

*Special year in number theory at IMSc*

## Course on Multiple Zeta Values IMSC 2011

*References with comments*

by

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- A list of references on Multiple Zeta Values and Euler Sums has been compiled by Michael Hoffman  
<http://www.usna.edu/Users/math/meh/biblio.html>  
 The different categories are
  - A. DOUBLE HARMONIC SERIES (55 references)
  - B. TRIPLE HARMONIC SERIES (9 references)
  - C. MULTIPLE HARMONIC SERIES/MULTIPLE ZETA VALUES (175 references)
  - D. MULTIPLE ZETA VALUES OVER FUNCTION FIELDS (7 references)
  - E. ALTERNATING SERIES (18 references)
  - F. MULTIPLE POLYLOGARITHMS/NESTED SUMS (55 references)
  - G. FINITE MULTIPLE HARMONIC SUMS (31 references)
 It has been updated on 03/04/2011.
  
- An excellent introduction to the subject (in French) is the Bourbaki Seminar by Pierre Cartier 10 years ago:
  - P. CARTIER, *Fonctions polylogarithmes, nombres polyzetas et groupes pro-unipotents*, Séminaire Bourbaki t. **42** 53<sup>e</sup> année, 2000–2001, Exposé No°885, Mars 2001. Astérisque No.**282** (2002), 137–173.  
[http://www.numdam.org/numdam-bin/fitem?id=SB\\_2000-2001\\_\\_43\\_\\_137\\_0](http://www.numdam.org/numdam-bin/fitem?id=SB_2000-2001__43__137_0)
 It includes an annotated bibliography with references to
  - (A) General and historical texts  
 (15 reference, from Euler to Whittaker and Watson to Cartier with 3 previous Bourbaki seminars).
  - (B) Introduction to polylogarithms and polyzeta numbers  
 (23 references, including to papers by Ecalle, Zagier, Hoffman...).
  - (C) Structure of the algebra of MZV  
 (17 references to Minh, Petitot, Ihara, Goncharov...).
  - (D) Openings  
 (6 references: Ball–Rivoal, Kontsevich–Zagier, Broadhurst, Cartier...).
  
- The reference to the well-known paper by Maxim Kontsevich and Don Zagier on *periods* is
  - M. KONTSEVICH AND D. ZAGIER, *Periods*, in Mathematics Unlimited–2001 and Beyond, Springer, Berlin (2001), 771–808.  
<http://www.ihes.fr/~maxim/publicationsfrançais.html>

- Two references giving a down-to-earth introduction to this topic:
  - M. WALDSCHMIDT, *Valeurs zêta multiples. Une introduction*, J. Théor. Nombres Bordeaux 12 (2000), 581–595.  
[http://www.numdam.org/numdam-bin/fitem?id=JTNB\\_2000\\_\\_12\\_2\\_581\\_0](http://www.numdam.org/numdam-bin/fitem?id=JTNB_2000__12_2_581_0)
  - M. WALDSCHMIDT, *Multiple polylogarithms: an introduction* in Number Theory and Discrete Mathematics (Chandigarh, 2000), A. K. Agarwal et. al. (eds.), Birkhäuser, Basel (2002), 1–12.  
<http://hal.archives-ouvertes.fr/hal-00416166/fr/>
- A conjecture due to Goncharov and Manin, which states that the periods of the moduli spaces  $\mathfrak{M}_{0,n}$  of Riemann spheres with  $n$  marked points are multiple zeta values, has been solved by Francis Brown:
  - F. BROWN, *Multiple zeta values and periods of moduli spaces  $\mathfrak{M}_{0,n}$* , Annales scientifiques de l'ENS **42**, fascicule 3 (2009), 371–489.  
[arXiv:math/0606419v1](http://arxiv.org/abs/math/0606419v1)
- The two recent papers by Francis Brown are:
  - F. BROWN, *On the decomposition of motivic multiple zeta values*.  
[arXiv:1102.1310](http://arxiv.org/abs/1102.1310)
  - F. BROWN, *Mixed Tate motives over  $\mathbf{Z}$* .  
[arXiv:1102.1312](http://arxiv.org/abs/1102.1312)

These two last mentioned papers include a number of references to papers by Deligne, Goncharov, Manin, Terasoma, Minh and Petitot, Soudères, Racinet.
- A formula conjectured by Brown which he needed to complete his proof has been established by Don Zagier
  - D. ZAGIER, *Evaluation of the multiple zeta value  $\zeta(2, \dots, 2, 3, 2, \dots, 2)$* , manuscript, 2011.

**This text can be downloaded on the internet at URL**

<http://www.math.jussieu.fr/~miw/articles/pdf/MZV2011IMScRef.pdf>

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