Connections with poles in differential geometry

(Philip Boalch, IsoMoDyn ANR opening meeting, April 2, 2026)

The isomonodromy story is part of “differential algebraic geometry” (or “algebraic gauge theory”), attaching a nonlinear algebraic differential equation to a meromorphic linear differential system. The six Painlevé equations are the simplest examples. For topologists and traditional differential geometers, isomonodromy (the theory of monodromy preserving deformations) is difficult since it involves meromorphic connections, i.e. connections with poles, and in turn the notion of “monodromy data” is generalized, leading to a generalization of the topological fundamental group, the Martinet-Ramis wild π_1 (or its finitely generated slices, the wild surface groupoids). I'll describe some of the simplest definitions and constructions that have proved useful and inspiring in this story so far, leading up to the topological symplectic structures, the wild mapping class groups and some of the diagrammatic ways to classify isomonodromy systems (Stokes diagrams, fission graphs, global Cartan matrices). If time permits I'll discuss the two apps on my webpage illustrating the “wave-particle duality” of the theory (Stokes diagrams versus global Dynkin diagrams).