On the maximal length of two sequences of integers in arithmetic progressions with the same prime divisors

by

R. Balasubramanian, M. Langevin, T.N. Shorey and M. Waldschmidt

§ 1. For a positive integer n, we denote $u(n)$ for the greatest square free divisor of n i.e.

$$u(n) = \prod_{p|n} p \ ,$$

$P(n)$ for the greatest prime factor of n and $\omega(n)$ for the number of distinct prime divisors of n. We understand that $u(1) = P(1) = 1$ and $\omega(1) = 0$. Erdös and Woods (see [1] for related literature) conjectured that there exists a positive integer k with the following property: if x, y are positive integers such that, for $1 \le i \le k$, the two numbers $x+i$ and $y+i$ have the same prime factors, then $x = y$. The solution of this problem would be of interest in logic. The only known example of positive integers (x, y, k) with $1 \le x < y$, $k \ge 2$ and

$$u(x+i) = u(y+i) \text{ for } 1 \le i \le k \tag{1}$$

arise from

$$u(2^h - 2) = u(2^h(2^h - 2)),\ u(2^h - 1) = u(2^h(2^h - 2) + 1), \quad (h \ge 2)$$

and

$$u(75) = u(1215) = 15, u(76) = u(1216) = 38.$$