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Indo–French Cooperation in Mathematics

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Abstract Links between mathematicians from India and France are old, strong and fruitful. We present a short survey of these relations. We start with a brief overview of the mathematical heritage of India. The stay of A. Weil as a Professor in Aligarh Muslim University from 1930 to 1931 was the first outstanding event in the Franco-Indian relationship. A few years later the French jesuit Father Racine came to India where he played a major role in the development of mathematics in this country, as well as in its relations with French mathematicians. Now, there are many collaborations and exchanges of visitors between both countries. This is one of the most flourishing bilateral cooperations and plays a unique role both for French and Indian mathematicians.

1 Indian tradition in mathematics

Mathematics has been studied in India since very ancient times. There are many references on the subject; among the reliable ones are the books of G. R. Kaye [5] in 1915, C. N. Srinivasiengar [12] in 1967, A. K. Bag [1] in 1979 and A. Weil [18] in 1984. References on the more recent period are the papers by R. Narasimhan [8] in 1991, V. S. Varadarajan [13] in 1998 and M. S. Raghunathan [9] in 2003.

According to the Hindu tradition the most important work on astronomy, *Surya Siddhānta*, is supposed to have been written more than two thousand years ago. However, it seems more likely that this treatise is only some 1200 years old.

As a matter of fact there is no proof of mathematical activity in India before 1500 B.C. The Indus civilisation (around 3000 B.C.) was discovered less than one century ago, the writings have not yet been deciphered; there are weights and objects which seem to be devoted to measure and hence may be to a numerical system, but nothing else is known yet.

The *Sulvasūtras* (*cord manuals*) were written by Baudhāyana, Apastamba and Katyayana not earlier than between the 8th and the 4th Century B.C.

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From 500 to 200 B.C. Vedic mathematics continued to be developed before declining and leaving place to the mathematics of the Jains: number theory, permutations and combinations, binomial theorem and, as always, astronomy.

A decimal number system existed in India in the 3rd Century B.C., during the time of the king Ashoka. On the pillars he erected are found inscriptions in *Brahmi* with early ancestors of our numerical decimal system of writing in the so–called *arabic* digits. This way of writing would be an indispensable tool for the later developments of mathematics in India: it enabled Jain mathematicians to write out very large numbers by which they were fascinated.

It is probably between the second and fourth Century A.D. that the manuscript Bakhṣhālī should be dated; it introduces algebraic operations, decimal notations, zero, quadratic equations, square roots, indeterminates and the minus sign.

The classical period of mathematics in India extends from 600 to 1200. The major works are named *Āryabhaṭīya*, *Pančasiddhāntikā*, *Āryabhaṭīya Bhasya*, *Mahā Bhāskariya*, *Brāhmaspuṭasiddhānta*, *Pāṭīgaṇita*, *Gaṇitasārasañgraha*, *Gaṇitalaka*, *Līlāvatī*, *Bijagaṇita*, and the authors are Āryabhaṭa I (476–550), Varāhamihira (505–587), Bhāskara I (∼600–∼680), Brahmagupta (598–670), Mahāvīracarya (∼800–∼870), Āryabhaṭa II (∼920–∼1000), Śrịdhara (∼870–∼930), Bhāskarācārya (Bhāskara II, 1114–1185).

Mahāvīra, a jain from the region of Mysore, one of the few Indian mathematicians of that time who was not an astronomer, wrote one among the first courses of arithmetic.

Bhāskara II (also named Bhāskarācārya, or Professor Bhāskara), belonged to the Ujjain school, like Varāhamihira and Brahmagupta. His treatise of astronomy called *Siddhāntaśiromaṇi* (1150) contains chapters on geometry (*Līlāvatī*) and on algebra (*Bījagaṇita*).

While the mathematical school was declining in the rest of India, it flourished in Kerala between the 14th and the 17th Century. After the work on astronomy and on series by Madhava of Sangamagramma (1350–1425), the four most important works on astronomy and mathematics from that period are *Tantrasañgraha* of Nīlakaṇṭha Somayaji (1444–1544), *Yuktibhasa* by Jyesthdeva, *Karana Paddhati* by Putamana
Somayaij and *Sadratnamala* by Sankara Varman. Another Indian mathematician from the 14th Century is Narayana (∼1340–∼1400), who studied Fibonacci–like sequences.

One of the main features of the mathematics from the Kerala School is the geometric treatment of algebraic problems. An example is the text *Karanamrta* by Citrabhanu written in 1530.

In his treatise on astronomy Madhava used a series for π and obtained 11 decimal digits, while Viète in 1579 could obtain only 9 decimal digits by computing the perimeter of a polygon with 393 216 sides. Three centuries before Newton, Madhava knew the expansions

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

and

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

The invention of infinitesimal calculus in India was motivated by the predictions of eclipses. Āryabhaṭa I, and later Brahmagupta, used the concept of instant motion. Bhāskarācārya used the derivative of the sine function to compute the angle of the ecliptic. These works were later pursued by the Kerala school.

One may consider Madhava as one of the founders of modern analysis.

Srinivasa Ramanujan (1887–1920) is a major character in Indian mathematics. Despite his passing away when he was not yet 33, he left behind him an important work whose originality is outstanding.

During his school years his unique subject of interest was mathematics and he neglected all other topics. Hence he failed the entrance exam at the University of Madras. In 1903 he acquired a book by G. S. Carr, *A synopsis of elementary Results in Pure and Applied Mathematics*, which had a strong influence on him and on his conception of mathematics. This book is a list of exercises without proofs and Ramanujan took each of these as a challenge. He devoted his energy to prove all the 6 165 theorems without any help. The main part of the *notebooks* [10] of Ramanujan is written in the same style, with statements without proofs. One may compare this with the style of the *sutras* used by ancient Hindu mathematicians.

From 1903 to 1914 while he had a hard time to find a job for supporting his living expenses, he wrote in his notebooks some 3 254 mathematical statements. Despite his important results on divergent series, he did not get recognition from established mathematicians.

In 1912 he obtained a position as a clerk in an office of the Port Trust in Madras. He received a grant from the University of Madras only in 1913. The next year he received an invitation from G. H. Hardy at Cambridge which he could accept thanks to this grant.

During the five years he spent in Cambridge he published 21 papers including five joint papers with Hardy. In 1916 he graduated from Cambridge University thanks to his memoir on highly composite numbers. In February 1918 he was awarded the prestigious *Fellowship of the Royal Society*. He was elected *Fellow of Trinity College* in October the same year. Shortly afterwards he became ill (may be from tuberculosis) and went back to India where he passed away on April 26, 1920.

His work is extremely original. According to Hardy [4] the main contribution of Ramanujan in algebra was on hypergeometric series (identities of Douglas-Ramanujan and Rogers-Ramanujan) and continued fractions. The *circle method*, one of the most powerful tools in analytic number theory, originates in his joint work with Hardy. His conjecture that the tau function defined by

$$\sum_{n\geq 1} \tau(n)x^n = x\prod_{n\geq 1}(1-x^n)^{24}$$

satisfies

$$|\tau(p)| \leq 2p^{11/2}$$

for any prime number p was proved only in 1974 by P. Deligne.

The equation of Ramanujan-Nagell is

$$x^2+7=2^n;$$

Ramanujan found five solutions in positive integers

$$1^2+7=2^3,\quad 3^2+7=2^4,\quad 5^2+7=2^5,\quad 11^2+7=2^7,\quad 181^2+7=2^{15},$$

and conjectured that these are the only ones; this was proved by T. Nagell in 1948.

In 1976 at the Trinity College (Cambridge) G. E. Andrews found, among files from the succession of G. N. Watson , the so–called *Lost Notebooks* of Ramanujan. There were 138 sheets of paper containing some 650 statements all written while Ramanujan was spending the last years of his life in India. In these texts he describes his discovery of what he called *Mock Theta Functions*, a hot research subject nowadays.

2 Indo–French connexion

The relations between mathematicians from France and from India are old. As was already mentioned, the first links were established by A. Weil in 1930, and shortly after that, Father Racine played a major role in the development of mathematical research in India.

2.1 André Weil

In 1929 Syed Ross Masood, Vice-Chancellor of Aligarh Muslim University, proposed a chair of French civilization to André Weil, who was recommended to him by a specialist of Indology, Sylvain Levi. A few months later this offer was converted into a chair of mathematics. Weil reached India in early 1930 and stayed there for more than two years. Among his Indian colleagues were T. Vijayaraghavan, D. Kosambi and S. Chowla ([17] Chap. IV) whose intellectual qualities he appreciated.

T. Vijayaraghavan, who later became the first director of the Ramanujan Institute in Madras (at that time it was independent of the department of mathematics of the University) is known for his study of the so–called *P.V. numbers*, which were studied by C. Pisot. The influence of Weil on Vijayaraghavan was important (cf. [8], p. 242).

Chowla later went to the University of Punjab and then migrated to the USA.

Kosambi became a noted historian - his book on the Maurya Empire is classical.

In [15], Weil gave a report on the situation of the universities in India in 1936. In his previous report [14] of 1931 at the Indian Mathematical Society he had suggested actions for the improvement of Indian mathematics. The conclusion of [15] deals with the potential of this country and the possibility for India to soon take one of the leading places in the international mathematical community (see also his comment p. 536 of [16]).

2.2 Father Racine

Father Racine (1897–1976) reached India in 1937 as a Jesuit missionary after having taken his Doctorate in Mathematics in 1934 under Élie Cartan. He taught mathematics first at St Joseph's College in Tiruchirappally (Trichy, Tamil Nadu) and from 1939 onwards at Loyola College (Madras). He had connections with many important French mathematicians of that time like J. Hadamard, J. Leray, A. Weil, H. Cartan. His erudition was clear from his lectures, his courses were research oriented in contrast with the traditional way of teaching which aimed only at leading the largest number of students to success in their exams. At that time with Ananda Rao, a noted analyst, and Vaidyanathaswamy, who had broader interests, Madras was the best place in India for studying mathematics and starting into research.

K. Ananda Rao (1893–1966), a former student of G. H. Hardy at Cambridge (he was there at the same time as Ramanujan), is the author of important contributions to the summability of Dirichlet series, and, most of all, he had brilliant research students: T. Vijayaraghavan (1902–1955), S. S. Pillai (1901–1950), Ganapathy Iyer, K. Chandrasekharan and C. T. Rajagopal (1903–1978) are among them.

R. Vaidyanathaswamy (1894–1960), a former student of E. T. Whittaker at Edinburg and of H. F. Baker at Cambridge, had a deep impact on Indian mathematics. His background was more abstract than the ones of his colleagues, and he was remarkably open-minded.

Father Racine and Vaidyanathaswamy were the promoters of modern mathematics. Instead of following the tradition by teaching only classical material, they also introduced in their courses the new mathematical structures which were developed in a systematic way at that time by Bourbaki. Father Racine also encouraged his students to read recent books, like the one of L. Schwartz on distributions. Further, he helped young mathematicians who did not find a position after their theses, like Minakshisundaram, one of the most gifted mathematicians of his generation, according to R. Narasimhan ([8], p. 251).

The list [11] of the former students of Father Racine who became well known mathematicians is impressive: Venugopal Rao, P. K. Raman, M. S. Narasimhan, C. S. Seshadri, Ramabhadran, K. Varadarajan, Raghavan Narasimhan, C. P. Ramanujam, Ramabhadran Narasimhan, Ananda Swarup, S. Ramaswamy, Cyril D'Souza, Christopher Rego, V. S. Krishnan and S. Sribala.

Father Racine encouraged his best students to join the newly founded Tata Institute of Fundamental Research (TIFR) in Bombay with K. Chandrasekharan and K. G. Ramanathan. This explains why so many mathematicians from that generation who were the leaders in TIFR came from Tamil Nadu.

2.3 The TATA institute

The creation of the Tata Institute of Fundamental Research in Bombay is due to a physicist of the Indian Institute of Science of Bangalore, Homi J. Bhabha (1909–1966), who had political support from J. Nehru and financial support from the Tatas, a Parsi industrial family which is still extremely powerful. At the end of the 19th Century a member of this family, Jamsetji Nusserwanji Tata, was at the origin of the creation of the Indian Institute of Science in Bangalore. The goal of Bhabha was for India to acquire nuclear power, and for this purpose it was necessary to create a research school in physics of high level; in turn, this objective made it necessary to create a strong mathematical research school.

K. Chandrasekharan (who was to emigrate to ETH Zürich later) joined the Tata Institute as early as 1948 and became the director. Thanks to his remarkable action as the head of this Institute, TIFR became a prestigious research institute. He had the ability of discovering the talented future scientists and he knew how to direct them towards suitable research topics, even when he was not himself a specialist. At the same time he succeeded in attracting to Bombay a large number of the best mathematicians of that time, who gave courses to the young students working on their PhD. With such a director the Tata Institute of Bombay soon had a strong international reputation.

Raghavan Narasimhan (who left later for Chicago), K. G. Ramanathan, K. Ramachandra in number theory, C. S. Seshadri (FRS, now Director of the Chennai Mathematical Institute), M. S. Narasimhan (who has been Director of Mathematics at I.C.T.P. (*International Center for Theoretical Physics*, also called *Abdus Salam Center*, in Trieste) in algebraic geometry, R. Sridharan, R. Parimala (invited lecturer at ICM2010) in commutative algebra, M. S. Raghunathan (FRS), Gopal Prasad, S. G. Dani, specialist of arithmetic groups, V. K. Patodi (Heat equation) are among the eminent mathematicians from TIFR. R. Balasubramanian, now Head of Mat. Science (IMSc *Institute of Mathematical Sciences*) in Chennai, is a former student of K. Ramachandra at TIFR Bombay.

Right after its creation, many influential French mathematicians visited the Tata Institute of Bombay and gave courses. In the 50's, L. Schwartz visited it several times, followed by H. Cartan, F. Bruhat, J. L. Koszul, P. Samuel, B. Malgrange, J. Dieudonné, P. Gabriel, M. Demazure, A. Douady and many others, invited by the Director of that time, K. Chandrasekharan. Later, at the end of the 60's, A. Weil and A. Grothendieck visited TIFR.

2.4 Indo–French cooperation in mathematics

The influence of French mathematicians on the development of mathematics in India has played a leading role in at least two topics: algebraic geometry in the 1960's and theoretical partial differential equations in the 1970's.

J.-L. Verdier was responsible of a PICS Inde (PICS = *Programme International de Coopération Scientifique*) of the CNRS (*Centre International de la Recherche Scientifique*) from 1986 to 1989. A report on this cooperation was published in the *Gazette des Mathématiciens* of the Société Mathématique de France (n° 49, juin 1991, pp. 59–61).

A second report dealing with the activities from 1986 and 1995 was published in the same *Gazette des Mathématiciens* (n° 71, 1997, pp. 62–65).

In applied mathematics also the cooperation between mathematicians from France and from India is quite strong. While J. L. Lions was at the head of INRIA (*Institut National de Recherche en Informatique et Automatique*) in Rocquencourt, he developed close relations with several Indian institutions: IISc (*Indian Institute of Science*) in Bangalore, IIT *Indian Institute of Technology* in Delhi, and most of all with the small group of mathematicians working on partial differential equations in the Bangalore section of TIFR. In September 1997 a Master of Scientific Calculus was created at the University of Pondicherry, thanks to a cooperation directed by O. Pironneau. The cooperation on *Scientific Calculus for Mechanics and Engineering* between the laboratory of Numerical Analysis of Paris VI and INRIA Rocquencourt in France and IISc Bangalore, TIFR Bangalore and IIT Delhi in India, started in 1975 and the agreements have been renewed in 1993; this program is supported by IFCPAR, the French Ministry of Foreign Affairs and the *Pôle de recherche commun Dassault-Aviation/Université Paris VI.*

Since 1995, many projects benefitted from different financial supports in all fields of science. Among them are the following ones.

- The *Indo–French Centre for the Promotion of Advanced Research* (IFCPAR, CEFIPRA in French) is a bilateral programme of scientific cooperation between India and France under the Department of Science and Technology, Government of India and the Ministry of Foreign Affairs, Government of France. Under the heading of Pure and Applied Mathematics the following are the ongoing projects:

  ARITHMETIC OF AUTOMORPHIC FORMS
  Three years (September, 2007 to August, 2010)

CONTROL OF SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS
Three years (February, 2008 to January, 2011)

and the following ones have been completed:

CONSERVATION LAWS AND HAMILTON JACOBI EQUATIONS
Three years (September, 2006 to August, 2009)

ADVANCED NUMERICAL METHODS IN NONLINEAR FLUID MECHANICS AND ACOUSTICS: NONLINEAR ANALYSIS AND OPTIMIZATION
Three years (March, 2006 to February, 2009)

ANALYTIC AND COMBINATORIAL NUMBER THEORY
Three years (October 2003 to, September 2006)

MATHEMATICAL TOPICS IN HYPERBOLIC SYSTEMS OF CONSERVATION LAWS
Four years (July 2002 to June, 2006)

STUDIES IN GEOMETRY OF BANACH SPACES
Three years (November 2001 to October, 2004)

ALGEBRAIC GROUPS IN ARITHMETIC & GEOMETRY
Three years (September 2001 to August, 2004)

NON-CUMULATIVE MARKOV PROCESSES AND OPERATOR SPACES
Three years (May, 2001 to April, 2004)

NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS AND CONTROL
Three years and six months (July, 1999 to December, 2002)

THEORETICAL STUDY OF ELECTRONIC AND MOLECULAR DYNAMIC
Three years and six months (March, 1999 to August, 2002)

GEOMETRY
Three years (May, 1997 to April, 2000)

NONLINEAR HYPERBOLIC AND ELLIPTICAL EQUATIONS AND APPLICATIONS
Three years (May, 1997 to April, 2000)

RIGOROUS RESULTS ON SCHRODINGER EQUATIONS AND FOUNDATIONS OF QUANTUM THEORY AND APPLICATIONS TO PARTICLE PHYSICS AND ASTROPHYSICS
Three years and six months (March,1999 to August, 2002)

ARITHMETIC AND AUTOMORPHIC FORMS
Three years (November, 1996 to October, 1999)

CHAOS, TURBULENCE AND COLLECTIVE RELAXATION IN NON-EQUILIBRIUM PLASMAS
Four years (December, 1995 to November, 1999)

INTEGRABILITY ASPECTS OF DISCRETE AND CONTINUOUS EQUATIONS
Three years (August, 1995 to July, 1998)

ASYMPTOTIC ANALYSIS IN PARTIAL DIFFERENTIAL EQUATIONS
Three years (February, 1995 to February, 1998)

GEOMETRY AND NUMBER THEORY
Three years (February, 1992 to January, 1995)

NONLINEAR HYPERBOLIC EQUATIONS AND APPLICATIONS
Three years (March, 1992 to February, 1995)

A STUDY OF SOME FACTORIZATION AND COMPOSITION PROBLEMS IN GRAPHS
Three years and six months (September, 1992 to February, 1996)

NUMERICAL MODELLING OF THE OCEAN-ATMOSPHERE SYSTEM WITH SPECIAL REFERENCE TO MONSOONS
One year (April, 1989 to March, 1990)

- The *Indo–French Institute of Mathematics* (IFIM = *Institut Franco–Indien de Mathématiques*) is a virtual institute which was created in 2003 with the support of NBHM (National Board for Higher Mathematics) and DST (Department of Science and Technology) on the Indian side and MAE (Ministère des Affaires Étrangères) and CNRS (Centre National de la Recherche Scientifique) on the French side. One of the main objectives is to provide financial supports for doctoral, postdoctoral and research positions.
- There are several MoU (*Memorandum of Understanding*) between French and Indian Universities. One of them involved the University of Pondicherry in India and the universities of Paris VI and Poitiers in France. During a number of years there were many scientific exchanges under this agreement with a strong support of the French Embassy in Delhi.

  Thanks to an agreement (MoU) between the CMI (*Chennai Mathematical Institute*) and ENS (*École Normale Supérieure*, rue d'Ulm, Paris), every year since 2000, some three young students from ENS visit CMI for two months and deliver courses to the undergraduate students of CMI, and three students from CMI visit ENS for two months. The French students are accommodated in the guest house of IMSc, which participates in this cooperation.

  Another MoU has been signed in 2009 between the University of Paris VI Pierre et Marie Curie and the two institutes of Chennai, CMI (*Chennai Mathematical Institute*) and IMSc (*Institute of Mathematical Sciences*). An item in this MoU follows a recommendation of the COPED (*Committee for Developing Countries*) of the French Academy of Sciences: each year, one full time teaching duty of a mathematician from Paris VI will be given in Chennai. In practice, two professors from Paris 6 will go to CMI each year to teach an graduate program for one term each.

  New IIT-s (*Indian Institute of Technology*) are being launched, one of them will be created in Rajasthan, and it will be supported by the French Government. An agreement with the University Paris Sud (Orsay) enables the teachers from that University to deliver their courses in this new IIT.
- Several other sources of funding enable senior mathematicians from India and France to visit the other country. Among them are supports from CNRS and the Ministry of Education in France, from the NBHM *(National Board of Mathematics)* in India. As an example, an agreement between CNRS and NBHM from 1999 to

2003 enabled each year some three mathematicians from each country to visit the other one, when no other support from one of the other programs was suitable. There are also agreements between the Académie des Sciences de Paris and the Indian National Science Academy (INSA) in New Delhi.

- A number of scholarships enable young mathematicians from India (as well as from other countries) to pursue their studies in France. This includes *cotutelle theses* (codirection). The website of the French Embassy gathers some information on this matter and proposes a number of links to several institutions in France: Campus France, Egide, CNOUS (*Centre National des uvres Universitaires et Scolaires*), ONISEP (a French provider of student career and job information), CEFI (*Centre de ressources et de prospective sur les Grandes écoles d'ingénieurs et de gestion, et sur les emplois d'encadrement*)...

  http://ambafrance-in.org/france_inde/spip.php?article4158

- A joint initiative of CIMPA (*Centre International de Mathématiques Pures et Appliquées*), SMF (*Société Mathématique de France*) and SMAI (*Société de Mathématiques Appliquées et Industrielles*) gave rise to a *wiki*–style website concerning mathematics in the world, with an emphasis on cooperations involving French mathematicians and mathematicians from developing countries:

  http://smf4.emath.fr/International/Projet-CIMPA-SMAI-SMF/consulte.php

- The *Online Survey of European Researchers in India*, an initiative of the Science & Technology Delegation of the European Union to India, has been launched in February 2010. It will lead a data base of European researchers who have links with Indian colleagues.
- The two French institutes, CSH (*Centre de Sciences Humaines*) and IFP (*Institut Français de Pondichéry*), are joint institutes of CNRS and MAE (*French Ministry of Foreign Affairs*) since 2007. Among their missions is the development of scientific international collaboration in the Indian subcontinent. Both institutes aim to develop cooperation in mathematics and statistics, including applications to social sciences and humanities.
- A Cyber University called FICUS (*French Indian Cyber-University in Sciences*) already started with the mathematics component called e-m@ths

  http://www.ncsi.iisc.ernet.in/disc/indo-french/emaths.htm

In France, e-m@ths is being funded within the frame of the *Campus Numériques* request for proposal. Its proceeds from the desire of the Applied Maths and Pure Maths communities to position themselves within the field of Information Technology for Education. It is an opportunity to gather together both communities through a joint project that could kick-start the development of collaborations in creation and utilization of learning materials. The purpose of the "e-m@th" project is the design of a bilateral curriculum of graduate type (Masters, first year of PhD) involving Indian and French institutions. The goal is the design, implementation and operation of education modules in applied mathematics for initial or continuing education. Simultaneously, the project aims at strengthening the links and collaborations at the research level between the scientific teams involved in the construction of the curriculum.

- The CIMPA, already mentioned, a non-profit international organization established in Nice (France) since 1978, whose aim is to promote international cooperation in higher education and research in mathematics and related subjects, particularly computer science, for the benefit of developing countries, organized several research schools in India. Here is the list:

  January 1996 Pondicherry University
  Nonlinear Systems
  org. Y. Kosmann-Schwarzbach, B. Grammaticos and K. M. Tamizhmani.

  September 2002 TIFR Mumbai (Bombay)
  Probability measures on groups: Recent Direction and trends, Tata Institute of Fundamental Research, Mumbai (Bombay),
  org. S. Dani, P. Gratzyck, Y. Guivarc'h.

  December 2002: Kolkata (Calcutta)
  Soft Computing approach to pattern recognition and image processing. Machine Intelligence Unit, Indian Statistical Institute, Calcutta,
  org. Ashish Ghosh, Sankar K. Pal.

  February 2003: Pondicherry,
  Discrete Integrable Systems, Pondicherry,
  org. Basil Grammaticos, Yvette Kosmann-Schwarzbach, Thamizharasi Tamizhmani.

  January 25 – February 5, 2005: IISc Bangalore,
  Security for Computer Systems and Networks.
  org. K. Gopinath and Jean-Jacques Lévy.

  January 2–12, 2008: IIT Bombay (Mumbai)
  Commutative algebra
  org. L. L. Avramov, M. Chardin, M. E. Ross, J. K. Verma, T. J. Puthenpurakal.

- A joint *Indo–French Conference in Mathematics* took place from December 15 to 19, 2008, at the Institute of Mathematical Sciences of Chennai. There were some 10 plenary lectures and 30 lectures in parallel sessions, half of them given by Indian mathematicians and the other half by French mathematicians.

  Since H. Cartan passed away a few days before this meeting (at the age of 104), two special lectures (by J. Oesterlé and C. S. Seshdari) were devoted to him the last day.
- The most important part of cooperation between France and India in mathematics is constituted by the new results proved by the joint works of mathematicians from both countries. We conclude with two such outstanding results.

  The first one is the final step to the determination of Waring's constant $g(4) = 19$ in 1986 by R. Balasubramanian, J.-M. Deshouillers and F. Dress [2,3]:

  Any positive integer is the sum of at most 19 *biquadrates.*

  The second one was called *Serre's Modularity Conjecture*, until it was finally proved in 2006 in a joint work by Chandrashekhar Khare and Jean-Pierre Wintenberger [6,7]:

  Let
$$\rho : G_{\mathbb{Q}} \to GL_2(F).$$

be an absolutely irreducible, continuous, and odd two-dimensional representation of $G_{\mathbb{Q}}$ over a finite field $F = \mathbb{F}_{\ell^r}$ of characteristic ℓ, There exists a normalized modular eigenform

$$f = q + a_2q^2 + a_3q^3 + \cdots$$

of level $N = N(\varrho)$, weight $k = k(\varrho)$, and some Nebentype character $\chi : \mathbb{Z}/N\mathbb{Z} \to F^*$ such that for all prime numbers p, coprime to $N\ell$, we have

$$\mathit{Trace}(\rho(\mathit{Frob}_p)) = a_p \quad \mathit{and} \quad \det(\rho(\mathit{Frob}_p)) = p^{k-1}\chi(p).$$

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