Arithmetic L-functions and Differential Geometric Methods (Regulators IV)

IMJ–PRG (Paris), May 23rd to 28th 2016

Organizers: P. Charollois, F. Déglise, G. Freixas, V. Maillot, X. Ma

Speakers

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Matt KERR
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Guido KINGS
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Andreas LANGER
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Michalis NEURURER
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Aurélien RODRIGUEZ
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Víctor ROTGER
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Ramesh SREEKANTAN
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Schedule of the talks

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**
9.00–10.00(*)	Esnault	Nori	Raskind	Niziol	Rotger	Patel
10.00–10.45	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
10.45–11.45	Bergeron	Brown	Besser	Langer	Balakrishnan	Asakura
11.45–14.00	Break	Break	———	Break	Break	Break
14.00–15.00	Raimbault	Kings	———	Bismut	Rodríguez	Ancona
15.00–15.45	Coffee	Coffee	———	Coffee	Coffee	Coffee
15.45–16.45	Quick	Neururer	———	Huber	Sreekantan	Dan-Cohen
17.00–18.00	———	———	———	Kerr	———	———

(*) On monday, the registration begins at 8.30. It will be followed by a short introduction before the first talk.

On thursday afternoon, starting from 18.15, we have the pleasure to offer a buffet to the participants.

Abstracts

Giuseppe Ancona
On the standard conjecture of Hodge type for abelian varieties

Let S be a surface and V be the finite dimensional $\mathbb{Q}$-vector space of divisors on S modulo numerical equivalence. The intersection product defines a bilinear form on V. We know since the thirties that it is of signature $(1, n)$. In the sixties Grothendieck conjectured a generalization of this statement for cycles on higher dimensional varieties. In characteristic zero this conjecture is a consequence of the Hodge Index Theorem. In positive characteristic, the only known result is on Lefschetz classes (cycles generated by divisors) on abelian varieties (Milne, 2002). We will show some partial results on exotic classes (those orthogonal to the Lefschetz ones) on abelian varieties over finite fields.

Masanori Asakura
Regulator of hypergeometric fibrations

We call a fibration $f : X \to \mathbb{P}^1$ a hypergeometric fibration if it satisfies some conditions, for example, f is smooth outside $t = 0, 1, \infty$ etc. Taking the base change $t \to a - t^n$, one gets fibrations $f_a : X_a \to \mathbb{P}^1$, which are parametrized by a. Then we discuss the Beilinson regulator of certain elements of the motivic cohomology groups $H^3_M(X_a, \mathbb{Q}(2))$. Regarding it as a function of a, the main result is that the regulator is a generalized hypergeometric function $3F2$. As a remarkable application, we can get a numerical sufficient condition for that $3F2$ is a logarithmic function (this gives a new formula of hypergeometric functions, as far as we know). This is a joint work with Noriyuki Otsubo.

Jennifer Balakrishnan
Iterated p-adic integrals and rational points on curves

I will discuss some new relationships between iterated Coleman integrals, motivated by the problem of explicitly finding rational points on curves, in the spirit of Kim's nonabelian Chabauty program. In particular, I will describe the link between p-adic heights and double integrals, as well as a p-adic analogue of the work of Goncharov and Levin, resulting in a new identity between triple Coleman integrals. This is joint work with Netan Dogra.

Nicolas Bergeron
Torsion homology growth in arithmetic groups

Arithmetic groups — that generalize the modular group — can have 'a lot' of torsion in their homology. Starting from the examples of congruence subgroups of the modular and Bianchi groups where homology reduces to abelianization, I will explain what 'a lot' means. I will then try to explain how this interacts with more classical question of number theory and geometry. This is joint work with Akshay Venkatesh and Mehmet Haluk Sengun.

Amnon Besser
Syntomic regulators for semi-stable curves and applications to the toric regulator

In past work, we proved several results computing syntomic regulators in terms of Coleman integration for varieties with good reduction. Recently, Nekovar and Niziol extended the theory of syntomic regulators to varieties with bad reduction. A natural conjecture would be that the results above continue to hold provided one replaces Coleman integration with Vologodsky integration. I will discuss the case of K_2 of curves. I will then discuss the relation between the syntomic regulator and the theory of the toric regulator developed by Raskind and myself. The former is supposed to be the logarithm of the latter. Consequently, we can conjecture formulas for the toric regulator by exponentiating the corresponding formulas for the syntomic regulator. I will discuss K_2 of curves and K_1 of certain surfaces.

Jean-Michel Bismut
Eta invariants and the hypoelliptic Laplacian

In previous work, we have viewed the evaluation of the semisimple orbital integrals associated with the heat kernel as the evaluation of a Lefschetz number, which can be obtained using cohomological methods. This way, we gave an explicit formula for such orbital integrals for reductive groups of arbitrary rank, that generalizes Selbergs formula for $SL_2(\mathbb{R})$.

The index theorem of Atiyah-Patodi-Singer asserts that for manifolds with boundary, the eta invariant, a global spectral invariant of the boundary, appears as its contribution to the index of the Dirac operator.

On locally symmetric spaces, using Selbergs trace formula, Moscovici and Stanton, extending earlier work by Milsson, gave a local expression of the eta invariant in terms of characteristic classes associated with the manifolds of closed geodesics.

In the talk, I will give the general construction of the hypoelliptic Laplacian, and explain how it can be used to recover the results of Moscovici-Stanton.

Francis Brown
Remarks on Kim's method for integral points on curves

Ishai Dan-Cohen
Towards an explicit motivic Chabauty-Kim approach to the unit equation

Over the course of the last 15 years or so, Minhyong Kim has developed a framework, based on Chabauty's method, for making effective use of the fundamental group to bound sets of solutions to hyperbolic equations; his approach opens a new avenue in the quest for an effective version of the Mordell conjecture. This motivates the problem of realizing the potential effectivity of Kim's framework in special cases by making "Chabauty-Kim theory" explicit. In the case of the unit equation, this problem may be approached via motivic methods. Using these methods we are able to describe an algorithm; its output upon halting is provably the set of integral points, while its halting depends on conjectures.

Hélène Esnault
Chern classes of crystals

The crystalline Chern classes of the value of a locally free crystal vanish on a smooth variety defined over a perfect field. We derive from this various consequences.

Annette Huber
Polylog for commutative group schemes

We report on joint work with Guido Kings. The polylog is arguably the single most important source for elements in motivic cohomology whose image under the various regulatator maps is computable. We generalize the construction to all commutative group schemes. It is simpler than the ones in the literature even in the classical cases. This are made possible by the advances in the development of the motivic machine, in particular a six functor formalism.

Matt Kerr
Higher normal functions and quantum curves

The first half of this talk will be a general description of the asymptotic behavior of variations of mixed Hodge structure arising from families of motivic cohomology classes, culminating in a "going-up theorem" for K-theory. Such variations arise in many places in physics, including local mirror symmetry, Feynman integrals, and most recently, the spectral theory of quantum curves. We will briefly describe this connection, and show how the general "going-up" result can be used to analytically verify predictions for limits of spectral traces arising from a conjecture of Marino. (Parts of this are products of work in progress and discussions with del Angel, Doran, Iyer, Lewis, Mueller-Stach, Patel, and Tang.)

Guido Kings
Regulators and the Eisenstein cohomology of Hilbert modular varieties

Harder has studied the Eisenstein cohomology of Hilbert modular varieties by analytic methods. This makes rationality results for the Eisenstein cohomology classes hard to obtain and gives no clue, whether these classes have a motivic construction.

For modular curves however, Beilinson was able to define motivic Eisenstein classes, whose images under the regulator from motivic cohomology to Betti cohomology generate the full Eisenstein cohomology. These classes have very important applications to special values of L-functions and to explicit reciprocity laws.

In this talk we will explain the generalization of Beilinson's result to Hilbert modular varieties. This gives rise to a motivic construction of Harder's Eisenstein cohomology classes (in all cohomological degrees), which makes their rationality properties evident.

Andreas Langer
p-adic deformation of motivic Chow groups

In their recent work on p-adic deformation of algebraic cycle classes Bloch, Esnault and Kerz give – in the framework of the p-adic variational Hodge-conjecture – an equivalent Hodge-theoretic

condition on the crystalline Chern class when an an algebraic cycle class lifts from char p to a pro-class in the continuous Chow group of a formal lifting over $W(k)$. In my talk I will present a relative version of their work. Starting with a projective smooth variety X over the ring $R = n$-truncated Witt- vectors of a perfect field, the classical Chow group is replaced by a motivic Chow group using a mixed characteristic version of the Suslin-Voevodsky complex and based on a definition of Bloch-Esnault-Kerz. It is still related to Milnor K-cohomology. I explain that their results on deforming algebraic cycles formally hold in a relative setting: one can give an equivalent Hodge-theoretic condition on the relative crystalline Chern class when an element in the motivic Chow group lifts to a pro-class on a formal lifting of X over $W(R)$. In the proof the relative de Rham-Witt complex and a relative version of syntomic cohomology play a crucial role.

Michalis Neururer
Products of Eisenstein series and Mahler measures

My talk will be about two related problems that involve products of Eisenstein series. The first is the question of which modular forms are linear combinations of products of two Eisenstein series. Writing a modular form as such a linear combination can be used in many applications, e.g. for calculating Fourier expansions at any cusp. In joint work with Martin Dickson we show that for many congruence subgroups all modular forms are linear combinations of products of Eisenstein series. I will present a proof of this result using the Rankin-Selberg method and modular symbols.

The second part of the talk will be about Boyd's conjectures about relations between Mahler measures and L-values of elliptic curves. In recent years a new method to prove Boyd's conjectures has been developed by Rogers-Zudilin and Brunault that uses products of Eisenstein series. I will talk about a joint project with Franois Brunault where we try to extend this method to study a connections between L-values of K3 surfaces and Mahler measures.

Wieslawa Niziol
Syntomic cohomology and its applications

As is well-known syntomic cohomology is a p-adic analog of Deligne cohomology. In particular, it an absolute p-adic Hodge cohomology and a target of p-adic regulator maps. I will review its definitions and its applications to p-adic Hodge Theory and motivic cohomology.

Madhav Nori
CM Hodge structures

Discussion of an attempt to represent certain Hodge classes on products $X \times A$, where A is an Abelian variety, by algebraic cycles. This is work in progress with Kapil Paranjape.

Deepam Patel
A Conjecture of Kato-Saito on Epsilon factors of Tensor Products

We recall a recent conjecture of Kato-Saito which gives a formula for the global ℓ-adic epsilon factor of a tensor product of a highly ramified sheaf with a lisse étale sheaf in terms of class field theory. Then we will discuss a proof of an analogous formula in the setting of D-modules on a smooth projective variety in characteristic zero. If there is time, we will also discuss the setting

of Berthelot's arithmetic D-modules with Frobenius. This is based on joint work with Tomoyuki Abe.

Gereon Quick
Generalized Deligne-Beilinson cohomology theories and regulator maps

I will report on joint work with Mike Hopkins on a construction of generalized Deligne-Beilinson cohomology associated to topological cohomology theories. This construction is natural in many ways. In particular, a morphism between two theories induces a morphism between the associated Deligne-Beilinson theories which one can interpret as a regulator map. I will also explain how this yields a new type of Abel-Jacobi invariant for algebraic cobordism cycles.

Jean Raimbault
Torsion in the homology of non-uniform lattices

Results of Bergeron–Venkatesh and Müller–Marshall show that the torsion subgroups of the homology groups of uniform arithmetic lattices in $SL_2(\mathbb{C})$, with coefficients in arithmetic modules, are "as large as possible". I will present how to deal with the obstacles that arise when trying to extend these results to nonuniform lattices (congruence subgroups of Bianchi groups). In particular I will talk about recent joint work with Jonathan Pfaff.

Wayne Raskind
p-adic Toric Regulators

This is joint work with Amnon Besser. Let K be a p-adic field and X a smooth projective variety over K. We generalize the"p-adic intermediate Jacobians" that were defined and studied by Raskind-Xarles to regulators taking values in the quotient of a multiplicative torus by a finitely generated abelian group, and relate them to other regulators such as *log*-syntomicand p-adic étale. We then reinterpret and in some cases refine conjectures onthe relationship between values of L-functions and such regulators.

Aurélien Rodríguez *The construction of a (weak) Arakelov Cobordism group.*

In the early 2000's Levine and Morel have given a geometric construction of an algebraic cobordism group defined for all smooth quasi projective varieties over a field k. We will show how we can refine their construction to build an Arakelov version of this group for Arakelov varieties over a number field, and how this integrates well in the general philosophy of Arakelov geometry.

Víctor Rotger
p-adic L-functions and p-adic regulators associated to rank two Selmer groups

I will describe a series of joint works with Bertolini, Darmon and Lauder in which we construct Euler systems for the Rankin convolution of pairs and triples of modular forms and relate them to p-adic L-functions. The deformation of these systems to weight one give rise to new applications to the conjecture of Birch and Swinnerton-Dyer in scenarios where the order of vanishing of the classical L-function is 0, 1 or 2.

Ramesh Sreekantan
Higher Chow Groups and Modular Forms

There are several results starting with the work of Hirzebruch and Zagier relating special subvarieties of Shimura varieties to coefficients of modular forms culminating in a rather general conjecture of Kudla. In this talk we will explain an approach to this question and discuss some results relating regulators of higher Chow cycles with Borcherds lifts of modular forms in a special case.

Participants

1. Giuseppe ANCONA
2. Masanori ASAKURA
3. Jennifer BALAKRISHNAN
4. Nicolas BERGERON
5. Daniel BERTRAND
6. Marie José BERTÍN
7. Amnon BESSER
8. Jean-Michel BISMUT
9. Francis BROWN
10. José Ignacio BURGOS
11. Victoria CANTORAL FARFAN
12. Pierre CHAROLLOIS
13. Ishai DAN-COHEN
14. Frédéric DÉGLISE
15. Rob DE JEU
16. Christopher DENINGER
17. Mladen DIMITROV
18. Hélène ESNAULT
19. Gerard FREIXAS
20. Javier FRESÁN
21. Oliver GREGORY
22. Annette HUBER
23. Max KAROUBI
24. Matt KERR
25. Guido KINGS
26. Bruno KLINGLER
27. Satoshi KONDO
28. Andreas LANGER
29. James LEWIS
30. Jin LIE
31. Eric LEICHTNAM
32. Xiaonan MA
33. Loïc MEREL
34. Michalis NEURURER
35. Wieslawa NIZIOL
36. Madhav NORI
37. Yuji ODAKA
38. Joseph OESTERLÉ
39. Deepam PATEL
40. Frédéric PAUGAM
41. Praneel SAMANTA
42. Gereon QUICK
43. Jean RAIMBAULT
44. Wayne RASKIND
45. Jishnu RAY
46. Joaquin RODRIGUES
47. Aurélien RODRIGUEZ
48. Víctor ROTGER
49. Saadouli SANA
50. Justin SCARFY
51. Ramesh SREEKANTAN
52. Masha VLASENKO