Esthetopies
Varieties of sensitive spaces

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It is difficult to write an explicit (and not tautological) definition on art or mathematics. It is like if the core of these fields was unsayable.

A nice observable for both fields is:

"A mathematical breakthrough usually represents a new way of thinking." W. Thurston

In the mathematical activity there is also quest of the true in a mysterious, but seducing world. The feeling of the mathematical object we have during our research, including the quest of something basic but not already seen by the others.

"We need to pay much more attention to communicating not just our definitions, theorems, and proofs, but also our ways of thinking." W. T.

Feeling a space

A mathematician wishes to define the spaces he meets, and to describe their properties. A musician cares about the sounds of a space, in particular those emerging while one touches it or when one speaks inside it. Others would love to draw it, photography the light which circulate over it or film it metamorphoses.

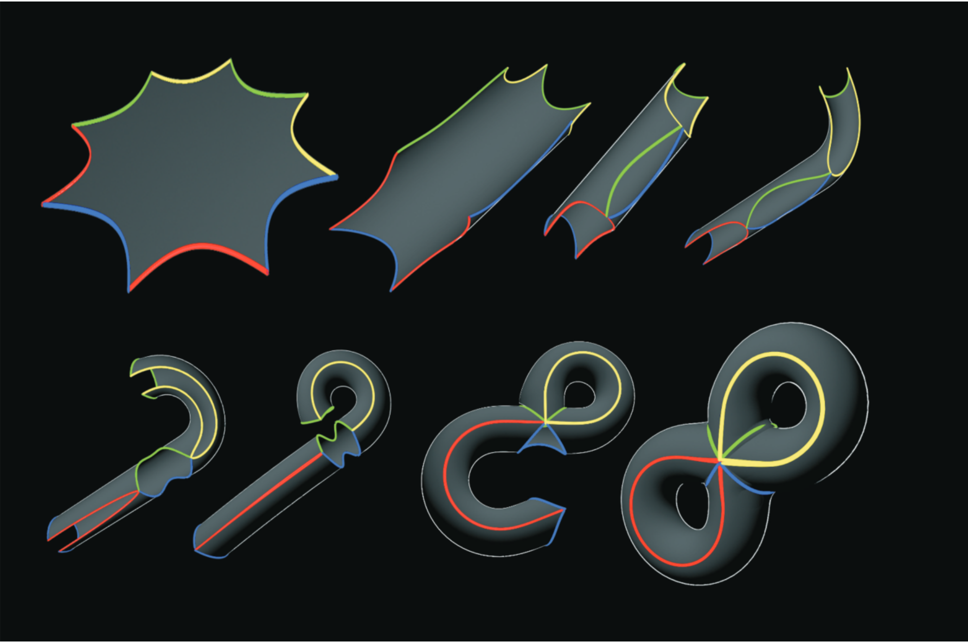
A space lives if it is visited. It can be visited for his utility ; but also for its own beauty and its mysteries. It might be traveled for his fertility: an entrance toward new spaces, news curiosities. Beloved for being savage and omnipresent, or classical and amazing... the discovery of a new space can be disturbing.

What is our subject?

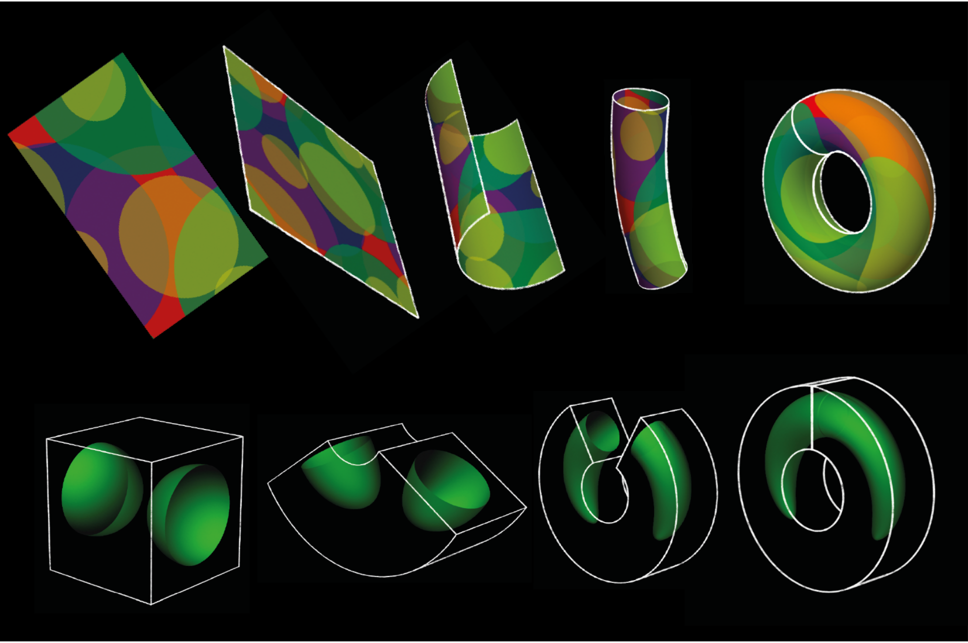
A curve is a shape which locally look like a straight line : Every point of a curve belongs to a segment.

A surface is a shape which locally looks like a (Euclidean) plane : Every point of a surface belongs to a disk.

We will be interested in spaces - *called 3-manifolds* - which locally look like the Eclidean space : Every point of a 3-manifold belongs to a ball.



Construction of a space from a polyhedron, screen shots of a film done with Pierre-Yves Fave 2013



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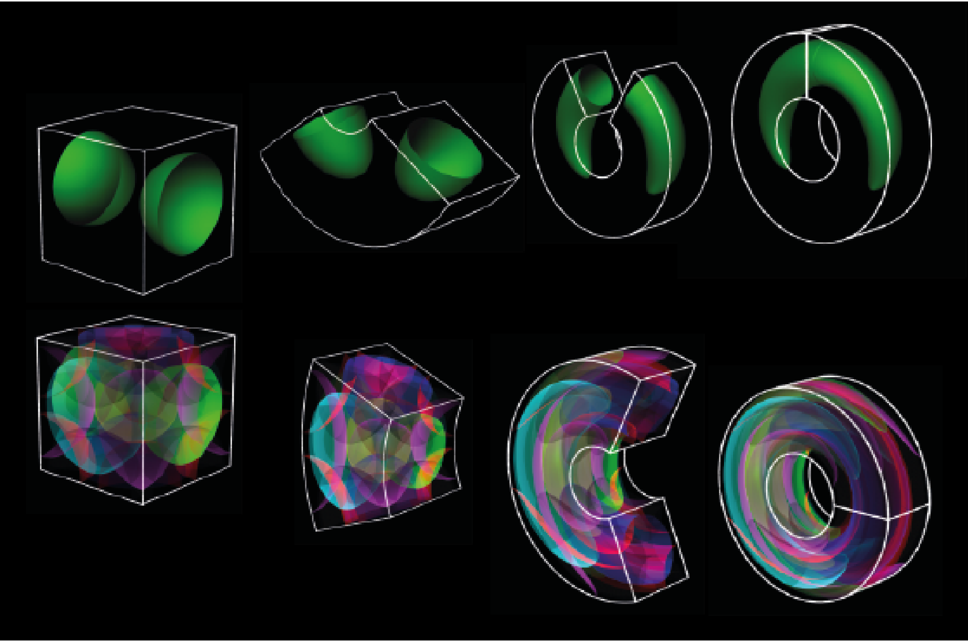
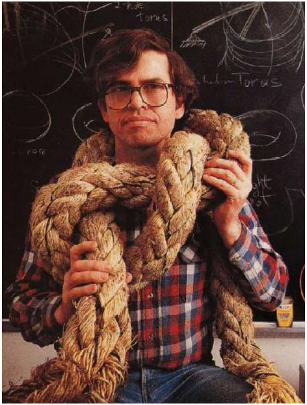


Construction of a space from a polyhedron, screen shots of a film done with
Pierre-Yves Fave 2013

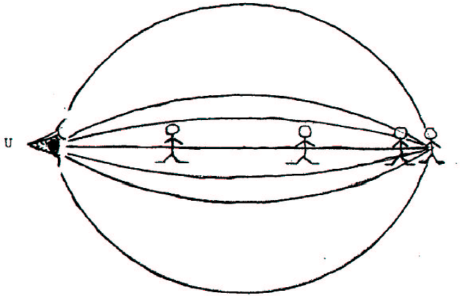
Problem :

How to feel/think these spaces although they cannot be made inside our space?



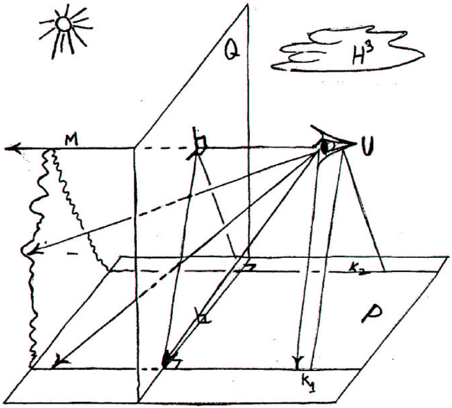
Following Thurston the best way to think of these spaces is to imagine them as large as a house and then go inside.

Thurston's drawing of perception in a Elliptic space.

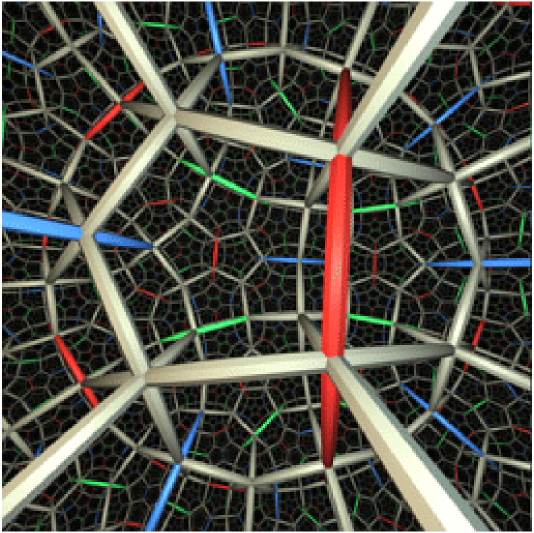


"It would nonetheless be distressing to live in elliptic space, since you would always be confronted with an image of yourself, turned inside out, upside down and filling out the entire background of your field of view."
W. P. Thurston, The Geometry and Topology of Three-Manifolds, chp 2.

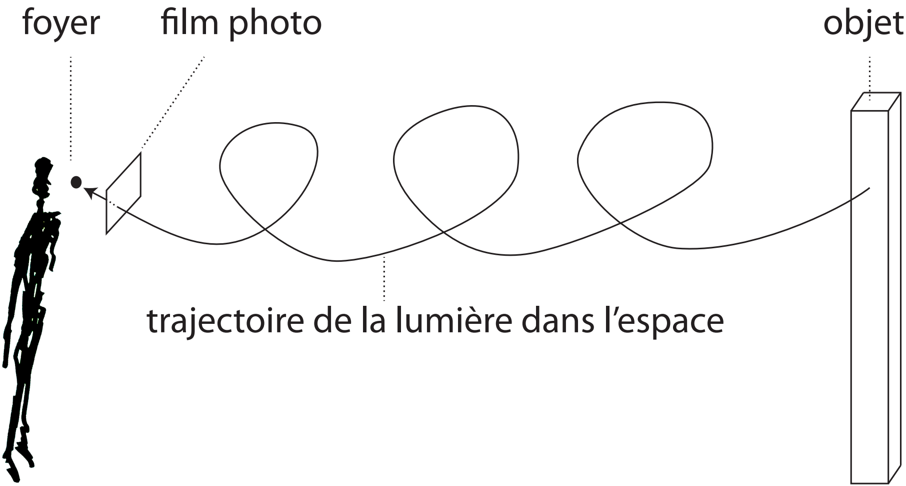
Thurston's drawing of the perception in the hyperbolic space $\mathbb{H}^3$.

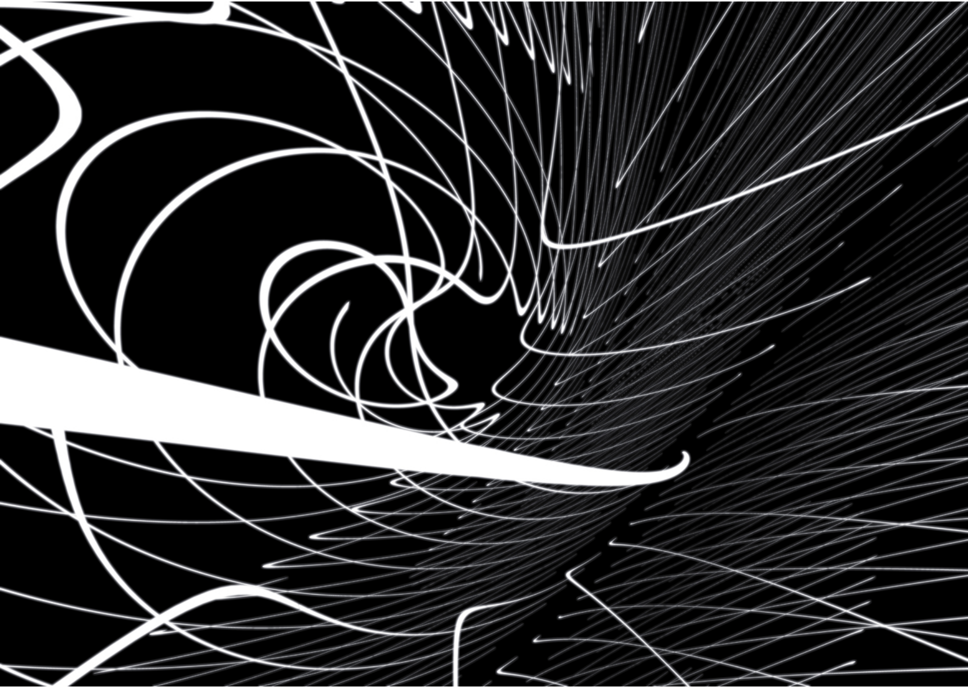


"A sphere in Euclidean space with radius r has constant curvature $1/r^2$. Thus, hyperbolic space should be a sphere of radius i."
W. P. Thurston, The Geometry and Topology of Three-Manifolds, chp 2.



State of the art, back to the Geometry Center in the 90's, screen shot of the movie 'not knot'

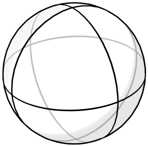

foyer
film photo
objet
trajectoire de la lumière dans l'espace

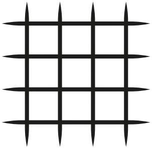


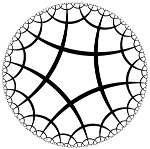
A straight line in a NIL manifold, P.B. 2014

For surfaces, they are 3 geometries.

$\mathbb{S}^2$ $\quad$ $\mathbb{E}^2$ $\quad$ $\mathbb{H}^2$.

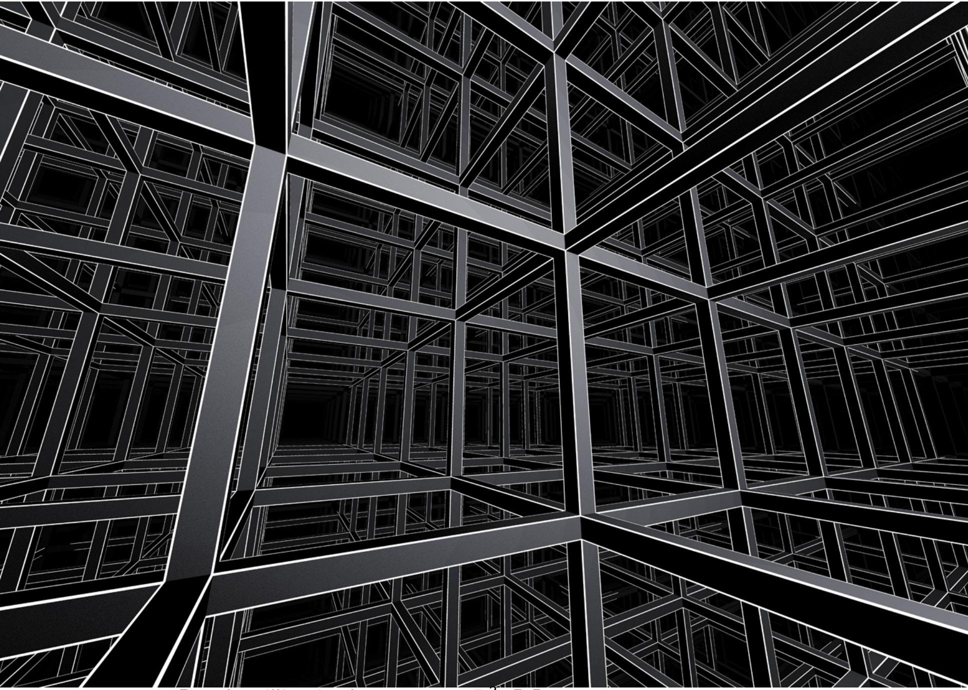




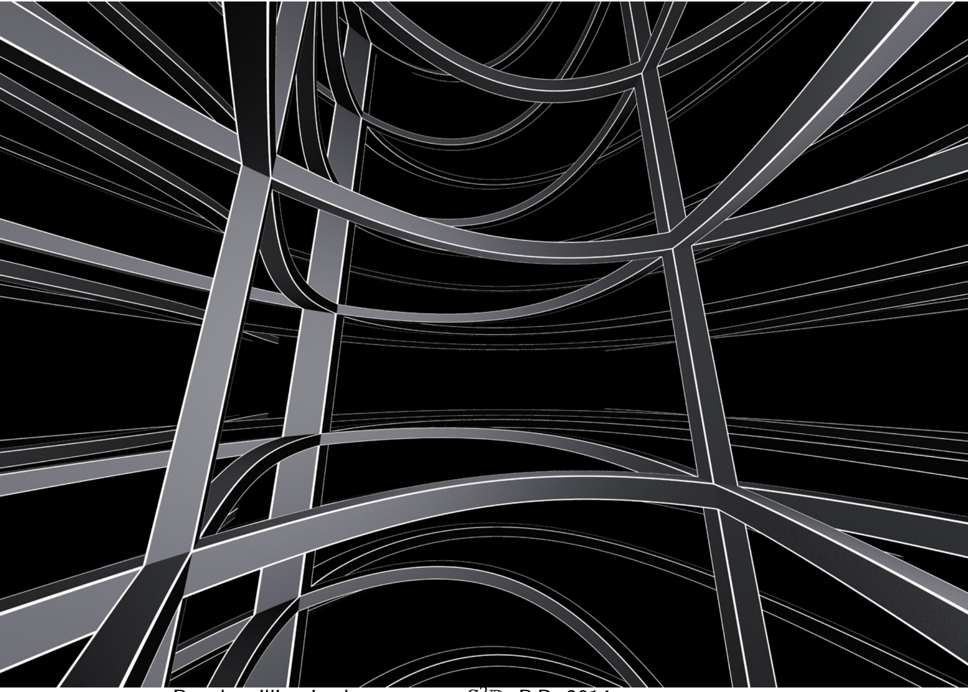


For 3-manifolds, they are 8 geometries.

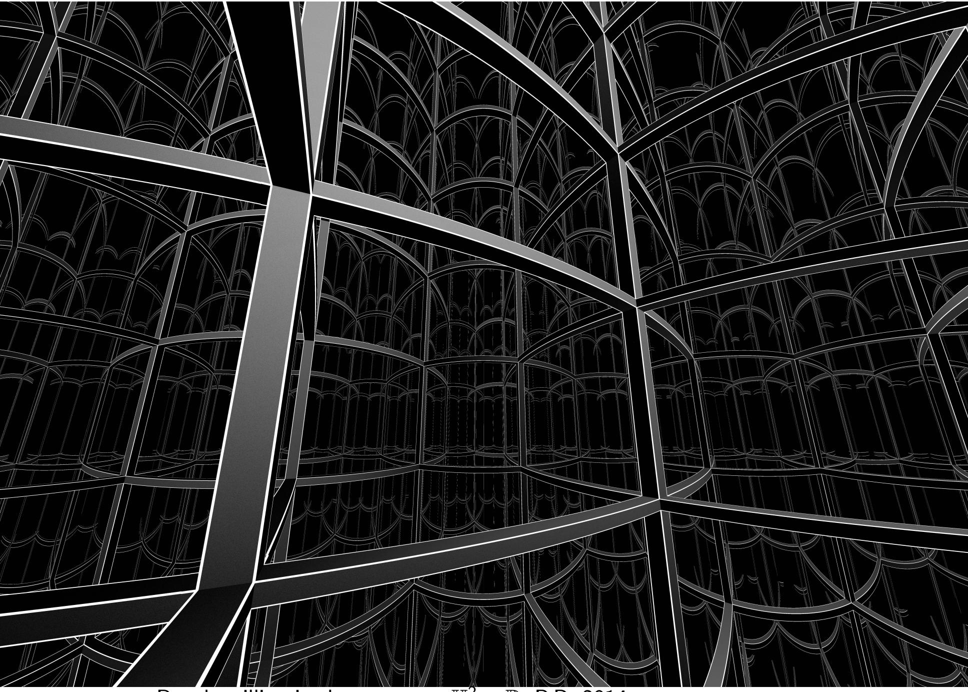
$$\mathbb{S}^3 \ , \ \mathbb{R}^3 \ , \ \mathbb{H}^3 \ , \ \mathbb{S}^2 \times \mathbb{R} \ , \ \mathbb{H}^2 \times \mathbb{R} \ , \ \mathbb{SOL} \ , \ \mathbb{NIL} \ , \ \widetilde{\mathbb{SL}}_2$$



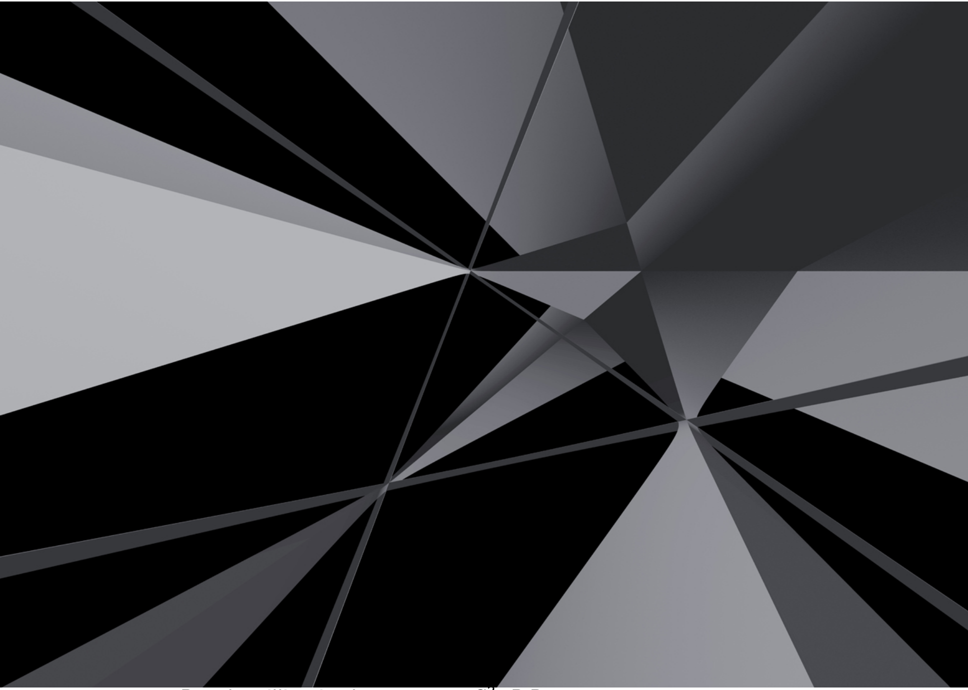
Regular tilling in the geometry $\mathbb{R}^3$, P.B. 2014



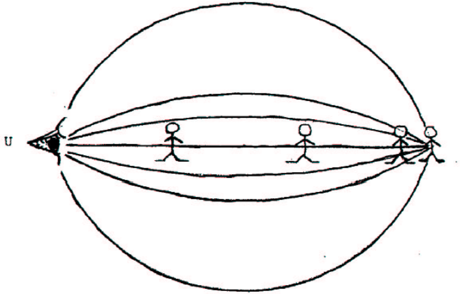
Regular tilling in the geometry $\mathbb{S}^2\mathbb{R}$, P.B. 2014



Regular tilling in the geometry $\mathbb{H}^2 \times \mathbb{R}$, P.B. 2014



Regular tilling in the geometry $\mathbb{S}^3$, P.B. 2014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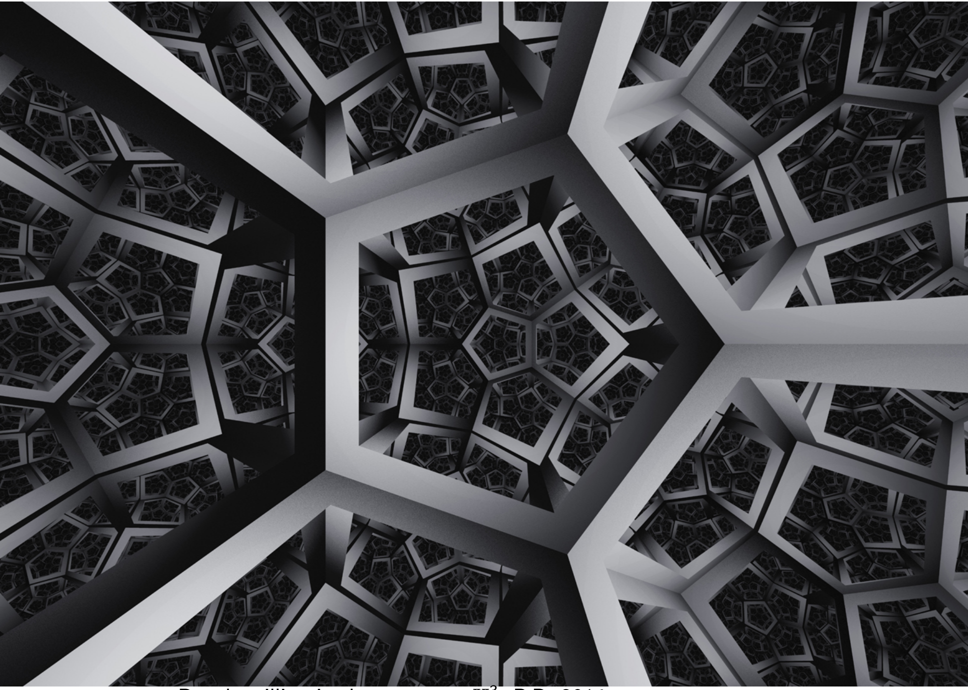
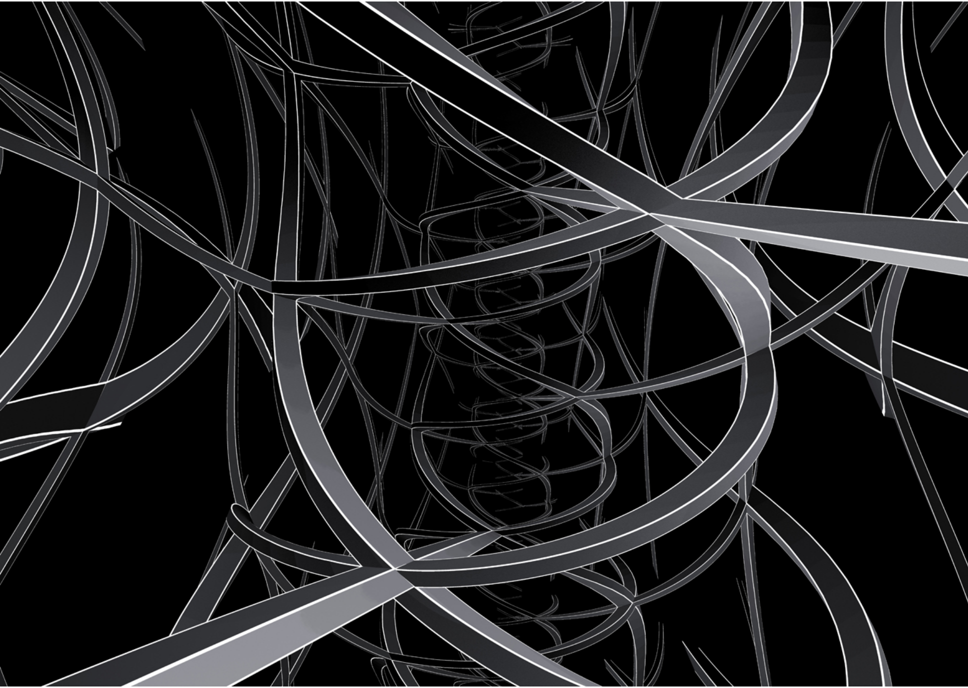
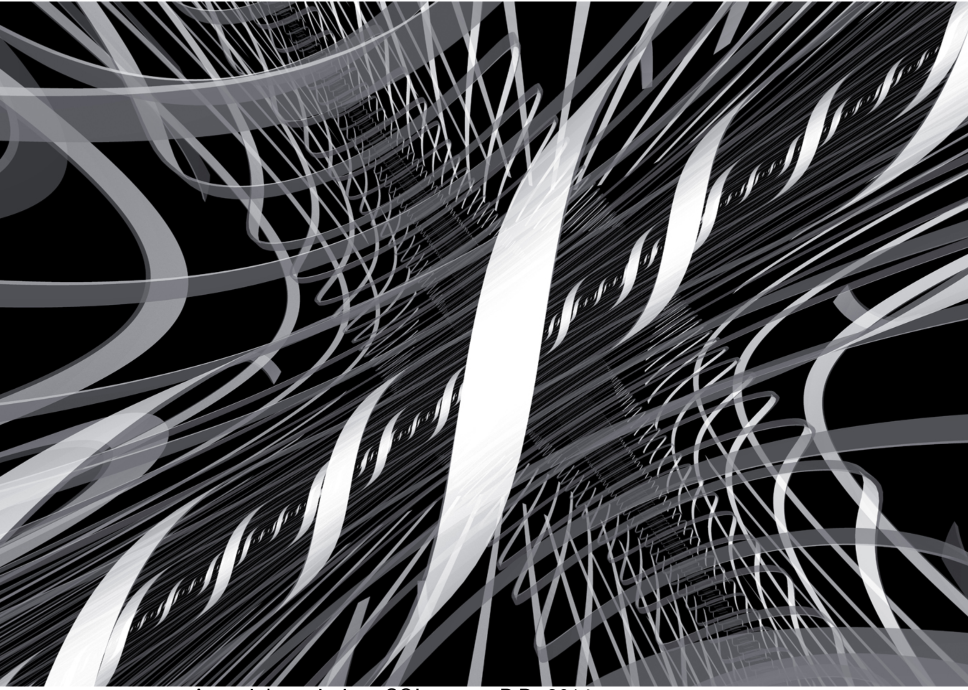


Regular tilling in the geometry $\mathbb{H}^3$, P.B. 2014

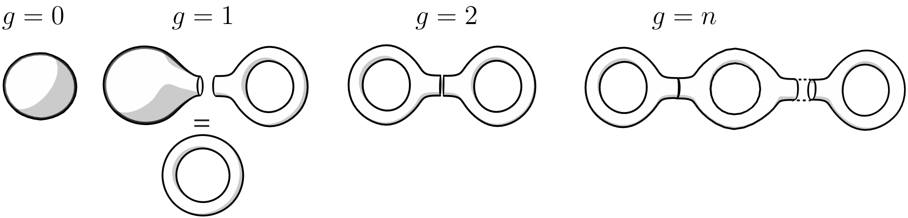


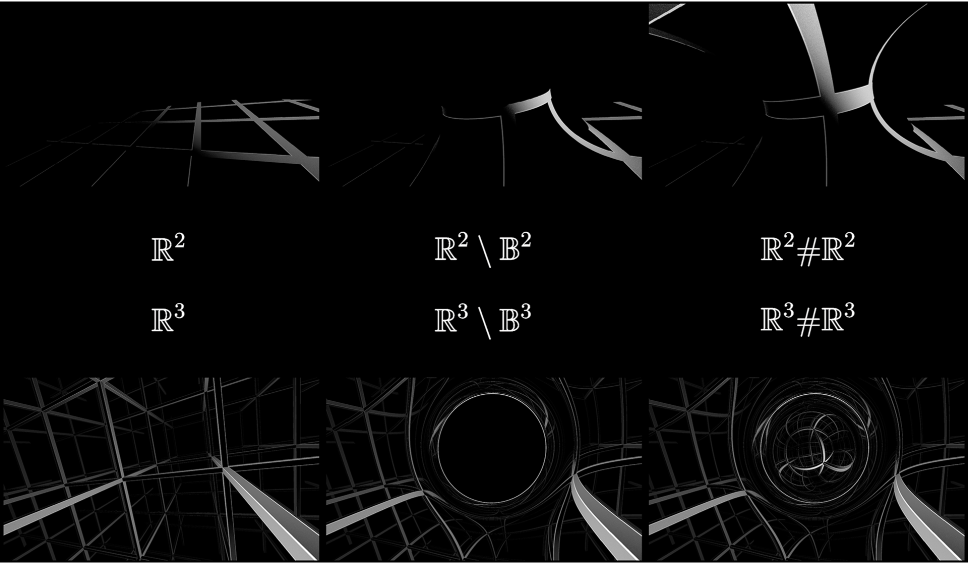
Regular tilling in a SOL geometry, P.B. 2014

Regular tilling in a NIL geometry, P.B. 2014

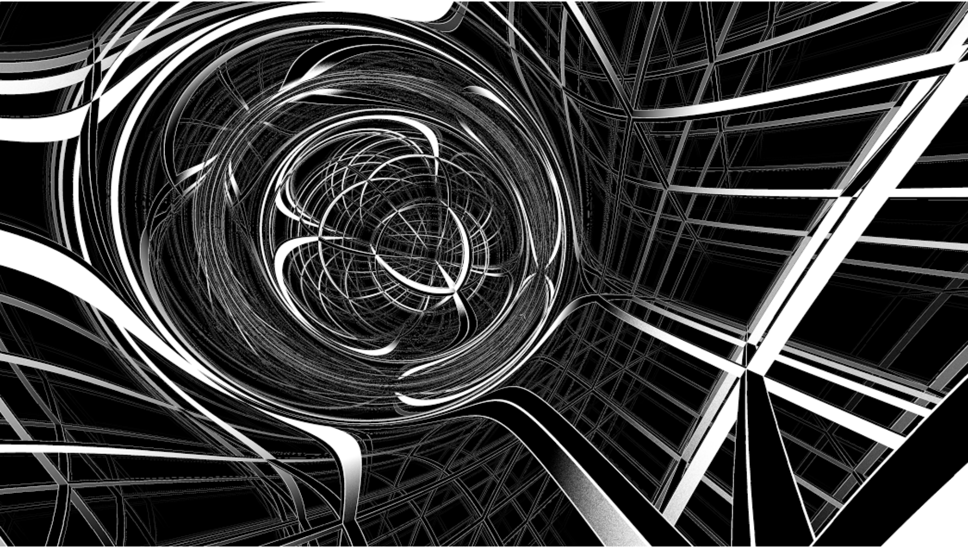


A straight strip in a SOL space, P.B. 2014


$g = 0$
$g = 1$
$=$
$g = 2$
$g = n$



Navigation in the connected sum $\mathbb{R}^3\#\mathbb{R}^3$, P.B. 2014



Navigation in the connected sum $\mathbb{R}^3 \# \mathbb{R}^3$, P.B. 2014

What are the sounds of those spaces?

Every space M displays infinitely many resonances frequencies $\omega_1, \omega_2, \dots, \omega_n \dots$ which depend on the geometry of M and the speed of the sound c.

($\star$) **Percussive space.** If M is made in an ideal material, when we hit it, the following sound appears:

$$P(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \cdots + \cos(\omega_n t) + \cdots$$

If the material of M is not ideal, there is a dissipation $D_n = D \cdot \omega_n^2$ depending on the frequency and on the material. The sound becomes:

$$P(t) = e^{-D_1 t} \cdot \cos(\omega_1' t) + \cdots + e^{-D_n t} \cdot \cos(\omega_n' t) + \cdots \quad \text{with } \omega_n' = \sqrt{\omega_n^2 - D_n^2}$$

($\star\star$) **Acoustic space.** When we put a sound E in the space, it spreads, reverberates and is heard as a new sound S. Actually S is a solution of:

$$\partial_t^2 S - c^2(1 + 2D\partial_t)\Delta S = E \ . \qquad (\star)$$

which is:

$$S(t) = \int_0^t G(t-s) \cdot E(s) ds \text{ with } G(t) = e^{-D_1 t} \cdot \frac{\sin(\omega_1' t)}{\omega_1'} + \cdots + e^{-D_n t} \cdot \frac{\sin(\omega_n' t)}{\omega_n'} + \cdot$$

A conversation in several space, with S. Krakoski and J. Royo-Letelier, 2016

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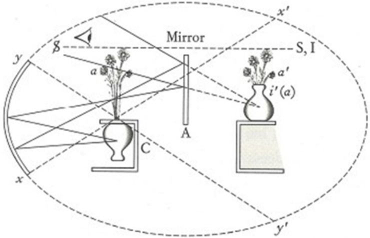
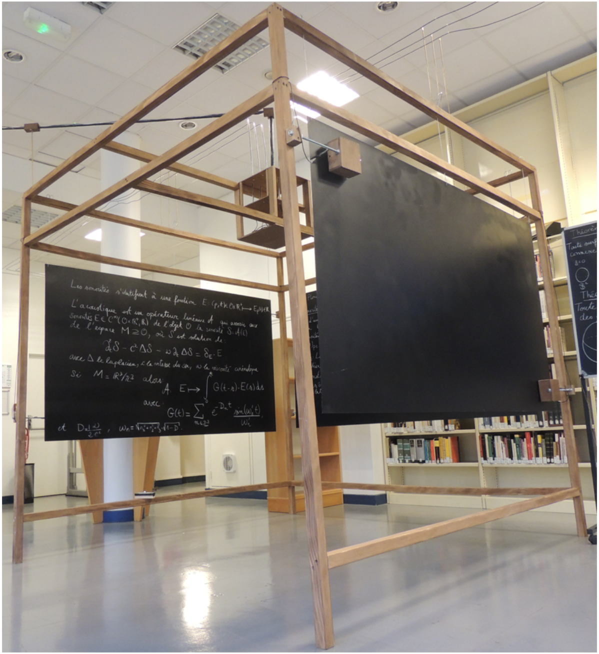
Does contemporary geometry might be interesting for humanities?

In the Lacannian school, the topology and the optic perception has been used as a conceptual/metamorphical framework to study the relationship between ego, subjectivity, symbolic, imaginary, real etc.

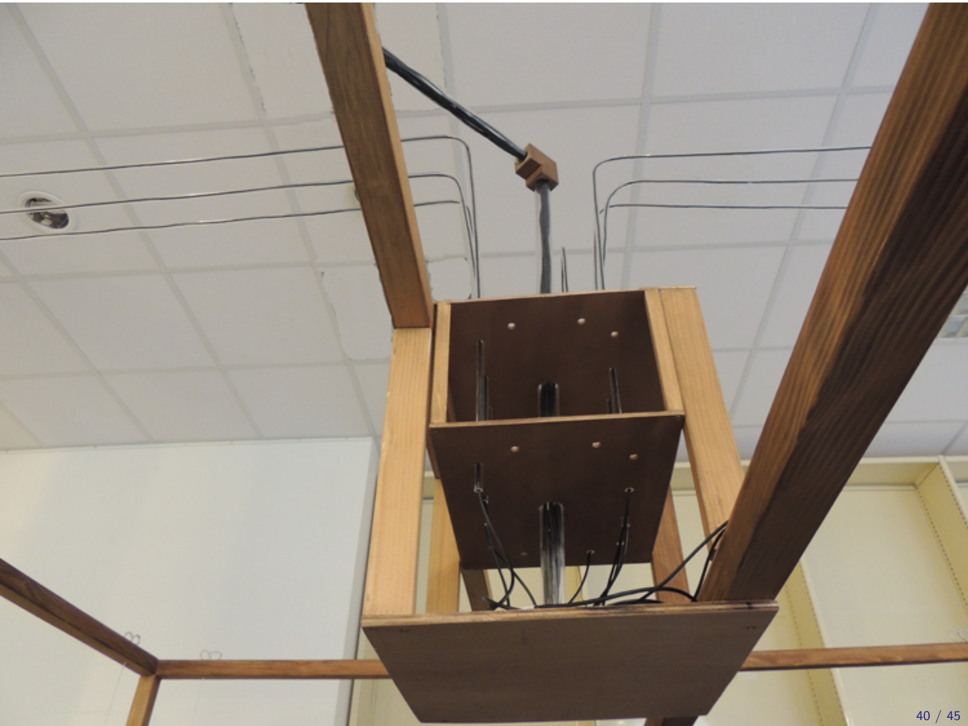


Subjectivity is nowadays 'scientifically' modeled trough machines learning. The algorithms involve in particular convolutions with functions (wavelet).

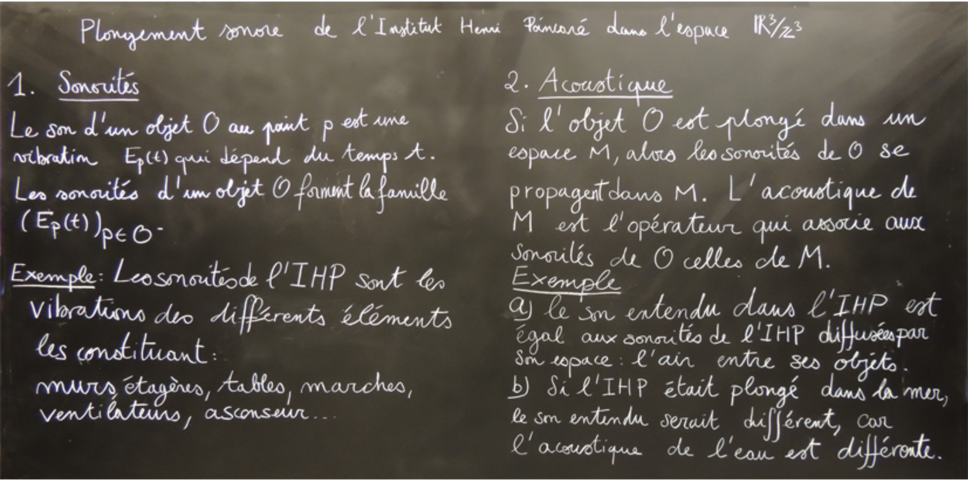
"An interesting phenomenon in spatial thinking is that scale makes a big difference. We can think about little objects in our hands, or we can think of bigger human-sized structures that we scan, or we can think of spatial structures that encompass us and that we move around in. We tend to think more effectively with spatial imagery on a larger scale: it's as if our brains take larger things more seriously and can devote more resources to them." W. Thurston

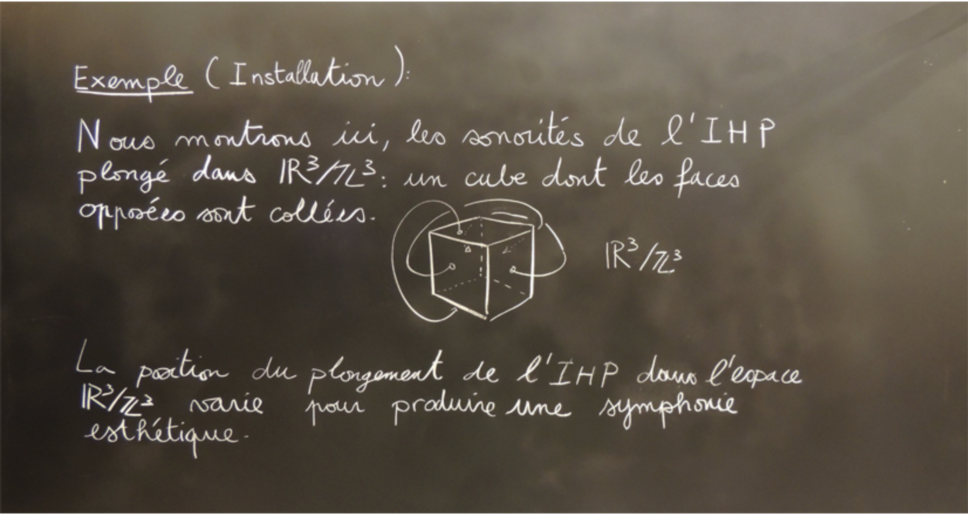


Embedding of the IHP into $\mathbb{R}^3/\mathbb{Z}^3$, with V. Martial and J. Royo-Letelier, 2017




Théorème (Klein, Poincaré,
$\mathbb{S}^2$
$\mathbb{E}^2$


Plongement sonore de l'Institut Henri Poincaré dans l'espace ℝ³/ℤ³
1. Sonorités
Le son d'un objet O au point p est une vibration Ep(t) qui dépend du temps t.
Les sonorités d'un objet O forment la famille (Ep(t))p∈O.
Exemple: Les sonorités de l'IHP sont les vibrations des différents éléments les constituant:
murs, étagères, tables, marches, ventilateurs, ascenseur...
2. Acoustique
Si l'objet O est plongé dans un espace M, alors les sonorités de O se propagent dans M. L'acoustique de M est l'opérateur qui associe aux sonorités de O celles de M.
Exemple
a) le son entendu dans l'IHP est égal aux sonorités de l'IHP diffusées par son espace: l'air entre ses objets.
b) Si l'IHP était plongé dans la mer, le son entendu serait différent, car l'acoustique de l'eau est différente.


Exemple (Installation):
Nous montrons ici, les sonorités de l'IHP
plongé dans $\mathbb{R}^3/\mathbb{Z}^3$: un cube dont les faces
opposées sont collées.
$\mathbb{R}^3/\mathbb{Z}^3$
La position du plongement de l'IHP dans l'espace
$\mathbb{R}^3/\mathbb{Z}^3$ varie pour produire une symphonie
esthétique.

Les sonorités s'identifient à une fonction $E : (p,t) \in O \times \mathbb{R}^+ \longmapsto E_p(t) \in \mathbb{R}$

L'acoustique est un opérateur linéaire $\mathcal{A}$ qui associe aux sonorités $E \in C^\infty(O \times \mathbb{R}^+, \mathbb{R})$ de l'objet O la sonorité $S = \mathcal{A}(E)$ de l'espace $M \supseteq O$, où S est solution de :

$$\partial_t^2 S - c^2 \Delta S - \nu\, \partial_t \Delta S = \delta_E \cdot E$$

avec Δ le laplacien, c la vitesse du son, ν la viscosité cinématique

Si $M = \mathbb{R}^3/\mathbb{Z}^3$ alors :

$$\mathcal{A} : E \longmapsto \int_0^t G(t-s) \cdot E(s)\, ds$$

avec :

$$G(t) = \sum_{n \in \mathbb{Z}^3} e^{-D_n t} \frac{\sin(\omega_n' t)}{\omega_n'},$$

et $D = \frac{1}{2} \frac{\nu}{c^2}$, $\omega_n = \sqrt{n_1^2 + n_2^2 + n_3^2} \cdot \sqrt{1 - D}$.