ADDENDUM TO "SIMPLY-LACED ISOMONODROMY SYSTEMS"

PHILIP BOALCH

The purpose of this note is to add a reference to [1]. The main prototype for the article [1] was a symmetry of the JMMS system of isomonodromy equations, stated in [1] Thm. 1.2 (and reviewed in [1] Appx. B). The correct attribution of this result looks to be as follows:

Theorem 1.2 (see Môri [4] (20) and Harnad [2])**.** *The permutation*

$$(W_0, W_\infty, P, Q, T_0, T_\infty) \mapsto (W_\infty, W_0, Q, -P, -T_\infty, T_0)$$

preserves the JMMS equations.

The relation between this symmetry and the Fourier–Laplace transform is also noted in [4] (16) (cf. [1] footnote in Appx. B, p.59).

Beware that in [1] Thm. 1.2 the JMMS equations are slightly more general than in [3, 4, 2] since it is only assumed that T_0, T_∞ are semisimple, not necessarily regular semisimple (and this led to the supernova extension of the relevant complete bipartite fission graphs).

References

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Université Paris Cité, Sorbonne Université, CNRS, IMJ-PRG, F-75013 Paris, France.
boalch@imj-prg.fr https://webusers.imj-prg.fr/~philip.boalch/

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