ABC of quasi-Hamiltonian geometry

(Waseda, Tokyo, December 2017)

Quasi-Hamiltonian geometry was introduced by Alekseev-Malkin-Meinrenken for compact groups in the 1990's as an algebraic way to construct symplectic moduli spaces of flat connections on Riemann surfaces, complementary to more analytic approaches (Atiyah-Bott, Segal, Meinrenken-Woodward), involving loop groups in the case of surfaces with boundary. In this theory the moduli spaces are constructed out of two simple pieces C (the conjugacy classes) and D (the double), via two operations called fusion and reduction. This process can be viewed as a kind of classical version of a 2d TQFT (producing symplectic manifolds rather than vector spaces). (It was later shown that D can be derived from certain examples of C.) I will discuss the world of complex quasi-Hamiltonian geometry and some of the examples that occur when one allows connections with "wild" boundary conditions, corresponding to algebraic connections with irregular singularities. In brief there are two new classes of simple pieces that occur, called A and B (the fission spaces and the reduced fission spaces). Lots of new algebraic symplectic manifolds, such as the wild character varieties, arise by fusing them together and reducing. I will describe these pieces and draw some pictures of the types of things that can occur, no doubt underlying some generalisation of a 2d TQFT.

Irregular isomonodromy

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By describing the Stokes data of an irregular connection on a curve in terms of a *Stokes local system* (as opposed the Deligne-Malgrange-Sibuya notion of *Stokes filtrations*) there is a very simple way (similar to that used in the tame case) to describe the notion of "monodromy preserving deformations" of such connections in the general case, with no restriction on the leading term of the connections. This amounts to an extension of the viewpoint of Kimio Ueno's master thesis and the subsequent paper of Jimbo-Miwa-Ueno from around 1980. Further, inspired by Simpson's geometric viewpoint for compact curves, one sees that, in the tame case, isomonodromy is purely geometric and can be viewed as a nonabelian Gauss-Manin connection (i.e. a nonlinear analogue of Picard-Fuchs equations), attached to any smooth family of complex curves. This viewpoint may be extended to the irregular case by defining the intrinsic notion of "irregular curves" (or "wild Riemann surfaces"), so that an isomonodromy connection is attached to any admissible family of irregular curves.