Logarithmic connections on parahoric bundles on wild Riemann surfaces.

(Philip Boalch, Brin, Maryland 2025)

I'll have a go at describing some of the background and motivation leading up to the definition of "good" meromorphic connections on parahoric bundles [4] extending the tame case [2] (tame logahoric connections). In particular I'll try to emphasize that the parahoric weights are needed to define the enriched Riemann-Hilbert correspondence, not just to define stability and parameterise harmonic bundles. The corresponding notion of "nonabelian Hodge space" [4] generalises the definitions in the fantastic work of Simpson [7] in two ways (any pole order and any G), encompassing their hyperkahler upgrade (Biquard, Konno, Nakajima, ... that led to [1]). The (wild) nonabelian Hodge spaces may be viewed as the noncompact, infinite energy, analogues of the spaces considered by Hitchin [6] involving harmonic bundles on compact Riemann surfaces. The full story involves upgrading the topological notion of local system to the notion of Stokes $\mathcal{G}$ local system for a local system of groups $\mathcal{G}$ [5]. If time permits I'll discuss examples of the representation theory of genus zero nonabelian Hodge spaces, for example the G_2 representations of the symmetric Fricke-Klein-Vogt Betti spaces [3].

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[1] Olivier Biquard and P.B., Wild non-abelian Hodge theory on curves, Compositio Math. 140 (2004), no. 1, 179–204. arXiv:math/0111098

[2] P.B. Riemann–Hilbert for tame complex parahoric connections, Transform. Groups 16 (2011), no. 1, 27–50, arXiv:1003.3177.

[3] P.B. and Robert Paluba, Symmetric cubic surfaces as G_2 character varieties, J. Algebraic Geom. 25 (2016), 607–631. arXiv:1305.6594.

[4] P.B. Wild character varieties, meromorphic Hitchin systems and Dynkin diagrams, (2018), Geometry and Physics: A Festschrift in honour of Nigel Hitchin, pp.433–454, arXiv:1703.10376.

[5] P.B. and Daisuke Yamakawa, Twisted wild character varieties, arXiv:1512.08091

[6] Nigel Hitchin, The self-duality equations on a Riemann surface, Proc. London Math. Soc. 55 (1987), no. 3, 59–126.

[7] Carlos Simpson, Harmonic bundles on noncompact curves, J. Am. Math. Soc. 3 (1990), 713–770.

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