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What did I do in my life as a mathematician? I could look at my list of publications and discuss some of my old results. However, the past counts for nothing. If I am not able to prove something new now, what I did before is worthless. So here I am, day after day, working for hours pursuing some infinitely distant goal.

I am trying to "understand." I am not trying to discover something new, but rather see the "essential reasons" why some results are true. I return to the source, in an attempt to discover "the mother of all formulae." Other mathematicians' new ideas and results are irritating. I would desire very much to show that there is a simple reason why "all of that" is true (at least when I was young I had that arrogance).

Sometimes I succeeded in finding "higher reasons" why a result was valid: an idea springs up from my past work and lands just there in front of me, ordering me to do something. Why was it so easy to understand the Plancherel formula for nilpotent groups and so hard for reductive groups? I was puzzled by this question for a long time.

Suddenly, an inner voice speaks to me and tells me it is not harder. And the voice continues to give me orders: "just add terms and employ the Poisson formula." The invisible protagonist disappears from the scene, leaving me all the work. Wondrous miracle, I see the bridge of light and the work is easy to do. I am enchanted. The result becomes a logical consequence of another fact I knew, and in a blitz, I can annex a small part of mathematics to "my world." But soon, this fleeting feeling of

satisfaction disappears and I realize that there are more profound cases that my insights are unable to explain: I "explained" Harish Chandra's Plancherel measure for reductive groups, so what about Plancherel measure for symmetric spaces? To deal with this more general case, my new idea is powerless. I am unable to prove this, so the value of what I proved before is nullified.

Today I can see a dim light on a problem that has been on my mind for a long time. This is the assertion: quantization commutes with reduction. It was a beautiful conjecture of Guillemin—Sternberg, which was clearly true, but revealed itself hard to prove in general. I was able to prove an easy case. A much more difficult case was then proved by another mathematician ten years ago, using surgery. For me, this method via cuts is ugly. I would have liked to prove this conjecture with my own methods. Long after the full proof was found, I kept reorganizing my own arguments in all possible ways. If I repeated them over and over, the difficulties were bound to disappear. But they did not. These ceaseless failed attempts left a scar. I do still hope to discover where exactly the difficulty was, and today I feel I know the very small hole where the difficulty was hiding. I think it can be grasped easily. Then, maybe, I will be able to formulate and prove the theorem in a much more general way. True, for that I need someone else's idea, but just recently, I used a brilliant idea of one of my students to explain a very similar phenomenon. I believe it can also be used to understand this case. Anyway, I will try. Tomorrow.