# On the distribution of solutions of Thue's equations

by *B.Brindza, Á.Pintér, A.van der Poorten and M.Waldschmidt*

**Abstract**

Let $F \in \mathbb{Z}[X, Y]$ be an irreducible binary form of degree $n \geq 3$ and let $m$ be a positive integer having $s$ distinct prime factors. Generalizing and improving several earlier results, Bombieri and Schmidt [B&S] showed that the equation

$$|F(x, y)| = m \tag{1}$$

has at most $cn^{s+1}$ solutions in coprime integers $x$ and $y$, where $c$ is an effectively computable absolute constant. A bound linear in the degree $n$ was obtained by Brindza [B] provided that $m$ is large enough. However, this bound is still exponential in $s$. Most likely, almost all the solutions of the equation (1) are small, and around $m^{1/n}$, if $m$ is large enough comparing with the height of $F$. The following theorem based upon Baker's-method and a gap principle, illustrates the tendency. For brevity, let $M$ denote the Mahler height of the binary form $F$ (cf. [B&S]).

**Theorem.** *The equation* (1) *has at most* $2n^2(s+1) + 13n$ *solutions with*

$$\max(|x|, |y|) \geq 21n^2 M^5 m^{\frac{1}{n-2} + \frac{1}{(n-1)^2}}$$

**References.**

[B&S] E. Bombieri and W. Schmidt. On Thue's equation; Invent. Math. **88** (1987), 69–81

[B] B. Brindza. On the generators of $S$-unit groups in algebraic number fields; Bull. Austral. Math. Soc. **43** (1991), 325–329.