Quivers at the boundary and global Cartan matrices.

(Philip Boalch, IBS Pohang South Korea, December 2025)

I'll start by recalling the quiver modularity theorem showing how the Nakajima quiver varieties for all the supernova quivers appear in 2d gauge theory as additive De Rham moduli spaces [CB, B08, HY]. In turn I will explain how the same quivers appear in terms of the Stokes arrows and wild monodromy relations [BY20], on the multiplicative/Betti side of the Stokes–Birkhoff–Riemann–Hilbert map, leading to a much more general story: the notion of graph (= doubled quiver) may be generalised to a diagram, and Douçot has defined a diagram (and thus a global Cartan matrix) for *any* meromorphic connection on the Riemann sphere [D21], generalising [BY20] (for connections that are tame at finite distance). Crucial use is made of work of Laumon and Malgrange on the local Fourier transform, and Martinet-Ramis on the wild fundamental group.

If time permits the recent work [B25, D25] will be discussed, aiming to classify the non-abelian Hodge graphs, i.e. the special "modular" quivers that appear in relation to the wild nonabelian Hodge moduli spaces.

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