# Real and Complex Wild Dynamics

**Dates:** May 16-22 2026
**Venue:** the Arctic University of Norway, Tromsø at this room

**Programme:**

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| **Monday May 18** | 09:30 – 10:00 | Opening |
| | 10:00 – 11:00 | Dmitry Turaev |
| | 11:00 – 11:30 | Coffee break |
| | 11:30 – 12:30 | Sébastien Biebler |
| | 12:30 – 14:30 | Lunch |
| | 14:30 – 15:00 | Nigel Yoccoz |
| | 15:00 – 16:00 | Mathieu Helfter |
| | 16:00 – 16:30 | Coffee break |
| | 16:30 – 17:30 | Maria Saprykina |
| **Tuesday May 19** | 09:30 – 10:30 | Sylvain Crovisier |
| | 10:30 – 11:00 | Coffee break |
| | 11:00 – 12:00 | Dmitrii Mints |
| | 12:00 – 12:30 | Magnar Johnsen |
| | 12:30 – 14:30 | Lunch |
| | 14:30 – 15:30 | Francisco Torres |
| | 15:30 – 16:00 | Coffee break |
| | 16:00 – 17:00 | Yann Delaporte |
| **Wednesday May 20** | 09:30 – 10:30 | Amie Wilkinson |
| | 10:30 – 10:45 | Coffee break |
| | 10:45 – 11:45 | Anna Florio |
| | 11:45 – 12:30 | Quick lunch |
| | 12:30 – 20:00 | Excursion to Sommarøy |
| **Thursday May 21** | 09:30 – 10:30 | Eva Miranda |
| | 10:30 – 11:00 | Coffee break |
| | 11:00 – 12:00 | Sebastian Perez |
| | 12:00 – 14:00 | Lunch |
| | 14:00 – 15:00 | Jaime Paradela Diaz |
| | 15:15 – 16:15 | Yushi Nakano |
| | 16:15 – 16:45 | Coffee break |
| | 16:45 – 17:45 | Dongchen Li |
| **Friday May 22** | 09:30 – 10:30 | Meysam Nassiri |
| | 10:30 – 11:00 | Coffee break |
| | 11:00 – 12:00 | Inmaculada Baldomá |
| | 12:00 – 14:00 | Lunch |
| | 14:00 – 15:00 | Alexandre Delplanque |
| | 15:00 – 15:30 | Coffee break |
| | 15:30 – 16:30 | Raphaël Krikorian |

**Titles and abstracts:**

Dlmitry Turaev (Imperial College)

*Ergodic averaging for partially-integrable systems.*

We discuss adiabatic invariants for partially-integrable Hamiltonians with slowly changing parameters.

Sébastien Biebler (Université Paris Cité)

*Newhouse phenomenon: prevalence and applications.*

In the 60s, in a mathematical optimistic movement aiming to describe a typical dynamical system, Smale conjectured the density of uniform hyperbolicity in the space of $C^r$-diffeomorphisms f of a compact manifold M.

In the 70s, Newhouse discovered an extremely complicated new phenomenon, resulting in an obstruction to Smale's conjecture. Specifically, he showed the existence of (nonempty) open sets $U$ of $C^2$-diffeomorphisms of a surface M such that a generic map $f$ in $U$ has infinitely many attracting periodic points.

In this talk, I will review some applications of this phenomenon, in particular in the holomorphic context (namely, the existence of wandering Fatou components for polynomial automorphisms of $\mathbb{C}^2$), and I will discuss the question of its typicality (in the smooth context and in the Kolmogorov sense).

This talk is based on joint works with Pierre Berger.

Nigel Yoccoz (The Arctic University)

*The fluctuations of small mammal populations: an ecological riddle leading to interesting mathematical dynamics*

Mathieu Helfter (ISTA)

*Rigidity of $\beta$-Mather function for generalized standard maps.*

The $\beta$-function describes the minimal average action associated with invariant measures of prescribed rotation number. In this talk, I will present rigidity phenomena for generalized standard maps defined by analytic even potentials exhibiting KAM phenomena on a fixed set of Diophantine rotation numbers. Using KAM theory in infinite-dimensional Fréchet spaces together with complex extensions of the parametrizations of the KAM curves, we show that, on finite-dimensional spaces of perturbations, the $\beta$-Mather function is generically injective. Moreover, nontrivial polynomial deformations of a generic potential are not $\beta$-preserving.

Maria Saprykina (KTH)

*KAM-rigidity for parabolic affine abelian actions on the torus.*

Two famous instances of local rigidity for $\mathbb{Z}^2$-actions are the classical KAM rigidity of Diophantine toral translations and smooth rigidity of hyperbolic or partially hyperbolic higher rank actions proved by Damjanovic and Katok. To complete the study of local rigidity of affine $\mathbb{Z}^2$-actions on the torus, we address the case of parabolic affine actions.

Consider an affine $\mathbb{Z}^2$-action $(a, b)$ on $\mathbb{T}^d$ generated by two commuting parabolic affine maps of the form $a(x) = A(x) + \alpha$, $b(x) = B(x) + \beta$, where $A, B \in SL(d, \mathbb{Z})$.

We say that the action $(a, b)$ is *tame KAM-rigid under volume-preserving perturbations* if there exists $\sigma \in \mathbb{N}$, $r_0 \geq \sigma$ and $\varepsilon > 0$ satisfying the following. If $r \geq r_0$ and $(F, G) = (a+f, b+g)$ is a smooth $\lambda$-preserving $\mathbb{Z}^2$-action such that

$$\|f\|_r \leq \varepsilon, \quad \|g\|_r \leq \varepsilon, \quad \widehat{f} := \int_{\mathbb{T}^d} f d\lambda = 0, \quad \widehat{g} := \int_{\mathbb{T}^d} g d\lambda = 0,$$

then there exists $H = Id + h \in \mathrm{Diff}^\infty_\lambda(\mathbb{T}^d)$ such that $\|h\|_{r-\sigma} \leq \varepsilon$ and

$$H \circ (a + f) \circ H^{-1} = a, \quad H \circ (b + g) \circ H^{-1} = b.$$

Let $\mathcal{T}(A, B)$ denote the set of possible translation parts $(\alpha, \beta)$ in the affine actions with linear part $(A, B)$, that is $\mathcal{T}(A, B) := \{\alpha, \beta \in \mathbb{R}^d \mid (A - Id)\beta = (B - Id)\alpha\}$.

We present the following dichotomy for a commuting pair $(A, B)$ of parabolic matrices, where $A$ is step-2 (i.e., $(A - Id)^2 = 0$):

($i$) either for any choice of $(\alpha, \beta) \in \mathcal{T}(A, B)$, the affine action $(a, b)$ has a rank one factor that is different from a nonzero translation, in which case the action is not KAM-rigid,

($ii$) or for almost every choice of $(\alpha, \beta) \in \mathcal{T}(A, B)$ the action $(a, b)$ is ergodic and tame KAM-rigid under volume preserving perturbations.

This is the result of a joint work with D. Damjanovic and B. Fayad.

Sylvain Crovisier (CNRS - Université Paris Saclay)

*Organizing the non-wandering set of mild dissipative diffeomorphisms of the disk.*

For mild dissipative of the disk, we will show that any ergodic measure is either metric isomorphic to an odometer or it is contained in an homoclinic class . That result will be used to decompose the non-wandering set into different maximal transitive pieces. The main technique used is a (new) closing lemma that we will outline the proof. This is a joint work with Enrique Pujals.

Dmitrii Mints (Imperial College)

*Corank-2 homoclinic tangencies*

Our research is aimed at studying the dynamics of multidimensional diffeomorphisms from the Newhouse domain, that is, open regions in the space of maps where maps with homoclinic tangencies are dense. We prove that in the space of smooth and real-analytic multidimensional maps in any neighborhood of a map such that it has a bi-focus periodic orbit whose invariant manifolds are tangent, there exist open regions (which are subdomain of the Newhouse domain) where maps with high order homoclinic tangencies of corank 2 (invariant manifolds forming the tangency have a plane of common tangent vectors) are dense and maps having universal two-dimensional dynamics are residual. This is a joint work with D. Turaev.

Magnar Johnsen (The Arctic University)

*Auroral and space weather research in Northern Norway.*

The upper atmosphere has been studied from Northern Norway for hundreds of years. We will go through the historic developments of this research from early pioneers, to cutting edge scientific infrastructure and the challenge of tackling the increasing threat of space weather towards modern infrastructure.

Francisco Torres (University of Seville)

*Invariants of coadjoint orbits of volume-preserving diffeomorphisms*

The Lie group of volume-preserving diffeomorphisms on a simply connected manifold acts on its dual Lie algebra (the space of exact two forms) by pull-back. In this talk we are interested in the orbits of this co-adjoint action and, more precisely, on functionals on 2-forms that are constant on the orbits. In 2D, the area form identifies 2-forms with functions, and A. Izosimov,

B. Khesin and M. Mousavi gave a complete classification of the invariants for a generic co-adjoint orbit in terms of the measured Reeb graph of the function. In 3D, the volume form identifies 2-forms with vector fields. The asymptotic linking number of the vector field is the paradigmatic example of a co-adjoint invariant. It has remarkable uniqueness properties: many other invariant functionals reduce to functions of helicity. I will present joint work with Robert Cardona and Julian Chaidez where we examine how severely this uniqueness can fail. On integral homology spheres, we show that for every C1-open set of nonvanishing exact fields of fixed helicity, some other global dynamical invariant is continuous and non-constant in that set; the Ruelle invariant if some field is non-Anosov, and topological entropy otherwise.

Yann Delaporte (Sorbonne University)

*Analytic pseudo-rotations on the sphere, the disk, and the cylinder: minimal ergodicity and emergence.*

The Approximation by Conjugacy method of Anosov and Katok was recently generalized to the analytic setting by P. Berger through the AbC Principle on the sphere, the disk, and the cylinder. This principle makes it possible to construct analytic pseudo-rotations exhibiting properties concerning *almost every orbits*, such as transitivity, ergodicity, or high emergence. However, it was not sufficient to obtain properties involving *all orbits* such as minimal ergodicity. To overcome this difficulty, we introduced a generalization of the AbC Principle, called the AbC$^*$ Principle.

In this talk I will first present the main results, together with an example illustrating the implementation of Anosov-Katok type construction within Berger's framework. I will then discuss the Principle itself and explain how the analytic aspects of the construction are achieved.

Finally, after describing the obstruction to obtaining minimal ergodicity, I will introduce the AbC$^*$ Principle. This new framework allows us to prove the existence of analytic pseudo-rotations exhibiting minimal ergodicity.

Amie Wilkinson (The university of Chicago)

*Centralizers gone wild*

The centralizer $Z(f)$ of a diffeomorphism $f : M \to M$ of a closed manifold $M$ is the group of all diffeomorphisms commuting with $f$; it is the collection of dynamical symmetries of $f$. The centralizer of $f$ always contains the group $< f >$ generated by $f$ as a normal subgroup, and conjecturally the two typically coincide (that is, "the generic diffeomorphism has only trivial symmetries"). But what happens when $Z(f)$ is bigger than $< f >$? I will discuss examples and results that explore this question, focusing on the interplay between the dynamics of f and the dynamics of $Z(f)$.

Anna Florio (Université Paris Dauphine)

*Genericity of transverse homoclinic points for analytic convex billiards.*

n a work with Inmaculada Baldomà, Martin Leguil and Tere M-Seara, we prove that a generic analytic strongly convex billiard has, for every rational rotation number, a hyperbolic periodic orbit with homoclinic intersections, all of which are transverse. As a consequence, we deduce that positive topological entropy is a generic property among trigonometric polynomial billiard tables.

Eva Miranda (UPC)

*The Shape of the Undecidable*

Chaos is often regarded as the ultimate obstruction to prediction. Since the pioneering work of Stephen Smale, the horseshoe has become the canonical geometric signature of chaos: a simple topological mechanism generating sensitive dependence on initial conditions, symbolic dynamics, and infinitely many periodic orbits. In this sense, the horseshoe may be viewed as the shape of chaos.

But is chaos truly the final frontier of unpredictability?

Recent developments suggest the existence of a deeper phenomenon: undecidability in dynamics. Beyond systems whose long-term behaviour is merely difficult to predict, there are dynamical systems for which certain natural questions are algorithmically impossible to answer. No computer program can decide, in full generality, whether a trajectory reaches a given region, whether a prescribed behaviour occurs, or whether a qualitative event will ever happen.

In this talk, I will present recent results showing how computational universality emerges in dynamical systems associated to physical systems, including billiards, fluid flows, and conservative systems. Undecidability thus appears as a new facet of wild dynamics, one that this workshop seeks to address. I will explain how symbolic mechanisms reminiscent of horseshoes can be enhanced to encode full computation, turning classical phase spaces into machines capable of simulating arbitrary algorithms.

I will also discuss the implications of these ideas for celestial mechanics. The question of the long-term stability of the Solar System has recently been addressed with remarkable mathematical precision, while the three-body problem has long stood as one of the great cradles of chaos, where horseshoes and intricate symbolic dynamics naturally arise. This raises a fundamental question: can gravitational systems also exhibit undecidability? Is the three-body problem not only chaotic, but algorithmically unpredictable in a deeper sense?

This perspective suggests a new hierarchy of complexity in dynamical systems: from integrability, to chaos, to universality, and finally to undecidability. If the horseshoe is the shape of chaos, what then is the shape of the undecidable?

Sebastian Perez (PUC Valparaiso)

*Robust heterodimensional cycles of co-index two.*

In dimension four, we study co-index two heterodimensional cycles, i.e., cycles associated with two saddles whose unstable indices differ by 2. In a partially hyperbolic setting, we introduce the notion of non-escaping for such cycles and prove that they can be C1 approximated by diffeomorphisms exhibiting robust heterodimensional cycles of co-index one and two.

This is joint work with P. Barrientos, L. Díaz, Y. Ki and C. Lizana.

Jaime Paradela Diaz (PSU)

*Robustly transitive behavior in symplectic dynamics*

We consider the direct product of two symplectomorphisms, one of which exhibits a basic set and the other one a non-degenerate elliptic equilibrium. Under a domination condition we show that a typical real-analytic deformation of this system displays large robustly transitive sets. As a corollary of our construction we also obtain new examples of real-analytic robustly transitive symplectomorphisms which are not uniformly hyperbolic. To establish these results we develop perturbation techniques to create blender horseshoes in the real-analytic setting and import ideas from control theory which show that, typically, these objects have a large domain of influence.

Yushi Nakano (Hokodai University)

*Complexity of Historic Behavior.*

An orbit has historic behavior if its empirical measures fail to converge. In this talk, I will discuss quantitative and structural refinements of this basic non-convergence, focusing on high pointwise emergence, the B1/B2 distinction for higher-order Birkhoff averages, and wild historic behavior defined via upper occupation capacities.

These notions detect different aspects of orbit statistics: the metric complexity of empirical-measure accumulation sets, the persistence of oscillations under higher-order Cesàro averaging, and the phase-space size of statistical carriers. I will describe symbolic constructions showing that they are independent: every combination of high/non-high emergence, B1/B2 type, and wild/non-wild behavior occurs.

I will then explain how such symbolic patterns can be realized on sets of positive Lebesgue measure in the Newhouse domain. If time permits, I will also discuss the non-convergence of Lyapunov exponents and non-physical-like behavior.

Dongchen Li (Fudan)

*Abundance of families of diffeomorphisms displaying infinitely many sinks for an open set of parameter values.*

Let $f$ be any $C^r$ $(2 \leq r < \infty)$ diffeomorphism of a smooth manifold $M$ of dimension $\geq 3$, which has a heterodimensional cycle connected to a homoclinic tangency. Let $C^r(\mathbb{B}^d \times M, M)$ be the space of $C^r$ families with parameters in the $d$-dimensional unit ball.

We show that, for any integer $d > 0$, there exists an open set $\mathbf{U} \subset C^r(\mathbb{B}^d \times M, M)$ arbitrarily close to the constant family $\{\tilde{f}_t = f\}$ such that, densely in $\mathbf{U}$, a family $\{f_t\}$ exhibits locally constant tangencies: for any $t \in \mathbb{B}^d$, $f_t$ has a tangency between the invariant manifolds of the same periodic point for all close $t$ values. The proof is based on the observation that a generalization of Berger's parablenders can be constructed near heterodimensional cycles.

As an application, we show that families $\{f_t\}$, which have at some $t_0$ a simple homoclinic tangency to a weakly dissipative saddle, generically display infinitely many sinks for an open set of parameter values. This is a joint work with Dmitry Turaev

Meysam Nassiri (IPN)

*Cantor Sets in Higher Dimensions.*

While all Cantor sets are topologically equivalent, their geometric properties vary drastically. This geometric diversity is further enriched as the dimension of the ambient space increases. What dictates the infinitesimal geometry of Cantor sets generated by smooth expanding maps? Under what conditions is it impossible to separate two intersecting Cantor sets via small perturbations? Furthermore, what precise role do their dimensions and regularities play in this stability problem? In this talk, we will address these fundamental questions in both $\mathbb{R}^n$ and $\mathbb{C}^n$ for all $n$. (Joint work with M. Zareh Bidaki)

Inmaculada Baldomá (UPC)

*Transverse homoclinic points in area preserving maps*

We study the existence of transverse homoclinic points in two settings of area preserving maps. The first one corresponds to the so-called generalized standard maps having a weak saddle and the second one deals with the hyperbolic islands appearing near elliptic points. We prove an asymptotic formula measuring the splitting of separatrices which turns out to be exponentially small in some singular parameter. Our methods are constructive and suitable to be implemented by computer assisted proofs.

This a join work with I. de Blasi, D. Gil, M. Guardia, P. Martin and T. Seara

Alexandre Delplanque (Sorbonne University)

*SRB measures for smooth surface endomorphisms.*

We are interested in the asymptotic behaviour of the orbits of smooth hyperbolic maps. In the 1970s, the works of Sinai, Ruelle, and Bowen showed that for Axiom A diffeomorphisms, Lebesgue-almost every point is in the basin of an SRB measure, meaning that its orbit is asymptotically distributed according to that measure. SRB measures are invariant measures whose conditionals along the unstable manifolds are absolutely continuous with respect to the Lebesgue measure.

For nonuniformly hyperbolic maps, the existence of SRB measures is conjectured by Viana. A recent result by Ben Ovadia and Burguet shows a version of this conjecture for smooth diffeomorphisms. In this talk, we explain how to obtain a similar version of this conjecture for noninvertible smooth surface maps with critical points.

Raphaël Krikorian (Ecole Polytechnique)

*Exotic rotation domains and Herman rings for quadratic Hénon maps.* Quadratic Hénon maps are polynomial automorphism of $\mathbb{C}^2$ of the form $h : (x, y) \mapsto (\lambda^{1/2}(x^2 + c) - \lambda y, x)$. They have constant Jacobian equal to $\lambda$ and they admit two fixed points. If $\lambda$ is on the unit circle (one says the map $h$ is conservative) these fixed points can be elliptic or hyperbolic. In the elliptic case, a simple application of Siegel Theorem shows (under a Diophantine assumption) that $h$ admits many quasi-periodic orbits with two frequencies in the neighborhood of its fixed points. Surprisingly, in some hyperbolic cases, S. Ushiki observed some years ago what seems to be quasi-periodic orbits though no Siegel disks exist. I will explain why this is the case. This theoretical framework also predicts and mathematically proves, in the dissipative case ($\lambda$ of module less than 1), the existence of (attractive) Herman rings. These Herman rings, which were not observed before, can be produced in numerical experiments.