The Ramanujan Journal, à paraître

Simultaneous Approximation and Algebraic Independence

by Damien ROY and Michel WALDSCHMIDT

Abstract

We establish several new measures of simultaneous algebraic approximations for families of complex numbers $(\theta_1, \ldots, \theta_n)$ related to the classical exponential and elliptic functions. These measures are completely explicit in terms of the degree and height of the algebraic approximations. In some instances, they imply that the field $\mathbb{Q}(\theta_1, \ldots, \theta_n)$ has transcendance degree ≥ 2 over $\mathbb{Q}$. This approach which is ultimately based on the technique of interpolation determinants provides an alternative to Gel'fond's transcendence criterion. We also formulate a conjecture about simultaneous algebraic approximation which would yield higher transcendance degrees from these measures.

AMS Classification Numbers

- *Primary* 11J85
- *Secondary* 11J82 11J89 11J91

Keywords

Simultaneous approximation, transcendental numbers, Algebraic independence, Approximation measures, Diophantine estimates, Liouville's inequality, Dirichlet's box principle, Wirsing's theorem, Gel'fond's criterion, interpolation determinants, absolute logarithmic height, exponential function, logarithms of algebraic numbers, Weierstraß elliptic functions, Gamma function.