Proposals for the syllabus of the Master Program in mathematics at Sherubtse (RUB)
By S. David, M. Waldschmidt

Semester 1

Topology – Calculus
Proposed content :

a) Topological spaces (interior, closure, boundary, basis for a topology, metric spaces, subspaces, continuity and homeomorphisms, product spaces).
b) Connectedness (path connectedness, connected components and path components, Cantor set).
c) Compact spaces (compact spaces in Euclidean spaces, Hausdorff spaces, Normal spaces, Lebesgue numbers, infinite products)
d) Quotient spaces.
e) Differentiability in a normed vector space , implicit functions theorem, local inversion theorem.
f) Higher order differentials, Taylor, extrema.
g) Differential forms, wedge products, Poincaré theorem, Frobenius theorem.

References :
**** *to be completed for topology by Balamurugan*
H. Cartan, Calcul Differentiel, Hermann 1957.

Differential geometry/algebraic topology
Proposed content :
Differential geometry :

a) differential varieties, morphisms of varieties.
b) Vector bundles, sections of vector bundles, tangent bundle, cotangent bundle.
c) Lie derivative
d) Differential forms, De Rham complex.
e) Integration on varieties.
f) Stokes theorem.

Algebraic topology :

a) homotopy, fundamental group
b) quotient spaces, glueing, compact spaces, operations of groups, topological groups, projective spaces, simplicial complex.
c) Groups defined by generators and relations, free abelian group, theorem of Seifert-Van Kampen.
d) Covering spaces, lifts of paths, fundamental group of a covering, automorphisms of coverings, galois coverings, universal covering, existence of coverings.
e) Fibrations, locally trivial bundles, homotopy groups and long exact sequence of homotopy, homogeneous spaces
f) Homological algebra: complexes, exact sequences, homology, simplicial homology.
g) Singular homology: Mayer-Vietoris exact sequence, homology of a cover.
h) Degree, Lefschetz numbers.

The course will cover part of the above material. Depending on the teacher, the program will put more emphasis on one topic or the other, dividing the 15 credits into 6+9 or 9+6.
References:

A. Hatcher, Algebraic Topology, Cambridge Univ. Press and Georgia University web site
M. Spivak, Differential Geometry.
MIT Open website 18.996 :
<http://ocw.mit.edu/OcwWeb/Mathematics/18-966Spring-2007/CourseHome/index.htm>

Advanced Probability

Course Description

1. Introduction: Basic probability models
2. Combinatorics
3. Random variables
4. Discrete and continuous probability distributions: binomial, geometric, hypergeometric, and Poisson distributions, uniform, exponential, normal, gamma and beta distributions
5. Laws of large numbers and central limit theorems for sums of independent random variables
6. Conditioning and martingales
7. Brownian motion
8. Elements of diffusion theory.

References

_ MIT open course ware 18.05, 18.175 and 18.440

<http://ocw.mit.edu/OcwWeb/Mathematics/18-05Spring-2005/CourseHome/index.htm>

<http://ocw.mit.edu/OcwWeb/Mathematics/18-175Spring-2007/CourseHome/index.htm>

<http://ocw.mit.edu/OcwWeb/Mathematics/18-440Fall-2005/CourseHome/index.htm>

Convex analysis, optimization, game theory
Proposed content :

a) Convex spaces in a normed vector space, extremal points, faces, convex functions, semi continuity.
b) Finite dimensional case (interior, compact case, convex cones, polyhedral convexes, linear programming and simplex algorithm)
c) General case (separation theorem, compact case, Hilbert space case).
d) Convex functions (continuity, differentiability, conjugation)
e) To be continued (*game theory*)

References :
MIT Open website 18.433 and 18.997
http://ocw.mit.edu/OcwWeb/Mathematics/18-433Fall2003/CourseHome/index.htm
http://ocw.mit.edu/OcwWeb/Mathematics/18-997Spring2004/CourseHome/index.htm

Semester 2

Functional analysis
Proposed content :

a) normed vector spaces, duality, Banach spaces, Baire's lemma
b) Hahn-Banach theorem, Banach Steinhaus theorem, open image, closed graph.
c) Hilbert spaces, parallelogram identity, projection theorem on a closed convex, projections on closed subspaces, duality in Hilbert spaces, unicity of a separable basis.
d) Dual norm, biduals, reflexive spaces, weak topology, preweak topology, compacity on the unit ball in the preweak topology. Relation between strong closed sets and weak closed sets.
e) Classical Banach spaces.
f) Compact operators, spectral theory, tensor products of Banach spaces
g) Sobolev spaces
h) Distribution
i) Fourier analysis
j) Introduction to wavelets.
k) Fixed point theorems
l) Banach algebras

References:
B.V. Liemayee, Functional Analysis
MIT Open website 18.327 :
<http://ocw.mit.edu/OcwWeb/Mathematics/18-327Wavelets--Filter-Banks-and-ApplicationsSpring2003/CourseHome/index.htm>

Number Theory
Course Description

The first part of this course is an elementary introduction to number theory with no algebraic prerequisites.

1. Primes, congruences
2. Quadratic reciprocity
3. Diophantine equations
4. Irrational numbers
5. Continued fractions
6. Elliptic curves.

The second part will be either Analytic Number Theory, or Algebraic Number Theory, depending on the choice of the teacher. It may also be a mixture of them. An additional theme running throughout the course can be the use of computer algebra to investigate number-theoretic questions.

Analytic number theory includes

1. Introduction to zeta and L-functions
2. Distribution results concerning prime numbers (e.g., Dirichlet's prime number theorem

in arithmetic progressions).

Algebraic number theory includes
1. Number fields, class numbers, Dirichlet's units theorem
2. Cyclotomic fields
3. Local fields, places, absolute values, valuations, completions; decomposition and inertia groups, ramification
4. Basic analytic methods
5. Basic class field theory
6. l-adic representations

References

_ T.M. Apostol, Introduction to Analytic Number Theory. Springer, 1976.
_ J.W.S. Cassels et J. Fröhlich, Algebraic Number Theory. Academic Press
_ S. Lang, Algebraic Number Theory, Addison-Wesley.
_ J. Neukirch, Algebraic Number Theory, Springer Verlag, 2000.
_ MIT open course ware 18.781, 18.785 and 18.786
<http://ocw.mit.edu/OcwWeb/Mathematics/18-781Spring2003/CourseHome/index.htm>
<http://ocw.mit.edu/OcwWeb/Mathematics/18-785Spring-2007/CourseHome/index.htm>
<http://ocw.mit.edu/OcwWeb/Mathematics/18-786Spring-2006/CourseHome/index.htm>

Advanced Statistics

Course Description

1. Introduction: statistical estimation and testing, decision theory, confidence intervals and linear regression.
2. Chi-square tests, nonparametric statistics, analysis of variance, regression, and correlation.
3. Large sample theory.
4. Asymptotic efficiency of estimates
5. Exponential families
6. Sequential analysis.

Matlab examples will be included.

References

_ MIT open course ware 18.466 and 18.443
<http://ocw.mit.edu/OcwWeb/Mathematics/18-466Mathematical-StatisticsSpring2003/CourseHome/index.htm>
<http://ocw.mit.edu/OcwWeb/Mathematics/18-443Fall-2006/CourseHome/index.htm>

_ Bristol course - Second year statistics (by Jonathan Rougier):
Course description:
<http://www.maths.bris.ac.uk/study/undergrad/current_units/unit/?id=211>
Course webpage:

<http://www.maths.bris.ac.uk/~mazjcr/stats2/home.html>

Advanced Financial Mathematics

Course Description

1. Corporate Finance,
2. Risk and Insurance,
3. Financial Econometrics,
4. Financial Derivatives, Mathematical Review
5. Discrete Time Finance, Continuous Time Finance
6. Risk Management
7. Computations in Finance
8. Optimisation Methods for Finance.

References

_ **University Purdue**

<http://www.gradschool.purdue.edu/programs/>

_ **Leeds University Business School**

Postgraduate Degrees

<http://lubswww.leeds.ac.uk/masters/index.php?id=144>

_ **King's College London**

Financial Mathematics

<http://www.mth.kcl.ac.uk/finmath/MSc.html>

<http://www.mth.kcl.ac.uk/finmath/courses.html>

Semester 3

Complex analysis and special functions

Course Description

A: Basics

1. Review of one complex variable: complex algebra and functions, analyticity, contour integration, Cauchy's theorem, singularities, Taylor and Laurent series, residues, evaluation of integrals
2. Fourier analysis and Laplace transforms. Mellin transform.
3. Series and products of analytic or meromorphic functions.
4. Exponential and logarithms.

B: Fundamental

1. The Gamma function.
2. Elliptic functions.
3. Power series solutions of differential equations. Ordinary points, Regular singular point, Indicial equations.

C: Advanced topics

1. Hypergeometric functions. Gauss Hypergeometric equation.
2. The Riemann zeta function, Hurwitz-Lerch zeta functions. Bernoulli polynomials
3. Legendre polynomials.
4. Bessel functions and Epstein zeta functions.
5. Theta functions
6. Euler digamma function.
7. Polylogarithms, multizeta values.

References

_ MIT open course ware 18.04 and 18.112
<http://ocw.mit.edu/OcwWeb/Mathematics/18-04Fall-2003/CourseHome/index.htm>
see also
<http://ocw.mit.edu/OcwWeb/Mathematics/18-112Fall-2006/CourseHome/index.htm>
_ Textbooks in Mathematics at geocities or 110mb
<http://www.geocities.com/alex_stef/mylist.html#FuncAn>
_ Robert B. Ash and W.P. Novinger, Complex Variables, <http://www.math.uiuc.edu/~r-ash/CV.html>
_ E.T Whittaker and G.N. Watson, A course of modern analysis. Cambridge Mathematical Library.
_ S. Kanemitsu and H. Tsukada, Vistas of special functions. World Scientific 2007.

Multivariate Complex Analysis

Course Description

A: Basics

1. Functions of one complex variable, Cauchy integral formula, Taylor series, analytic continuation
2. Inhomogeneous Cauchy Riemann equation, Riemann equation in one variable
3. Introduction to functions of several complex variables, analytic continuation, singularities of holomorphic functions
4. Subharmonic and plurisubharmonic functions

B: Fundamental

1. The inhomogeneous Cauchy-Riemann equation in several variables, Hartog's Theorem.
2. Applying Hartog's Theorem, The Dolbeault Complex, Exactness of the Dolbeault complex on polydisks
3. The holomorphic version of the Poincare Lemma
4. The inverse function Theorem and the implicit function Theorem for holomorphic mappings

C: Advanced topics

1. Schwarz Lemma in several variables.
2. Harmonic theory on complex manifolds
3. The Hodge decomposition theorem
4. The hard Lefschetz theorem
5. Vanishing theorems.

Some results and tools on deformation and uniformization of complex manifolds may also be discussed.

Further topic which may be covered in this course include: L2 estimates of Hörmander, Kähler Manifolds, Elliptic Operators and Pseudo-differential Operators, Hodge Theory on Kaehler Manifolds, Geometric Invariant Theory.

References

_ MIT open course ware 18.117

<http://ocw.mit.edu/OcwWeb/Mathematics/18-117Spring-2005/CourseHome/index.htm>
_ M. Waldschmidt, chap. 7 of Groupes algébriques et nombres transcendants, Astérisque.

Riemann Surfaces *(6 or 9 credits)*

Course Description

A: Basics

1. Definitions and examples
2. Analytic aspects of Riemann surfaces: multivalued functions
3. Algebraic aspects of Riemann surfaces: algebraic curves

B: Fundamental

1. Coverings. Euler characteristic. Riemann Hurwitz formula.
2. Classification of Riemann surfaces: Elliptic, Parabolic, Hyperbolic
3. Hurwitz's automorphisms theorem
4. Riemann-Roch and uniformization theorems.

References

_ H. Farkas and I. Kra, Riemann surfaces, Springer-Verlag, 1980.
_ O. Forster, Lectures on Riemann surfaces, Springer-Verlag, 1981.
_ R. Narasimhan, Compact Riemann surfaces, Birkhauser

Discrete mathematics, cryptography and coding theory

Course Description

A: Basics

1. Complements on logic and sets, number theory, combinatorics and graph theory
2. Finite fields
3. Languages, combinatorics on words

B: Fundamental

1. Introduction to cryptography
2. Introduction to error correcting codes

C: Advanced topics

Depending on the taste of the teacher, this course will cover some of the following items

1. Further aspects of cryptography
2. Further aspects of coding theory.
3. Advanced graph theory:

- Valency, Walks, Paths and Cycles, Hamiltonian Cycles and Eulerian Walks, Trees, Spanning Tree of a Connected Graph
- Weighted graphs: Minimal Spanning Tree
- Search algorithms: Depth-First Search, Breadth-First Search, The Shortest Path Problem
- Digraphs: Introduction, Networks and Flows, The Max-Flow-Min-Cut Theorem

4. Applied combinatorics on words.

The application areas include core algorithms for text processing, natural language processing, speech processing, bioinformatics, and several areas of applied mathematics such as combinatorial enumeration and fractal analysis. There are also applications of the shuffle algebra to multizeta values with connections to number theory and mathematical physics.

References

_ László Lovász and Kati Vesztergombi, Discrete Mathematics,1999
<http://www.cs.elte.hu/~lovasz/dmbook.ps>
_ W.W.L. Chen, Discrete Mathematics, 201 pp. (web edition, 2008)
<http://www.maths.mq.edu.au/~wchen/lndmfolder/lndm.html>
_ M. Lothaire
Combinatorics on Words, Cambridge Mathematical Library, 1997
Algebraic Combinatorics on Words, Cambridge University Press, 2002
Applied Combinatorics on Words, Encyclopedia of Mathematics and its Applications (No. 105), Cambridge University Press, 2005

Commutative algebra
Proposed content :

a) background on rings (radicals, local rings, Nakayama's lemma)
b) primary decomposition, associated primes (primary submodules, ideals, primary decomposition, associated primes, localization, artinian rings)
c) integral extensions (integral elements, integrality and localization)
d) valuation rings (general properties, extension theorems, discrete valuation rings)
e) completion (graded rings, completion of a module, Krull intersection theorem)
f) dimension (Hilbert Samuel polynomials, dimension theorem, affine *k*-algebras
g) depth (systems of parameters, regular sequences)
h) homological methods (homological dimension, Tor and dimension)
i) regular local rings

References :
Jacobson, Basic Commutative Algebra
Samuel- Zariski, Commutative Algebra, I+II
Matsumura, Commutative Algebra

Lie theory
Proposed content :
A) *Basics*
a) Topological groups and Lie groups, exponential map.
b) Lie algebras, structure theorems for Lie algebras (Killing form, roots systems, Weyl groups, Poincaré-Birkoff-Witt).
c) Haar Measure.
d) Linear representations of Lie algebras.
e) Applications to classical groups, closed subgroups of GL(n,**R**).
B) *Fundamentals*
a) Semi-simple Lie groups
b) Symmetric spaces
c) Algebraic groups
C) *Advanced topics*:
either
a) Affine Lie algebras, Kac-Moody algebras,
or
b) Lattices in Lie groups

Some references:
V.G. Kac, *Infinite dimensional Lie algebras*, Cambridge University Press, Third Edition
J. Fuchs, C. Schweigert, *Symmetries, Lie algebras and Representations : A Graduate Course for Physicists*, Cambridge Monographs on Mathematical Physics
M. Wakimoto, *Lectures on Infinite Dimensional Lie Algebras*, World Scientific
U. Ray, *Automorphic forms and Lie superalgebras*, Springer
M. Raghunathan : Discrete subgroups of Lie groups, Springer (1972)
R. Zimmer : Ergodic theory and semisimple groups,Birkhauser (1984)
G. Margulis : Discrete subgroups of semisimple Lie groups, Springer (1991)
D. Witte-Morris : Ratner's theorems on unipotent flows, Chicago LMS(2004)
Y. Benoist: Five lectures on lattices, SMF (2007)
R. Carter : Lie algebras of finite and affine type, Cambridge 2005
J. Humphreys : Introduction to Lie algebras and representation theory, Springer, 1978

Dynamical systems
Proposed content :

A) *Basics*

a) differentiable aspects : hyperbolicity ; linearization ; invariant manifolds ; structural stability
b) topological and combinatorial aspects: entropy ; rotation number ; symbolic dynamics and coding
c) ergodic aspects : invariant measures ; recurrence ; ergodicity ; mixing ; ergodic theorem ; Sinai-Ruelle-Bowen measures ; unique ergodicity.

B) *Fundamentals*

a) chaotic dynamics : for example hyperbolic dynamics in the motion of a forced pendulum. Exponential divergence of the trajectories ;
b) ergodic dynamics : for example equirepartition and results from combinatorial number theory. Weak versions of Szemeredi's theorem.

C) *Advanced topics*

a) Markov partitions of dynamical systems
b) topological entropy, variational principle
c) hyperbolic automorphisms of the torus
d) homeomorphisms of the annulus, Poincaré-Birkoff theorem, applications to differential equations.
e) Twist applications

Some references : Devaney, M. Hirsch, S. Smale *Differential equations, dynamical systems, an Introduction to chaos*.
M. Brins, G. Stuck, *Introduction to Dynamical Systems,* Cambridge university press 2002

A.Katok, B. Hasselblatt, *Introduction to the modern theory of dynamical systems*, Cambridge University press, 1995

E. Akin, *The general topology of dynamical systems*, AMS

(also: Barnsley, *Fractals Everywhere)*

Harmonic analysis
Proposed content :

A) *Basics*

a) Hardy-Littlewood maximal functions, differentiations of measures, real and complex interpolation
b) Sobolev spaces, Poincaré inequalities.
c) Lipschitzian functions, differentiability
d) Geometric measure theory

B) *Fundamentals*
a) BMO spaces, BV spaces
b) Carleson measures
C) *Advanced topics*
a) Calderon-Zygmund operators.
b) Littlewood-Paley theory
c) Theorem T(b).

Some references :

E. M. Stein, Singular integrals and differentiability properties of functions, Princeton university press 1970.

K. Falconer, The geometry of fractal sets, Cambridge University Press 1984.

P. Mattila, *Geometry of sets and measures in Euclidean space*, Cambridge Studies in Advanced Mathematics 44, Cambridge University Press 1995.

F. Morgan, *Geometric measure theory, A beginner's guide,* Academic Press 1988.

E. Giusti, *Minimal surfaces and functions of bounded variation,* Monographs in mathematics, 80. Birkhauser Verlag. Basel-Boston, Massachussets, 1984.

J. Duoandikoetxea, *Fourier analysis,* Graduate Studies in Mathematics, 29, American Mathematical Society, Providence, RI, 2001.

L. Hörmander, *The Analysis of Linear Partial Differential Operators I,* Springer-Verlag, 256.

C. Sogge, *Fourier integrals in classical analysis*, Cambridge Tracts in Mathematics, 105, Cambridge University Press, Cambridge, 1993.

E. M. Stein, *Harmonic analysis: real-variable methods, orthogonality, and oscillatory integrals,* with the assistance of Timothy S. Murphy, Princeton Mathematical Series, 43, Monographs in Harmonic Analysis, III, Princeton University Press, Princeton, NJ, 1993.

COMMENTS

★ *Before the program is launched the references (both on the internet and for the books) should be updated.*

★ *For the first two semesters, we propose that the internal assessment counts for 25% and the final exam for 75%. For the third semester, the final exam will count for 100%.*

★ *Concerning Semeter 3, for each course, part A (Basics) will give 6 or 9 credits, part B plus one topic in part C will give 9 or 6 credits, the total being 15. The coordinator of the Master Program, upon the advice of the teachers team, will announce as early as possible (anyway before the course starts) his choice between 6+9 or 9+6 as well as the topic(s) in part C which will actually be covered.*

★ *When the BSc program will be revised, the content of the course Topology-Calculus (now in semester 1) should be included in the new syllabus at the BSc level. The slot which will be free at the first semester of MSc could be filled either with the course on commutative algebra, or with a course on applied mathematics.*

Kanglung, December 4, 2008.

Some internet references - December 2008

Ottawa:
Undergraduate Studies Calendar
<http://www.uottawa.ca/academic/info/regist/calendars/courses/MAT.html>

Texas Austin:
Mathematics Course Descriptions
<http://www.ma.utexas.edu/dev/math/Courses/Descriptions/>

Nottingham:
Mathematics BSc Hons
<http://www.nottingham.ac.uk/ugstudy/course.php?inc=course&code=000308>
School of Mathematical Sciences *Degree Courses*
<http://www.maths.nottingham.ac.uk/admissions/undergraduate/degree_courses/>

MIT's open courseware
<http://ocw.mit.edu/OcwWeb/web/home/home/index.htm>
Mathematics is here:
<http://ocw.mit.edu/OcwWeb/Mathematics/index.htm>

Digital Library of Georgia databases
<http://www.galileo.usg.edu/guest/>

Georgia Institute of Technology: a source of free textbooks:
<http://www.math.gatech.edu/%7Ecain/textbooks/onlinebooks.html>
In particular, this is good for first year linear algebra:
<http://joshua.smcvt.edu/linalg.html/>

Some individual **Bristol courses**
[a] Introduction to Group Theory (by Harald Helfgott):
<http://www.maths.bris.ac.uk/~mahah/aut2006.html>
[b] Second year statistics (by Jonathan Rougier):
Course description:
<http://www.maths.bris.ac.uk/study/undergrad/current_units/unit/?id=211>
Course webpage:
<http://www.maths.bris.ac.uk/~mazjcr/stats2/home.html>

A list of available **Textbooks in Mathematics**
<http://www.geocities.com/alex_stef/mylist.html>

Scanned **books on the Internet**
<http://www.eknigu.com/lib/>
<http://matchast.ru/M_Mathematics.php>
<http://www.justpasha.org/math/links/books/online.html>

Lecture Notes of **William Chen**
<http://www.maths.mq.edu.au/~wchen/ln.html>

Number Theory
<http://www.numbertheory.org/ntw/lecture_notes.html>
<http://www.numbertheory.org/book/>

Linear Algebra
Jim Hefferon, Linear Algebra
<http://joshua.smcvt.edu/linearalgebra/>

WWL Chen, Linear Algebra
<http://www.maths.mq.edu.au/~wchen/lnlafolder/lnla.html>
Edwin H. Connell, Elements of Abstract and Linear Algebra
<http://www.math.miami.edu/~ec/book/>
Alex Postnikov, Linear Algebra
<http://web.mit.edu/18.06/www/>

Updated : December 4, 2008