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ERRATUM

Un phénomène de Hartogs dans les variétés projectives

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Let V be a projective manifold, $\dim V \geq 2$. Let Z be an open subset in V which is pseudoconcave (see [1]). Let $\mathcal{M}(Z)$ be the field of meromorphic functions on Z. According to Andreotti [1], $\mathcal{M}(Z)$ is a finite extension of $\mathcal{M}(V)$. Hence meromorphic functions on a pseudoconcave domain in a projective manifold are algebraic. In [3], Theorem 3.2.1, we claimed that they are rationals. This claim is false. Counterexamples can be found in [6], Chapter 5, page 199 or in [2], chapter 9, example 9.1.

In order to state a Hartogs' theorem on a projective manifold, we must take care of algebraic multivaluedness. We state corrected versions of main theorem 3.2.4 in [3] and of Corollary 6 of [4]. First, we work with a pseudoconcave domain Z such that $\mathcal{M}(Z) \simeq \mathcal{M}(V)$. Then, we use a primitive element to reduce to the first case.

Theorem *Let V be a projective manifold,* $\dim V \geq 2$*. Let U be an open subset in V such that $V \backslash \bar{U}$ is pseudoconcave in the sense of Andreotti and the boundary of U is connected. Let H be the maximal compact reduced divisor in U* (*see* [3]). *Assume meromorphic functions on $V \backslash \bar{U}$ are rationals. Let $F \to V$ be a holomorphic vector bundle. Then any meromorphic section of F defined on a connected neighborhood W of ∂U extends as a meromorphic section of F over U. Moreover if that section is holomorphic on W, then its extension is holomorphic on $U \backslash H$.*

Proof In the proof of theorem 3.2.4 of [3] line 34, use the hypothesis that meromorphic functions on $V \backslash \bar{U}$ are rationals instead of using theorem 3.2.1. □

The second corrected version works for general pseudoconcave domains.

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Theorem *Let V be a projective manifold,* $\dim V \geq 2$. *Let Z be a pseudoconcave domain in the sense of Andreotti with ∂Z connected. Assume $U = V \setminus \bar{Z} \neq \emptyset$. Then there exists a branched covering $\Pi : X \to V$ with a section S on Z such that meromorphic functions on Z are pull backs by S of rational functions on X (Theorem of Andreotti). Let $\tilde{U} = X \setminus \bar{S}(Z)$. Let $F \to V$ a holomorphic vector bundle. If α is a meromorphic section of F over a connected neighborhood W of ∂Z, then α is the pulback by $S_{|W}$ of a meromorphic section of $\Pi^*(F)$ defined on a neighborhood of $\tilde{U}$. Hence, $\mathcal{M}(W)$ is a finite algebraic extension of $\mathcal{M}(U)$.*

Proof According to Andreotti's theorem (see [1]), $\mathcal{M}(Z)$ is a finite algebraic extension of $\mathcal{M}(V)$. Let $f \in \mathcal{M}(Z)$ be a primitive element. Let $\Gamma_f \subset V \times \mathbb{P}^1$ be its graph. It is contained in a unique irreducible algebraic set $V_1 \subset V \times \mathbb{P}^1$ of the same dimension (for f is algebraic). Let $\alpha_1 : V_2 \to V_1$ be the normalization and let $V_2 \to X \to V$ be the Stein factorization of $\pi_V \circ \alpha_1$ (with $\pi_V : V \times \mathbb{P}^1 \to V$ the canonical projection). Then X is normal (for V_2 is) and $\pi : X \to V$ is a finite map. Let $I_f \subset Z$ be the indeterminacy of f. Then f determines a section of π over $Z \setminus I_f$ (a normal space). Since V is normal and $\operatorname{codim} I_f = 2$, this section extends through I_f as a section $S : Z \to X$ of Π. Hence X is smooth along this section. Let $\mu : \tilde{X} \to X$ be a desingularisation of X which is biholomorphic on $S(Z)$. Then $\tilde{X}$ is projective and $\mu^{-1} \circ S(Z)$ is a pseudoconcave domain whose meromorphic functions are rationals. Let $F \to V$ be a vector bundle and let s be a meromorphic section of F over W. Let W' be the connected component of $(\mu \circ \Pi)^{-1}(W)$ which contains $(\mu^{-1} \circ S)(Z \cap W)$. Then W' is a neighborhood of $\partial \tilde{U}$. Moreover, $(\mu^{-1} \circ S)_*(s_{|W \cap Z})$ extends as $(\pi \circ \mu)^*(s)_{|W'}$, a section of $(\pi \circ \mu)^*(F)$. We apply the first theorem to this data on $\tilde{X}$. Since μ is biholomorphic outside an analytic set of codimension at least two, the first assertion is proved. Since $\Pi^{-1}(U) \subset \tilde{U}$ and $\Pi_{|\Pi^{-1}(U)} : \Pi^{-1}(U) \to U$ is finite and proper, the second assertion is proved. □

Addendum. In the proof of Proposition 3.1.3 of [3], we assumed C irreducible. However, the proof works for any *effective* divisor with pseudoconcave complement. Hence Grauert's criterion for contractibility [5] implies that the support of the divisor with pseudoconcave complement may be blown down as claimed.

References

1. Andreotti, A.: Théorème de dépendance algébrique sur les espaces complexes pseudo-concaves. Bull. Soc. Math. France **91**, 1–38 (1963)
2. Bădescu, L.: Projective geometry and formal geometry, Mathematics Institute of the polish Academy of Sciences. Mathematical Monographs (New Series), 65. Birkhauser-Verlag (2004)
3. Dingoyan P.: Un phénomène de Hartogs dans les variétés projectives. Math. Zeit. **232**, 217–240 (1999)
4. Dingoyan, P.: Monge-Ampère currents over pseudoconcave spaces Math. Ann. **320**, 211–238 (2001)
5. Grauert, H.: Über Modifikationen und exzeptionelle analytische Mengen. Math. Ann. **146**, 331–368 (1962)
6. Hartshorne, R.: Ample Subvarieties of algebraic varieties. In: Lecture Notes in Math., vol. 156. Springer-Verlag (2004)