Seminar in Number Theory, HKUST, February 2006
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First lecture: Wednesday, February 8, 2006, 4-5 pm Room 3006 Lift 3
Transcendental numbers: definition, examples
Algebraic numbers are complex numbers which are roots of a polynomial with integer coefficients. The other complex numbers are called transcendental. Given a number, to decide whether it is rational, or algebraic irrational, or else transcendental is most often a difficult problem. We give a few examples of transcendental numbers.

Second lecture: Wednesday, February 8, 2006, 5-6 pm Room 3412 Lift 17 & 18
Diophantine approximation and applications
The earliest examples of transcendental numbers were given in 1844 by J. Liouville: there are real numbers which can be very well approximated by rational numbers. The quality of approximation of a given number (either real or complex) by rational or algebraic numbers is the subject of Diophantine approximation theory. Concrete applications will be described.

Third lecture: Monday, February 13, 2006, 3-4 pm Room 2404 Lift 17 & 18
The early history of transcendental number theory
After J. Liouville (1844) gave the first examples of transcendental numbers, G. Cantor provided another proof, Hermite proved the transcendence of *e*, F. Lindemann the transcendence of pi, until K. Weierstrass completed the picture by proving the so-called Lindemann-Weierstrass Theorem. In 1900, D. Hilbert proposed a list of 23 problems at the International Congress of Mathematicians in Paris, the seventh of which is related with transcendental number theory.

Fourth lecture: Monday, February 13, 2006, 4-5 pm Room 2404 Lift 17 & 18
Hilbert's seventh problem and its developments
The solutions by A.O. Gel'fond and Th. Schneider of Hilbert's seventh problem led to several developments of the theory: Schneider started the transcendence theory in connexion with elliptic curves and abelian varieties, Gel'fond provided Diophantine estimates which paved the way to Baker's theory.

Fifth lecture: Monday, February 20, 2006, 3-4 pm Room 2404 Lift 17 & 18
The six exponentials theorem and the four exponentials conjecture
The four exponentials conjecture is one of the main challenges of transcendental number theory. Only a special case has been settled, namely the six exponentials theorem. We give a complete proof of it.

Sixth lecture: Monday, February 20, 2006, 4-5 pm Room 2404 Lift 17 & 18
Higher dimensional diophantine problems
The six exponentials theorem states that some matrices, the entries of which are logarithms of algebraic numbers, have rank at least 2. A generalization, due to D. Roy, is the strong six exponentials theorem, dealing with matrices whose entries are linear combinations of *1* and logarithms of algebraic numbers. The linear subgroup theorem is a far reaching generalization in higher dimension.

Seventh lecture: Monday, February 27, 2006, 3-4 pm Room 2404 Lift 17 & 18
The role of complex conjugation in transcendental number theory
One of Ramachandra's contributions to transcendental number theory in 1968 involves a trick related with complex conjugation. This idea has been pushed further, especially by G. Diaz, who obtained quite recently new transcendence results.

Eighth lecture: Monday, February 27, 2006, 4-5 pm Room 2404 Lift 17 & 18
Conjectures and open problems
Schanuel's conjecture includes most of what can be expected on diophantine properties of values of the exponential function and logarithms, from the qualitative point of view. Generalizations have been considered by Y. André and C. Bertolin, which include a conjecture due to A. Grothendieck. On the quantitative side, several open problems describe the situation from a conjectural point of view, including the abc-conjecture as well as conjectures by S. Lang and M. Waldschmidt.