumbai), T.R. Ramadas (ICTP, Trieste ; CMI, after autumn 2013), S. Ramanan (CMI, Chennai), V. Srinivas (TIFR, Mumbai).

May 17, 2013 CEFIPRA

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