Errata

Pierre Schapira

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Deformation quantization modules

By M. Kashiwara and P. Schapira

Astérique, Société Math. France **345** (2012)

As pointed out by **François Petit**, the proof of Theorem 1.6.6 is not complete. We have proved that $\mathcal{N} \otimes^L_{\mathcal{A}} \mathcal{M}$ is concentrated in degree 0 if $\mathcal{N}$ has no $\hbar$-torsion. It remains to consider the case where $\mathcal{N}$ is of $\hbar$-torsion. Hence $\mathcal{N}$ is the union of the $\mathcal{N}_k$'s ($k \in \mathbb{N}$) where $\hbar^k u = 0$ for each $u \in \mathcal{N}_k$. By induction (similar arguments are used all along Chapter 1), one reduces to the the case $k = 1$. Then $\mathcal{N} \otimes^L_{\mathcal{A}} \mathcal{M} \simeq \mathcal{N} \otimes^L_{\mathcal{A}_0} \mathcal{M}/\hbar\mathcal{M}$ and the result follows since $\mathrm{gr}_\hbar \mathcal{M}$ is $\mathcal{A}_0$-flat by hypothesis.

Categories and Sheaves

By M. Kashiwara and P. Schapira

Grundlehren der Math. Wiss. **332** Springer-Verlag (2006).

Many detailed comments and corrections on this book have been written by Pierre-Yves Gaillard and we warmly thank him here. See "About *Categories and Sheaves*" *https://vixra.org/abs/1602.0067*

(i) As pointed out by **Joseph Oesterlé,** *if $\mathcal{C}$ is a $\mathcal{U}$-category, it is not clear that the functor $\mathrm{Hom}_{\mathcal{C}}(\,\bullet\,, X)$ takes its values in $\mathcal{C}^\wedge$ for $X \in \mathcal{C}$. Indeed, $\mathcal{C}^\wedge$ is the category of functors from $\mathcal{C}^{\mathrm{op}}$ to $\mathcal{U}$-**Set** and for any $Y \in \mathcal{C}$, the set $\mathrm{Hom}_{\mathcal{C}}(Y, X)$ is $\mathcal{U}$-small but does not necessarily belongs to $\mathcal{U}$. To overcome this difficulty, choose a universe $\mathcal{V}$ which contains $\mathcal{U}$ and such that $\mathrm{Hom}_{\mathcal{C}}(Y, X) \in \mathcal{V}$ for all $X, Y \in \mathcal{C}$. Then $\mathrm{Hom}_{\mathcal{C}}(\,\bullet\,, X)$ is a well defined functor from $\mathcal{C}^{\mathrm{op}}$ to $\mathcal{V}$-**Set**. By Lemma 1.3.11, there exists a functor from $\mathcal{C}^{\mathrm{op}}$ to $\mathcal{U}$-**Set** isomorphic to $\mathrm{Hom}_{\mathcal{C}}(\,\bullet\,, X)$ and this functor is unique up to unique isomorphism.*

(ii) As pointed out by **J. Climent Vidal,** *Definition 5.2.1 (i) p. 117 is not correctly formulated, and should be replaced by*

Definition 5.2.1 *A system of generators in $\mathcal{C}$ is a family of objects $\{G_i\}_{i\in I}$ of $\mathcal{C}$ such that I is small and a morphism $f\colon X \to Y$ in $\mathcal{C}$ is an isomorphism as soon as $\mathrm{Hom}_{\mathcal{C}}(G_i, X) \to \mathrm{Hom}_{\mathcal{C}}(G_i, Y)$ is an isomorphism for all $i \in I$.*

Contrarily to what is asserted, this is not equivalent in general to saying that the functor $\prod_{i\in I} \mathrm{Hom}_{\mathcal{C}}(G_i, \bullet\,)$ is conservative (in fact, $\mathrm{Hom}_{\mathcal{C}}(G_i, X)$ may be empty) and a category may admit small coproducts and a system of generators without admitting a generator. However, the two assertions are equivalent if $\mathcal{C}$ is additive.

(iii) As pointed out by **Nicolas Fort,** *in Remark 7.1.18 p. 156, one should replace $\mathcal{S}_X$ with $\mathcal{S}_X^{\mathrm{op}}$ (three times).*

(iv) In Exercise 8.23 (iii) p. 206, one has to replace the category $\mathrm{coh}(\mathcal{J})$ by the subcategory of $\mathcal{J}$-pseudo-coherent objects.

(v) As pointed out by **Prof. Vincent Beck,** *Definition 11.3.12 p. 282 differs from the corresponding one in SGA4 and gives rise to some mistakes in the relation (11.7.3) as well as in Exercise 11.11. Hence, Definition 11.3.12, p. 282, should be replaced by:*

Definition 11.3.12 *Let $F\colon \mathcal{C}^{\mathrm{op}} \to \mathcal{C}'$ be an additive functor. We define the* *functor* $\mathrm{C}(F)\colon (\mathrm{C}(\mathcal{C}))^{\mathrm{op}} \to \mathrm{C}(\mathcal{C}')$ *by setting:*

$$\mathrm{C}(F)(X^\bullet)^n = F(X^{-n}), \qquad d^n_{\mathrm{C}(F)(X)} = (-1)^{n+1}F(d_X^{-n-1}).$$

As a consequence, one has to replace the definition of the differential d'' *in Examples 11.6.2 p. 290. On p. 290, l. 17, one should read:*

$$d''^{n,m} \quad = \quad \mathrm{Hom}_{\mathcal{C}}((-1)^{m+1}d_X^{-m-1}, Y^n).$$

(vi) p. 390, in Definition 16.1.2, GT3, line 3:"belongs to $\mathcal{S}\mathrm{Cov}_V$*" should be replaced by "belongs to* S*".*

(vii) p. 474, l. 3: "Proposition 19.4.5" should be replaced by "Proposition 19.4.6".

(viii) In the bibliography,

reference [3], "Etale homology" should be replaced by "Etale homotopy".

reference [11], "Cohomologies" should be replaced by "Cohomology".

reference [23], "präsentibare" should be replaced by "präsentierbare".

reference [49], "R. Par" should be replaced by "R. Paré".

(ix) As pointed out by **Pierre-Yves Gaillard,**

(a) the two lines after Diagram (8.7.4), page 200, are not correct and should be replaced as follows:

Let us take an epimorphism $Z \xrightarrow{f} \mathrm{Ker}(Y' \xrightarrow{v} X')$*. Then the condition that* $K(\alpha)$ *is an isomorphism is equivalent to the fact that the sequence* $Z \oplus Y \to X \oplus Y' \to X' \to 0$ *is exact. This complex ...*

(b) in Theorem 5.2.6, one should add the hypothesis that $\mathcal{C}$ *admits finite projective limits in order to apply Proposition 5.2.3 in the proof (to admit kernels is in fact enough).*

Other corrections *see Pierre-Yves Gaillard "About* Categories and Sheaves*" https://vixra.org/abs/1602.0067*

Sheaves on Manifolds

By M. Kashiwara and P. Schapira

Grundlehren der Math. Wiss. **292** *Springer-Verlag (1990) 2nd reprint (2002)*

(i) All along the book, for a morphism of manifolds $f\colon Y \to X$*, the word "smooth" means "submersive". This is in particular the case in Proposition 5.4.5.*

(ii) In Remark 1.12.5, one has to assume that $\mathcal{I}$ *is filtrant in order that the functor* $\varinjlim$ *is exact.*

(iii) In Exercise I.35 (ii), correct as follows:

$$\mathrm{Hom}_{\mathcal{C}^\vee}(\text{“}\varinjlim_i\text{”}\, X_i, \text{“}\varinjlim_j\text{”}\, Y_j) \simeq \ \dots$$

(iv) In the proof of Corollary 2.4.8: delete the two zeroes at the bottom of the second and third columns.

(iv) As remarked by **Yuichi Ike***, in Proposition 2.7.2 one has to replace the set* $Z_s = \overline{\bigcap_{t>s}(U_t \setminus U_s)}$ *given in condition (iii) by the set* $Z_s = \bigcap_{t>s} \overline{(U_t \setminus U_s)}$.

(v) In Corollary 3.7.3, one has to assume that for any $x \in X$*, the set* $\{t \in R^+; \mu(x,t) \in U\}$ *is contractible.*

(vi) As remarked by **Takahiro Saito** *and* **Tomohiro Asano***, the hypothesis of Exercise II. 12 (ii) is too weak. Compare with Lemma 1.1.6 of Deformation quantization modules (M. Kashiwara and P. Schapira) Astérisque, Soc. Math. France, 345 (2012).*

Microdifferential systems in the complex domain

By P. Schapira

Grundlehren der Math. Wiss. **269** *Springer-Verlag (1985)*

As pointed out by **Fred Van Oystaeyen,** *Definition 1.1.2 of Chapter II is too weak to imply that* gr(A) *is Noetherian and Proposition 1.1.7 is not correct. One has to replace Definition 1.1.2 by the following one:*

Definition 1.1.2 *The filtered ring A is Noetherian if any subobject of a module of finite type in the category of filtered A-modules is of finite type.*

A list of corrections suggested by **Alexander Getmanenko***:*

– *P.14, line 12, correct as follows:*

Proposition 2.1.2. *For all positive* t, t', t'' *with* $t' + 2nt'' < 2^{-n}t$, *...*

– *P.14, line 19, correct the first member of the inequality as follows*

$$\frac{1}{\alpha!\beta!}(|D_x^\alpha D_\xi^\beta h|_{K_\rho})(s/\sqrt{2n})^{|\alpha|+|\beta|} \leq |h|_{\bar{K}_{\rho+s}} \leq \ldots$$

– *P.14, line 21, correct the first member of the inequality as*

$$N_0(P, K_\rho, \frac{s}{2n}) \leq \ldots$$

– *P.15, line 6 from the bottom, correct the power of* t *on the LHS as follows:*

$$\ldots t^{2k+|\alpha|+|\beta|} < \infty.$$

– *P.16, line 2, drop* 2^n *in front of* t*, like this:*

$$\sum_j N_0(P^j, \{0\} \times K, t)\varepsilon'^j < \infty.$$

– *P.16, lines 4-5, replace two instances of* t_1 *by* t

– *P.16, line 7 is correct, but the factor of* 2 *is not necessary on the RHS*

– *P.16, line 7 and until the end of the proof of prop.2.1.4, rewrite the argument as follows:*

$$\sum_{\alpha,\beta} \frac{1}{\alpha!\beta!}(|D_x^\alpha D_\xi^\beta h|_{\overline{B_\varepsilon}\times K})(t/2)^{|\alpha|+|\beta|} \leq \sum_{\alpha,\beta} \frac{1}{\alpha!\beta!}(|D_x^\alpha D_\xi^\beta h|_{\{0\}\times K})(t/2)^{|\alpha'+\beta|}(\varepsilon+t)^{\alpha_1},$$

where we write the multiindex $\alpha = (\alpha_1, \alpha')$.

Thus

$$\sum_{\alpha,\beta,k} \frac{2(2n)^{-k}}{\alpha!\beta!k!}(|D_x^\alpha D_\xi^\beta p_{-k}|_{\overline{B_\varepsilon}\times K})(t/2)^{|\alpha+\beta|+2k} \le$$

$$\le \sum_{\alpha_1} \frac{(\varepsilon+t)^{\alpha_1}}{\alpha_1!} \sum_{(\alpha',\beta,k)} \frac{2(2n)^{-k}}{\alpha'!\beta!k!}(|D_{x'}^{\alpha'} D_\xi^\beta D_{x_1}^{\alpha_1} p_{-k}|_{\{0\}\times K})(t/2)^{|\alpha'|+|\beta|+2k} \le$$

$$\le \sum_j (\varepsilon+t)^j N_0(P^j, \{0\}\times K, t/2)$$

which completes the proof. $\square$

– *P.18, line 9 from the bottom should have:* $L(\mathsf{A}_t(\rho) \times \mathsf{A}_t^p, \mathsf{A}_t(\rho))$

– *P.23, line 13 : Take* $s = s' + \frac{(1-s')}{k+1}$, *then* $1 - s = \frac{k}{k+1}(1-s')$, *and* $||w_{k+1}(x_1)||_{s'} \le \cdots$