DEPENDENCE OF LOGARITHMS ON COMMUTATIVE ALGEBRAIC GROUPS *

Michel WALDSCHMIDT †

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Abstract

A well known conjecture states that linearly independent logarithms of algebraic numbers are algebraically independent over the rational number field. So far, it is not yet known that there exist two algebraically independent logarithms of algebraic numbers. On the other hand, D. Roy has shown that the above conjecture is equivalent to a conjectural description of the rank of matrices whose entries are either algebraic numbers or else logarithms of algebraic numbers. From this point of view, half of the conjecture is known: one deduces from the theorem of the linear subgroup that the actual rank of such a matrix is at least half the conjectural rank.

We consider a similar question for commutative algebraic groups. A partial result can be proved (using the theorem of the algebraic subgroup) ; we also study a generalization to commutative algebraic groups of the conjecture on algebraic independence of (usual) logarithms.

§1. The mutiplicative group: usual logarithms

Here is the main conjecture for (usual) logarithms of algebraic numbers:

Conjecture 1. – *Let $\ell_1, \ldots, \ell_n$ be complex numbers which are linearly independent over the field $\mathbb{Q}$ of rational numbers. Assume that the n numbers*

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