

2017  
M2 Mathématiques, Paris Diderot  
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### Solution de l'exercice 8 feuille de TD5

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**Show that the long exact sequence for the pair  $(I \times Y, \partial I \times Y)$  breaks up into split short exact sequences**

$$0 \rightarrow H^n(I \times Y; R) \rightarrow H^n(\partial I \times Y; R) \rightarrow H^{n+1}(I \times Y, \partial I \times Y; R) \rightarrow 0.$$

**Solution.** The composite of inclusions  $\{0\} \times Y \xrightarrow{j} \partial I \times Y \xrightarrow{i} I \times Y$  induces an isomorphism in cohomology, since  $I \times Y$  deformation retracts to  $\{0\} \times Y$ . That is,  $H^n(j) \circ H^n(i)$  is an isomorphism so  $H^n(i)$  is injective which induces the short exact sequence. Let us show that it splits. There exists a retract  $r : I \times Y \rightarrow \{0\} \times Y$  such that  $rij = Id$  and  $ijr$  is homotopic to the identity. Hence  $H^*(jr) \circ H^*(i) = Id$  that is  $H^*(i)$  admits a retract. The short exact sequence then splits. Namely  $r(t, y) = (0, y)$ .