Masaki Kashiwara and Algebraic Analysis

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Awards

Masaki Kashiwara[1] was recently awarded the prestigious Abel Prize in Mathematics, an honor granted annually by the King of Norway on the recommendation of the Abel Committee, which is appointed by the Norwegian Academy of Science and Letters. The prize commemorates Niels Henrik Abel (1802–1829), a brilliant Norwegian mathematician often compared to Evariste Galois, who, like Galois, died very young. The Abel Prize is considered the mathematical equivalent of the Nobel Prize, which does not exist in mathematics for reasons that remain unclear, either scientific or sentimental (rumor has it that Alfred Nobel's beloved may have had a fondness for the mathematician Mittag-Leffler). In 2018, Masaki already obtained two major prizes, the Chern prize, awarded by the International Mathematics Union (IMU) at the International Congress of Mathematicians (ICM) in Rio de Janeiro, and the Kyoto prize.

Masaki could (and arguably should) have received the Fields Medal at the 1982 ICM. If he did not, it is likely because his work was too innovative to be fully understood at the time. He was working on the general theory of $\mathcal{D}$-modules and their microlocalization, $\mathcal{E}$-modules, proving deep results (that we shall describe later) which were clearly above the level of the average mathematician, including the Fields committee.

Mikio Sato

Masaki Kashiwara was a student of Mikio Sato, and I will first write a few words about Sato (see [Sch07] for a more detailed exposition). The story begins long ago, in the late fifties, when Sato created a new branch of mathematics –now called "Algebraic Analysis"– by publishing two papers on hyperfunction theory [Sat59] and then developed his vision of analysis and linear partial differential equations (LPDEs) in a series of lectures at the university of Tokyo in the 60s. Sato's idea is to define hyperfunctions on a real analytic manifold M as cohomology classes supported on M of the sheaf $\mathcal{O}_X$ of holomorphic functions on a complexification X of M. Hyperfunctions can then be represented as "boundary values" of holomorphic functions defined in tuboids in X with wedge on M. Understanding where these boundary values come from naturally led Sato (see [Sat70]) to define his microlocalization functor and, as a byproduct, the analytic wavefront set of hyperfunctions. This is the starting point of microlocal

[1] These notes partially overlap the papers [Sch07, Sch18, Sch21, Sch25].

Figure 1: Mikio Sato and Masaki Kashiwara

analysis. Lars Hörmander immediately understood the importance of Sato's ideas and adapted them to the C^∞-setting by replacing boundary values of holomorphic functions with the Fourier transform (see [Hör71]). In those early days, trying to understand real phenomena by complexifying a real manifold and looking at what happens in the complex domain was a totally new idea. And using cohomology of sheaves in analysis was definitely a revolutionary vision.

Like Grothendieck, Mikio Sato is a visionary of mathematics, and both continue to influence large parts of modern mathematics and mathematical physics.

Master's thesis and the SKK paper

Masaki Kashiwara, born in 1947, showed exceptional mathematical talent from an early age. In his master's thesis, dated 1970 and published in English in [Kas95], he introduces and develops the theory of $\mathcal{D}$-modules. Of course, a $\mathcal{D}$-module is a module (right or left) over the non commutative sheaf of rings $\mathcal{D}_X$ of holomorphic finite order differential operators on a given complex manifold X. And, as it is well known[2], a finitely presented module M over a ring R is the intrinsic way to formulate what is a finite system of R-linear equations with finitely many unknowns. Hence, a coherent $\mathcal{D}_X$-module $\mathcal{M}$ on X is (locally) nothing but a system of linear partial differential equations with holomorphic coefficients. Locally on X, it can be represented, non uniquely, by a matrix of differential operators.

In his thesis, Masaki defines the operations of inverse or direct images for $\mathcal{D}$-modules. In particular he extends the classical Cauchy-Kowalevski theorem to general systems of LPDEs. Consider the contravariant functor $\mathcal{S}ol$, which to a $\mathcal{D}_X$-module $\mathcal{M}$ on

[2] According to Mikio Sato (personal communication), at the origin of this idea is the mathematician and philosopher of the 17th century, E. W. von Tschirnhaus.

the manifold X, associates the complex (indeed, an object of the derived category of sheaves of $\mathbb{C}$-vector spaces) $R\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{O}_X)$ of its holomorphic solutions. The Cauchy-Kowalevski-Kashiwara theorem essentially asserts that the functor $\mathcal{S}ol$ commutes with the inverse image functor, under a non-characteristic hypothesis.

Hence Masaki Kashiwara may be considered as the founder of analytic $\mathcal{D}$-module theory[3], a theory which is now a fundamental tool in many branches of mathematics, from number theory to mathematical physics.

The seventies are, for analysts, the era of microlocal analysis. As mentioned above, the starting point was the introduction by Sato of the microlocalization functor and the analytic wavefront set. These ideas were then systematically developed in the famous paper [SKK73] by Mikio Sato, Takahiro Kawai and Masaki Kashiwara, a paper usually referred to as the SKK-paper. Now the framework is no more that of $\mathcal{D}$-modules but that of $\mathcal{E}$-modules, where $\mathcal{E}_X$ is the sheaf of microdifferential operators on the cotangent bundle T^*X. Many fundamental tools are constructed in this paper, in particular quantized contact transforms, and two fundamental results are proved.

First, the involutivity of characteristics of microdifferential systems. This was an open and fundamental question which, at that time, had only a partial answer due to Quillen, Guillemin and Sternberg [GQS70]. Later, a purely algebraic proof was given by Gabber [Gab81])[4].

The second result is a classification at generic points of any system of microdifferential equations. Roughly speaking, it is proved that, generically and after a quantized contact transform, any such system is equivalent to a combination of a partial de Rham system, a partial Dolbeault system and a Hans Lewy's type system.

This paper has had an enormous influence on the analysis of partial differential equations (see in particular [Hör83, Sjö82]) although very few people have read it.

The Riemann-Hilbert correspondence

Since the characteristic variety of a coherent $\mathcal{D}$-module is involutive (one would now say "co-isotropic"), it is natural to look at the extreme case, when this variety is Lagrangian. One calls such systems "holonomic". They are the higher-dimensional version of classical ordinary differential equations (ODE). Among ODEs, a particularly important subclass consists of Fuchsian equations, i.e., equations with regular singularities. Roughly speaking, the classical Riemann-Hilbert correspondence (R-H correspondence, for short) is based on the following question:

> Given a finite set of points $\{a_1, \dots, a_n\}$ on the Riemann sphere $\mathbb{P}^1(\mathbb{C})$ and a representation $\rho\colon \pi_1(\mathbb{P}^1(\mathbb{C}) \setminus \{a_1, \dots, a_n\}) \to GL_m(\mathbb{C})$, does there exist a unique (up to gauge equivalence) Fuchsian system of linear ODEs whose monodromy representation is equivalent to ρ?

In 1975 Kashiwara (see [Kas75]) proved that the contravariant functor $\mathcal{S}ol$–previously defined–when restricted to the derived category of $\mathcal{D}$-modules with holonomic cohomology, takes its values in the derived category of sheaves with $\mathbb{C}$-constructible cohomology.

[3]Joseph Bernstein [Ber71]) introduced algebraic $\mathcal{D}$-modules independently at the same time.

[4]Although many people refer to this result as Gabber's theorem, it is in fact due to SKK.

In the same paper, he showed that if one starts with a $\mathcal{D}$-module "concentrated in degree 0", then the resulting complex satisfies what would later (in [BBD82]) be called the perversity conditions, as formulated by Beilinson, Bernstein, Deligne and Gabber.

Moreover, already in 1973, Kashiwara had provided in [Kas73] a formula for computing the local index of the complex $\mathcal{S}ol(\mathcal{M})$ in terms of the characteristic cycle of the holonomic $\mathcal{D}$-module $\mathcal{M}$ and his formula already involved the notion of "local Euler obstruction" later introduced independently by MacPherson [Mac74].

To define a higher-dimensional analogue of Fuchsian ODEs, Kashiwara, together with Toshio Oshima [KO77], introduced the notion of regular singularities along a smooth involutive manifold. In 1978[5], he then precisely formulated the concept of a regular holonomic $\mathcal{D}$-module as well as the Riemann-Hilbert correspondence, namely an equivalence of categories between the derived category of $\mathcal{D}$-modules with regular holonomic cohomology and the derived category of sheaves with $\mathbb{C}$-constructible cohomology. He resolves this conjecture in 1980 (see [Kas80, Kas84]) by constructing a quasi-inverse to the functor $\mathcal{S}ol$, the functor $\mathrm{T}\mathcal{H}om$ of tempered cohomology. For a constructible sheaf F, the object $\mathrm{T}\mathcal{H}om\,(F, \mathcal{O}_X)$ is computed by applying the functor $\mathcal{H}om(F, \bullet)$ to the Dolbeault resolution of $\mathcal{O}_X$ by differential forms with distribution coefficients.

Naturally, Kashiwara's work followed Pierre Deligne's landmark monograph [Del70], in which he solves the R-H problem for regular connections. Deligne's work deeply influenced the microlocal approach to the R-H correspondence developed by Masaki jointly with Kawai [KK81].

A different proof of the R-H correspondence appeared later in [Meb84] which unfortunately led to a long-standing and unpleasant controversy over priority (see [Sch22]). However, a brief inspection of the papers [Kas75, Ram78, Kas80] should clearly settle the matter.

More recently, Kashiwara, in collaboration with Andrea D'Agnolo, proved that the category of (not necessarily regular) holonomic $\mathcal{D}$-modules can be fully faithfully embedded into the category of ind-sheaves (as introduced and developed in [KS01]) after adding a variable, following a method initiated by Dmitry Tamarkin [Tam08]. Their work builds on deep results by Mochizuki [Moc09, Moc11] and Kedlaya [Ked10, Ked11], which allow one to reduce the study of such modules to those associated with $\exp\phi$, for ϕ meromorphic. See [KS16] for a detailed exposition, and [Kas16] for a more accessible approach.

Microlocal sheaf theory

I started working with Masaki around 1978. From 1982 to 1990, we introduced and developed the microlocal theory of sheaves (see [KS82, KS85, KS90]), the core idea being to apply Sato's vision to sheaves.[6] This theory emerged from a joint paper (see [KS79])

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[5]This historical fact is mentioned in [Ram78]

[6]For a historical overview of sheaf theory, see the article by Christian Houzel in [KS90]. Sheaves are a mathematical tool of fundamental importance: they enable us to handle the local/global dichotomy, compute the obstructions that prevent passage from local to global, and define objects which are locally trivial but not globally trivial, such that the orientation sheaf or the Chern classes. They were created

Figure 2: Masaki Kashiwara and Pierrre Schapira

in which we solved the Cauchy problem for microfunction solutions of hyperbolic $\mathcal{E}$-modules on a real analytic manifold. The main concept is that of microsupport of sheaves which gives a precise meaning to the notion of propagation. On a real manifold M, for a (derived) sheaf F, its microsupport–or singular support–is a closed conic subset of the cotangent bundle T^*M that describes the codirections in which sections of F cannot be extended. The microsupport of sheaves is, in a sense, the real analog of the characteristic variety of coherent $\mathcal{D}$-modules on complex manifolds. The functorial properties of the microsupport are very similar to those of the characteristic variety of $\mathcal{D}$-modules. The precise link between these two notions is a result that asserts that the microsupport of the complex $\mathcal{S}ol(\mathcal{M})$ of holomorphic solutions of a coherent $\mathcal{D}$-module $\mathcal{M}$ is nothing but the characteristic variety of $\mathcal{M}$. Moreover-and this is one of the main results of the theory-the microsupport is coisotropic. As a by-product, one obtains a completely different proof of the involutivity of characteristics of $\mathcal{D}$-modules.

Microlocal sheaf theory has found significant applications across various fields of mathematics. In particular, Dmitry Tamarkin, David Nadler and Eric Zaslow (see [Tam08, NZ09, Nad09]) noticed that this theory was closely related to the so-called Fukaya category and could be an efficient tool in symplectic topology (see *e.g.,* [Gui23]). It has also influenced knot theory through a result of [GKS12] which shows that the category of simple sheaves along a smooth Lagrangian submanifold is a Hamiltonian isotopy invariant (see *e.g.,* [STZ17]).

More recently, Alexander Beilinson [Bei15] adapted the definition of the microsupport of sheaves to arithmetic geometry and the theory is currently being developed, in particular by Takeshi Saito (see [Sai17]).

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by the French mathematician Jean Leray (1906–1998) around 1945 while he was a prisoner in Austria. Leray, a specialist in fluid mechanics (and also a precursor of distribution theory through his weak solutions), was determined not to engage in mathematics that could be used during wartime. This theory was made more accessible by Henri Cartan in the 1950s and was later deeply generalized and reformulated in the language of category theory by Grothendieck.

Other results on $\mathcal{D}$-modules and related topics

Beyond the R-H correspondence and microlocal sheaf theory, Masaki Kashiwara has obtained many other fundamental results related to $\mathcal{D}$-modules and microlocal analysis.

(i) Masaki proves in [Kas76] the rationality of the zeros of the b-function of Bernstein-Sato by using Hironaka's theorem and adapting Grauert's direct image theorem to $\mathcal{D}$-modules.

(ii) Motivated by the theory of holonomic $\mathcal{D}$-modules, Masaki proves in [Kas82] the codimension-one property of quasi-unipotent sheaves.

(iii) Masaki gives a fundamental contribution to the theory of "variation of (mixed) Hodge structures" (see for example [Kas85b, Kas86]).

(iv) Microlocal sheaf theory naturally leads Kashiwara to extend his previous work on complex analytic Lagrangian cycles to the real setting. In [Kas85a] he defines the characteristic cycle of an $\mathbb{R}$-constructible sheaf and gives a new index formula. He also gives in this context a remarkable and unexpected local Lefschetz formula with applications to representation theory (see [KS90, Ch. IX § 6]).

(v) In [BK81], Masaki and Daniel Barlet endow regular holonomic $\mathcal{D}$-modules with a "canonical" good filtration.

(vi) Together with Pierre Schapira, he introduced in [KS01] and developed the theory of ind-sheaves as well as the Grothendieck subanalytic site on a subanalytic space. This framework enables one to construct the subanalytic sheaf of temperate holomorphic functions, an essential tool in the study of irregular holonomic $\mathcal{D}$-modules.

(vii) In [KS12], Kashiwara and Schapira carried out a systematic study of DQ-modules (deformation quantization modules). On a complex Poisson manifold X, one can construct-following Kontsevich-an algebroid stack, locally equivalent to a $\star$-algebra, i.e., the ring $\mathcal{O}_X[[\hbar]]$ endowed with a star product. The authors established finiteness, duality, and perversity results within this framework.

(viii) A classical theorem of complex geometry (Frisch-Guenot, Siu, Trautmann) asserts that, on a complex manifold X, any reflexive coherent sheaf defined on the complementary of a complex subvariety of codimension at least 3 extends as a coherent sheaf across this subvariety. The "codimension-three conjecture" is an analog statement for holonomic microdifferential modules when replacing X with a Lagrangian subvariety of the cotangent bundle. This extremely difficult conjecture was recently proved by Masaki together with Kari Vilonen in [KVi14].

(ix) Kashiwara's book on $\mathcal{D}$-modules [Kas03] contains a lot of original and deep results. In this book he develops in particular the theory of the microlocal b-function (after [SKKO80]) and the holonomy diagrams, a tool for their calculation.

(x) The book on category theory [KS06], written with P. Schapira, sheds new light on a very classical subject and contains a great deal of original results.

Other fields of mathematics and mathematical physic

I will not present here his numerous contributions to mathematical physic, group representation and quantum groups-in particular, his discovery of crystal basis.

Masaki Kashiwara has authored over 300 publications and collaborated with more than 70 co-authors. While he remains highly productive, the next generation of the Sato school is already in place. Notably, Takuro Mochizuki resolved the immense difficulties posed by the study of irregular holonomic modules, a work for which he was awarded the prestigious Breakthrough Prize in 2021. And besides Mochizuki, there are also many very young Japanese promising mathematicians working in these directions, such as Tomohiro Asano, Tatsuki Kuwagaki, Yuichi Ike, etc.

Masaki Kashiwara is a scholar who has profoundly shaped mathematics since the 1970s. His deep and lasting impact will endure.

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