1ST WORKSHOP ON WILD DYNAMICAL SYSTEMS

Banyuls sur Mer, France, 2020

31/08 — 04/09

ERC project 818737 – Emergence of wild differentiable dynamical systems

Monday	Tuesday	Wednesday	Thursday	Friday
9h40 Opening	9h30 Daniel Peralta-Salas	9h30 Frédérique Faure	9h30 Dmitry Turaev (V)	9h30 Yushi Nakano (V)
10h Francois Ledrappier				10h15 Polina Vytnova (V)*
11h15 2' short presentation	11h00 Anna Florio	11h00 Julia Slipantschuk	11h Mathieu Helfter	11h00 Ivan Shilin (V)
11h30 Cagri Sert				
				14h Enrique Pujals (V)
17h Sebastien Biebler	17h Charles Fougeron		17h Agnieszka Zelerowicz (V)	

(V) : video conference
* : To be confirmed

Organizing and scientific committee : Pierre Berger, Pierre-Antoine Guihéneuf

`https://webusers.imj-prg.fr/~pierre.berger/1_Workshop_on_wild_dynamics/`

FRANCOIS LEDRAPPIER - Robustness of Liouville measure under a family of unstable diffusion

We consider the geodesic flow on a closed connected C^∞ Riemannian manifold and the diffusion $X + \epsilon\Delta^u$, where Δ^u is the Laplacian along the unstable manifolds. As ϵ goes to 0, the unique stationary measure converge to Liouville measure. This is a joint work with Lin Shu (Peking Univ.).

CAGRI SERT - Equilibrium measures of affine fractals

We will start by giving an overview of dimension theory and equilibrium measures of similarity and affine fractals making connections with repellers of expanding maps. In a second part, we will explain a dimension gap result yielding in particular a sort of converse to a classical result of Hutchinson : under an irreducibility assumption, an affine fractal admitting a Bernoulli equilibrium measure with matching Hausdorff dimension is a similarity fractal. In the last part, I will explain some ways to turn around this gap by consideration of subsystems. Based on joint works with Ian Morris.

SÉBASTIEN BIEBLER - Polynomial automorphisms of $\mathbb{C}^2$ with a wandering Fatou component

In a joint work with Pierre Berger, we prove the existence of a locally dense set of real polynomial automorphisms of $\mathbb{C}^2$ displaying a wandering Fatou component. This answers a problem raised by Bedford and Smillie (1991). I will also discuss the statistical behavior of the points inside the component. In particular, this behavior is historical with a stretched exponential emergence.

DANIEL PERALTA-SALAS - A panoramic view of steady Euler flows

The motion of an ideal fluid flow in equilibrium is governed by the stationary Euler equations, a non-linear system of first order PDEs. The same equations describe magnetohydrodynamic equilibria in plasma reactors. In 1965, V.I. Arnold proved a remarkable theorem describing the dynamics of steady Euler flows : either they are laminar (a sort of integrability) or they are Beltrami fields, which may exhibit different features of chaos. How wild the dynamics of a Beltrami flow can be compared with a generic volume-preserving field was the object of several conjectures by Arnold. In this talk I will review the current state of the art of this subject, emphasizing some open problems and lines of future research.

ANNA FLORIO - Explosion phenomena in topological dynamics

In the 70s phenomena of Ω-explosions were studied by Palis, Shub-Smale, Nitecki ... They concern discontinuous jumps in the size of the non-wandering set. For a homeomorphism on a compact metric space, several notions of recurrent sets can be introduced, such as the chain recurrent or the generalized recurrent set. Can explosion phenomena also occur for these sets? We present some examples and discuss some criteria characterizing such explosions. Joint work with Olga Bernardi and Jim Wiseman.

CHARLES FOUGERON - Dynamics of simplicial systems and fractal geometry

A simplicial system defines a dynamical system on a graph which generalise (multidimensional) continued fraction algorithms as well as Rauzy induction on interval exchange maps. Using random walks techniques I will explain a graph criterion to show ergodicity of such systems. Moreover it provides us with a convenient tool to prove exponential tail results and start a thermodynamical study of their dynamics.

I will finish by explaining some applications to computing fractal dimensions of some spaces like spaces of real numbers with bounded continued fractions expansion and Rauzy gasket.

FRÉDÉRIQUE FAURE - What is the spectrum of the geodesic flow on a negatively curved manifold ?

In the 80's, D. Ruelle, D. Bowen and others have introduced probabilistic and spectral methods in order to study deterministic chaos ("Ruelle resonances"). For example, a geodesic flow on a strictly negative curvature Riemannian manifold is chaotic : each trajectory is strongly unstable and its behavior is unpredictable. A smooth probability distribution evolves also in a complicated way since it acquires higher and higher oscillations. Nevertheless this evolution is predictable in the sense of distributions and converges towards equilibrium. Following this approach and use of microlocal analysis, one obtains that long time fluctuations of classical probabilities are described by an effective quantum wave equation. This may be surprising because there is no added quantization procedure. We will explain the concepts and results using different simple models. Joint work with Masato Tsujii.

JULIA SLIPANTSCHUK - Lagrange approximation of transfer operators

This talk will be about approximation of spectral data of transfer operators associated to holomorphic map-weight systems using an effective numerical scheme based on Lagrange interpolation. In particular, for one-dimensional systems satisfying certain complex contraction properties, spectral data of the approximants converge exponentially to the spectral data of the transfer operator.

DIMITRY TURAEV (V) - Lorenz-like attractors in orientation-reversing three-dimensional Henon maps

We investigate bifurcations of a triply degenerate orbit with multipliers $(-1, i, -i)$ and show that these bifurcations can lead to the birth of discrete Lorenz attractors. The problem is reduced to the study of a normal form – a system of three differential equations with $\mathbb{Z}^4$-symmetry, for which we establish the existence of both the classical Lorenz attractor and a new type of Lorenz attractor (which emerges due to bifurcations of non-hyperbolic periodic orbits).

MATHIEU HELFTER - Notions and comparisons of scales of a measure

My master thesis, supervised by Pierre Berger, aims to generalise some classical results from dimension theory to what we introduce as *scales*. Scales' framework allows to open out some results about Hausdorff, packing, covering and quantization dimensions of distributions. The main result shows that the quantization scale of a measure is an upper bound of its packing-scale. Scales are motivated by the study local emergence introduced by Pierre Berger and allow to compare *orders* of measures and borelians.

AGNIESZKA ZELEROWICZ (V) - Large intersection classes for pointwise emergence

Recently, P. Berger introduced a concept of metric emergence to quantitatively study the non-uniqueness of statistics (such as Newhouse or KAM phenomena). Later, S. Kiriki, Y. Nakano, and T. Soma introduced a concept of pointwise emergence to quantitatively study non-existence of averages, and constructed a residual subset of the full shift with high pointwise emergence. In this talk I will consider the set of points with high pointwise emergence for topologically mixing subshifts of finite type. I will present my joint work with Yushi Nakano, where we show that this set has full topological entropy, full Hausdorff dimension, and full topological pressure for any Hölder continuous potential. Furthermore, we show that this set belongs to a certain class of sets with large intersection property.

YUSHI NAKANO (V) - Irregular sets with high emergence

Recently, P. Berger introduced a concept of metric emergence to quantitatively study the non-uniqueness of statistics (such as Newhouse or KAM phenomena). In this talk, we consider its "pointwise" version to quantitatively study the set of points without time averages (called the irregular set, or the set with historic behavior). We will see that the set of points with high emergence is remarkably large in several viewpoints for abundant dynamical systems : (i) There is a dense subset of any Newhouse open set each element of which has high pointwise emergence on a positive Lebesgue measure set (joint work with S. Kiriki and T. Soma). (ii) Any topologically mixing subshift of finite type has high pointwise emergence on a residual set with full topological entropy and full Hausdorff dimension (joint work with A. Zelerowicz).

IVAN SHILIN (V) - Attractors of direct products

When a direct product of two dynamical systems is considered, it is always tempting to assume that the attractor of the product coincides with the direct product of their attractors. Although this holds, indeed, for so-called maximal attractors (which are simply intersections of the iterates of dissipative domains), this is not true in general for several other types of attractors, namely Milnor, statistical, and minimal attractors (defined using a measure on the phase space), even for systems as nice as smooth vector fields in two dimensions. The talk will be devoted to the corresponding counterexamples. The example for minimal attractors can be tweaked to make it have an analogous property for the supports of SRB-measures.

Joint work with Stanislav Minkov.

ENRIQUE PUJALS (V) - Dynamics of Henon maps with zero entropy

I will focus on surface diffeomorphisms with zero entropy : can the dynamics of these 'simple' systems be described? How does it bifurcate to positive entropy systems?

These questions will be discussed for a class of volume-contracting surface diffeomorphisms whose dynamics is intermediate between one-dimensional and general surface dynamics. It includes the dynamics of any Hénon diffeomorphism with Jacobian smaller than $1/4$.

In particular,it is shown that any Hénon map with zero entropy and Jacobian smaller than $1/4$ 'can be renormalized'. As a consequence, we obtained a two-dimensional version of Sharkovsky's theorem about the set of periods of interval maps.