Market attention and Bitcoin price modeling: theory, estimation and option pricing

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Outline of the talk

- Introduction and motivation
- Definition of the model
- Statistical properties
- Parameter estimation
- No arbitrage conditions and option pricing
- Concluding remarks

Introduction

- BitCoin is the first decentralised digital currency, which provides a solution to the problem of trust in a currency system.
- Opposite to traditional transactions, which are based on trust in financial intermediaries, this system relies on network, on fixed rules and on cryptography.
- The entire system is based on an open source software created in 2009 by a computer scientist known under the pseudonym Satoshi Nakamoto.

(Nakamoto, 2008)

Introduction

Both currency and technology behind the system, the blockchain, have gained much attention in the last few years (Böhme et al., 2015; Yermack, 2015).

Strengths	Weaknesses
Independent currency	Cyber-attacks
No inflation/deflation policy	High volatility
Pseudonymous-irreversible payments	Pseudonymous-irreversible payments

Introduction

In this work we don't focus on the blockchain technology but rather on the price formation and dynamics of the digital currency.

- Recent literature on cryptocurrencies states that BitCoin price is driven by attention or sentiment indicator.
- Possible proxies for attention on the BitCoin system are the number of Google searches, Wikipedia requests, or more traditional factors as the volume of transactions.

(Bukovina and Martiček, 2016; Ciaian et al., 2016; Kristoufek, 2013, 2015)

Introduction

In recent times several markets for derivatives on Bitcoin were born on appropriate website:

- `https://deribit.com`;
- `https://coinut.com`.

Besides and more important, the Chicago Board Options Exchange (CBOE) has launched standardized Future contracts on the cryptocurrency in December 2017.

Introduction

Deribit | BTC Index 6298.11 | Status | Time (UTC) 09-10 13:54:39

- BTC Futures
- BTC Options
- Insurance Fund
- Documentation
- Information
- Join Our Chat
- Blog

BTC Options click on any price to open order form

☑ Last ☑ Implied Volatility ☑ Volume ☐ Open Interest ☑ Δ|Delta ☐ Positions

Last	Size	IV	Bid	Ask	IV	Size	Vol	Δ|Delta	Strike
Calls						Underlying: Index($6298)			14-09-18
0.0005	-	-	- -	Ƀ0.0015 $9.44	110.1%	4.7	0.2	0.01	7750
Calls						Underlying: BTC-28SEP18($6254)			28-09-18
0.0140	8.1	85.4%	Ƀ0.0095 $59.40	Ƀ0.0115 $71.90	90.0%	1.0	53.0	0.12	8000
0.0100	23.0	89.8%	Ƀ0.0060 $37.52	Ƀ0.0075 $46.90	94.4%	1.0	9.8	0.08	8500
0.0070	3.1	92.1%	Ƀ0.0035 $21.87	Ƀ0.0055 $34.37	100.4%	4.7	15.2	0.05	9000
0.0020	23.0	98.7%	Ƀ0.0015 $9.37	Ƀ0.0030 $18.74	109.7%	3.1	60.4	0.03	10000
0.0030	0.1	109.1%	Ƀ0.0010 $6.25	Ƀ0.0025 $15.62	123.7%	12.1	10.2	0.02	11000
0.0010	4.2	113.6%	Ƀ0.0005 $3.12	Ƀ0.0015 $9.37	129.4%	3.1	16.3	0.01	12000
0.0010	-	-	- -	Ƀ0.0020 $12.50	148.0%	12.1	-	0.00	13000
0.0005	-	-	- -	Ƀ0.0010 $6.25	147.0%	5.0	-	0.00	14000
0.0005	-	-	- -	Ƀ0.0015 $9.37	165.2%	7.3	0.1	0.00	15000
0.0005	-	-	- -	Ƀ0.0010 $6.25	199.9%	14.1	-	0.00	20000
Calls						Underlying: BTC-28SEP18($6254)			26-10-18
0.0385	5.9	77.9%	Ƀ0.0315 $196.97	Ƀ0.0355 $221.98	81.6%	4.2	0.1	0.23	8000
0.0270	10.4	79.2%	Ƀ0.0225 $140.68	Ƀ0.0265 $165.69	83.5%	5.4	-	0.18	8500
0.0195	7.3	80.9%	Ƀ0.0165 $103.20	Ƀ0.0200 $125.09	85.4%	5.2	2.0	0.14	9000
0.0180	8.6	83.8%	Ƀ0.0130 $81.29	Ƀ0.0160 $100.05	88.2%	7.3	2.2	0.11	9500
Calls						Underlying: BTC-28DEC18($6251)			28-12-18
0.0690	2.0	80.3%	Ƀ0.0650 $406.23	Ƀ0.0700 $437.48	82.9%	4.0	-	0.30	8750
0.0475	6.4	82.5%	Ƀ0.0430 $268.75	Ƀ0.0480 $300.00	85.7%	9.3	1.0	0.21	10000
0.0220	12.5	86.0%	Ƀ0.0200 $124.86	Ƀ0.0245 $152.96	90.2%	5.0	3.0	0.11	12500
0.0140	5.2	90.7%	Ƀ0.0115 $71.79	Ƀ0.0145 $90.52	94.8%	7.4	6.0	0.07	15000
0.0080	12.6	96.9%	Ƀ0.0045 $28.10	Ƀ0.0075 $46.83	104.5%	12.6	-	0.03	20000

Figure: Screenshot of the website www.deribit.com on September 10, 2018

Goals

Motivated by the quoted papers and the increasing interest in Bitcoin derivatives, our main goals are:

1. To develop a model in continuous time to describe the dynamics of Bitcoin price given the dynamics of suitable proxies for attention/sentiment.

2. To provide an estimation method for model parameters and to fit the model using market data.

3. To apply the model for derivative pricing purposes looking for close pricing formula.

Model specification

Given a filtered probability space $(\Omega, \mathcal{F}, \mathbf{P})$ endowed with $\mathbb{F} = \{\mathcal{F}_t,\ t \geq 0\}$, we assume that:

$$\begin{cases} \mathrm{d}S_t = \mu_S P_{t-\tau} S_t \mathrm{d}t + \sigma_S \sqrt{P_{t-\tau}} S_t \mathrm{d}W_t, \quad S_0 = s_0 \in \mathbb{R}_+, \\ \mathrm{d}P_t = \mu_P P_t \mathrm{d}t + \sigma_P P_t \mathrm{d}Z_t, \quad P_t = \phi(t),\ t \in [-L, 0], \end{cases} \tag{1}$$

where $S = \{S_t,\ t \geq 0\}$ is the BitCoin price process and $P = \{P_t,\ t \geq 0\}$ is a stochastic factor, representing market attention on the BitCoin itself.

Note that:

- factor P affects both the drift and the diffusion coefficients of the price dynamics;
- a possible delay τ is also considered between the confidence index and its effect on BitCoin price changes.

Model specification

We have:

- $(W, Z) = \{(W_t, Z_t),\ t \geq 0\}$ is a standard bi-dimensional Brownian motion on $(\Omega, \mathcal{F}, \mathbf{P})$, which is $\mathbb{F}$-adapted;
- $\phi : [-L, 0] \rightarrow [0, +\infty)$ is a continuous (deterministic) initial function;
- $\mu_P \in \mathbb{R} \setminus \{0\}$, $\sigma_P \in \mathbb{R}_+$, $\mu_S \in \mathbb{R} \setminus \{0\}$, $\sigma_P \in \mathbb{R}_+$ are constant model parameters.
- $\tau \in \mathbb{R}_+$ is a possible delay between the confidence factor and its effect on price changes.

Assuming that $\tau < L$ and that P is observed in the period $[-L, 0]$ makes the bi-variate model jointly feasible.

Model specification

Given that P is a Geometric Brownian motion with well-known solution, it is easy to prove the following:

Theorem

The bi-variate stochastic delayed differential equation defined in eq. (1) has a continuous, $\mathbb{F}$-adapted, unique solution $(S, P) = \{(S_t, P_t),\ t \geq 0\}$ given by

$$S_t = s_0 \exp\left(\left(\mu_S - \frac{\sigma_S^2}{2}\right)\int_0^t P_{u-\tau}\mathrm{d}u + \sigma_S \int_0^t \sqrt{P_{u-\tau}}\mathrm{d}W_u\right), \quad t \geq 0,$$

$$P_t = \phi(0)\exp\left(\left(\mu_P - \frac{\sigma_P^2}{2}\right)t + \sigma_P Z_t\right), \quad t \geq 0.$$

In particular, if $\phi(0) \geq 0$, then $P_t \geq 0$ **P**-a.s. for all $t \geq 0$.

Model specification

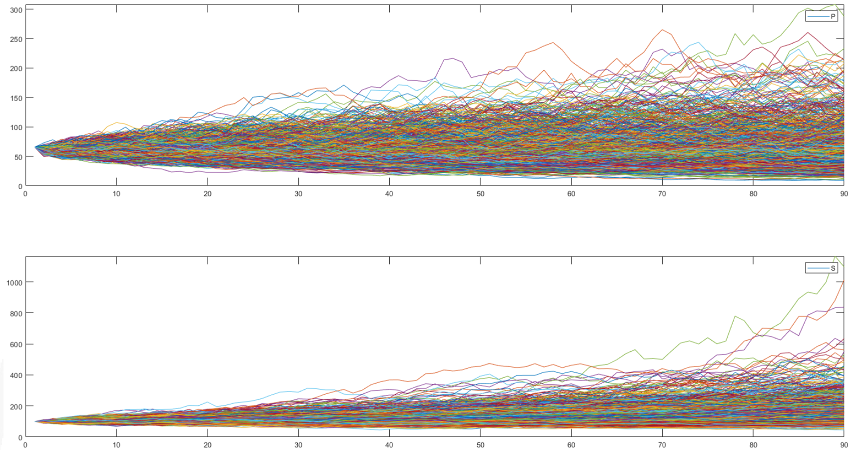

Figure: Simulation of 1000 paths for three months with a daily frequency. Parameter value are set to $\mu_S = 0.0386$, $\sigma_S = 0.0996$, $\mu_P = 0.5164$, $\sigma_P = 1.0652$ and $\tau = 1$ day.

Cumulative attention

Define the *cumulative attention process* $A^\tau = \{A_t^\tau,\ t \geq 0\}$, associated to the attention index P, as follows:

$$\begin{aligned} A_t^\tau &:= \int_0^t P_{u-\tau} \mathrm{d}u = \int_{-\tau}^0 \phi(u) \mathrm{d}u + \int_0^{t-\tau} P_u \mathrm{d}u \\ &= A_\tau^\tau + \int_0^{t-\tau} P_u \mathrm{d}u, \quad t \geq 0. \end{aligned}$$

By applying known results for the geometric Brownian motion, $\mathbb{E}\left[A_t^\tau\right]$ and $\mathrm{Var}[A_t^\tau]$ may be written in terms of model parameters μ_P, σ_P and τ. The feasibility of having $\tau \neq 0$ makes calculations trickier than "standard" case.

Note that both $A_0^\tau = 0$ and A_t^τ turns out to be deterministic for $t \in [0, \tau]$, so that $\mathbb{E}\left[A_t^\tau\right] = \int_{-\tau}^{\tau-t} \phi(u) \mathrm{d}u$ and $\mathrm{Var}[A_t^\tau] = 0$.

Statistical properties

The solution $(S,P) = \{(S_t, P_t),\ t \geq 0\}$ of eq. (1) has the following properties, for $t \geq 0$:

(i)

$$\mathbb{E}\left[\log\left(S_t\right)\right] = \log\left(s_0\right) + \left(\mu_S - \frac{\sigma_S^2}{2}\right)\mathbb{E}\left[A_t^\tau\right];$$

$$\mathrm{Var}\left[\log\left(S_t\right)\right] = \left(\mu_S - \frac{\sigma_S^2}{2}\right)^2 \mathrm{Var}[A_t^\tau] + \sigma_S^2 \mathbb{E}\left[A_t^\tau\right],$$

(ii) the conditional distribution of S_t, given $\mathcal{F}^P_{T-\tau}$, is Log-Normal (m, v) and we have $m = \log\left(s_0\right) + \left(\mu_S - \frac{\sigma_S^2}{2}\right) A_t^\tau$ and $v = \sigma_S^2 A_t^\tau$.

The proof of (i) is straightforward; the key point for proving (ii) is that $Y_t := \int_0^t \sqrt{P_{u-\tau}}\mathrm{d}W_u$, conditional on $\mathcal{F}^P_{t-\tau}$, is a $\mathcal{N}\left(0, \sigma_S^2 A_t^\tau\right)$, that only depends on cumulative attention A_t^τ up to time t.

Parameter estimation

Let us fix a discrete observation step Δ and consider the discrete time process $\{S_0, S_i, i \in N\}$ where $S_i := S_{i\Delta}$. Define the corresponding logarithmic returns process $\{R_i, i \in N\}$,

$$R_i = \log S_i - \log S_{i-1}.$$

Under the assumptions of model (1)

$$R_i = \left(\mu_S - \frac{\sigma_S^2}{2}\right) \int_{(i-1)\Delta}^{i\Delta} P_{u-\tau} du + \sigma_S \int_{(i-1)\Delta}^{i\Delta} \sqrt{P_{u-\tau}} dW_u.$$

Parameter estimation

If we define

$$Y_i = \int_{(i-1)\Delta}^{i\Delta} \sqrt{P_{u-\tau}} dWu \qquad \text{and} \qquad A_i = \int_{(i-1)\Delta}^{i\Delta} P_{u-\tau} du$$

we may write

$$R_i = \left(\mu_S - \frac{\sigma_S^2}{2} \right) A_i + \sigma_S Y_i.$$

In this way, under model assumptions, the vector of discretely observed logarithmic returns $\mathbf{R} = (R_1, R_2, \ldots, R_n)$, conditionally on $\mathbf{A}$, is jointly normal with covariance matrix $\sigma_S^2 Diag(A_1, A_2, \ldots, A_n)$.

Parameter estimation

The application of Bayes's rule makes it possible to obtain the likelihood function of $(\mathbf{R}, \mathbf{A})$:

$$\mathcal{L}_{\mathbf{R},\mathbf{A}}(\theta; \mathbf{r}, \mathbf{a}) = f_{A_1}(a_1) \prod_{i=2}^{n} f_{(A_i|A_{i-1})}(a_i)$$

$$\cdot \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_S^2 a_i}} \exp\left(-\frac{1}{2} \frac{\left(r_i - \left(\mu_S - \frac{\sigma_S^2}{2} \right) a_i \right)^2}{\sigma_S^2 a_i} \right).$$

The probability distribution functions $f_{A_1}(.)$ and $f_{(A_i|A_{i-1})}(.)$ for $i = 2, 3, \ldots, n$ are not available in closed form so it is for the likelihood $\mathcal{L}_{\mathbf{R},\mathbf{A}}(\theta; \mathbf{r}, \mathbf{a})$. Though, by applying the outcomes in Levy (1992), it is possible to find a closed formula approximating their value.

Parameter estimation

Theorem

Given the realized sample $(\mathbf{r}, \mathbf{a})$, the log-likelihood $\log \mathcal{L}_{\mathbf{R},\mathbf{A}}(\theta; \mathbf{r}, \mathbf{a})$ of θ can be approximated by

$$\log \mathcal{L}_{\mathbf{R},\mathbf{A}}(\theta; \mathbf{r}, \mathbf{a}) = \sum_{i=1}^{n} \left[\log \frac{1}{\sqrt{2\pi\sigma_S^2 a_i}} - \frac{1}{2} \frac{\left(r_i - \left(\mu_S - \frac{\sigma_S^2}{2} \right) a_i \right)^2}{\sigma_S^2 a_i} \right]$$
$$+ \log \frac{1}{a_1 \nu_1 \sqrt{2\pi}} - \frac{(\log(a_1) - \alpha_1)^2}{2(\nu_1)^2}$$
$$+ \sum_{i=2}^{n} \left[\log \frac{1}{a_i \nu_i \sqrt{2\pi}} - \frac{(\log(a_i) - \alpha_i)^2}{2(\nu_i)^2} \right]$$

where α_1, ν_1 and α_i, ν_i for $i = 2, \ldots, n$ can be expressed in terms of μ_P, σ_P, P_0 and Δ.

Parameter estimation

Parameters are estimated by maximizing the log-likelihood approximation,

$$(\widehat{\mu}_P, \widehat{\sigma}_P, \widehat{\mu}_S, \widehat{\sigma}_S) = \arg \max_{\substack{\mu_P, \sigma_P \\ \mu_S, \sigma_S}} \log \mathcal{L}_{\mathbf{R},\mathbf{A}}(\theta; \mathbf{r}, \mathbf{a}).$$

In this case the methodology is referred to as Quasi-Maximum likelihood (QML).

In order to empirically check the finite-sample behavior of the suggested QML estimation method we perform a simulation study and we find that all model parameters are properly estimated.

Estimation of the Delay Parameter

The approach proposed above is not feasible if τ is unknown since it affects the definition of the discrete process A_i.
For this reason we adopt a two step estimation procedure known as profile likelihood.

- Split $\theta := (\mu_P, \sigma_P, \mu_S, \sigma_S, \tau)$ in $\theta = (\tau, \lambda)$, with $\lambda = (\mu_P, \sigma_P, \mu_S, \sigma_S)$.
- Define

$$\mathcal{L}_p(\tau) = \max_{\lambda} \mathcal{L}(\tau, \lambda) = \mathcal{L}(\tau, \widehat{\lambda}_\tau),$$

  where $\widehat{\lambda}_\tau$ is the maximum likelihood estimate of λ for fixed τ.

(Davison, 2003; Pawitan, 2001)

Estimation of the Delay Parameter

- In this setting the estimate for τ is

$$\widehat{\tau} = \arg\max_{\tau} \log \mathcal{L}_p(\tau).$$

- The confidence region for τ is the set

$$\mathcal{C}_\alpha := \left\{ \tau : \mathcal{L}_p(\tau) \geq \mathcal{L}_p(\widehat{\tau}) - \frac{1}{2} c_p(1 - 2\alpha) \right\},$$

with $c_p(\alpha)$ is the α quantile of the χ^2_p distribution.

Model fit to market data

We consider

- Time series data from from January 1, 2015 to December 31, 2017;
- $\Delta = 1/52$ (weekly) for R_i and $\delta = \Delta/7$ (daily) for P_i;
- the process A by proper integration of the process P.

By applying the profile quasi maximum likelihood we obtain the following estimates

$P =$ *volume of transactions*

Variable	Fit. value	Std. error
τ	1.0000	-
μ_P	0.5164	0.6238
σ_P	1.0652	0.0607
μ_S	0.0386	0.0086
σ_S	0.0996	0.0057

$P =$ *Google Search Volume Index*

Variable	Fit. value	Std. error
τ	7.0000	-
μ_P	1.8336	0.6855
σ_P	1.1665	0.0667
μ_S	0.4416	0.1232
σ_S	0.4432	0.0253

No arbitrage conditions and risk neutral probability measure

- Fix a finite time horizon $T > 0$ and assume the existence of a money market account, whose value process $B = \{B_t,\ t \geq 0\}$ is $B_t = \exp\left\{\left(\int_0^t r(s)\mathrm{d}s\right)\right\}$, $t \geq 0$, where $r : [0, +\infty) \to \mathbb{R}$ is a bounded, deterministic function representing the instantaneous risk-free interest rate.
- To exclude arbitrage opportunities, we need to check that the set of all equivalent martingale measures for the BitCoin price process S is non-empty.
- Indeed, we show that it contains more than a single element, since P does not represent the price of any trade-able asset, and therefore the underlying market model is incomplete.

No arbitrage conditions and risk neutral probability measure

Theorem

Assume that the model in eq. (1) holds; then every equivalent martingale measure $\mathbf{Q}$ for S defined on $(\Omega, \mathcal{F}_T)$ has the following density

$$\left.\frac{\mathrm{d}\mathbf{Q}}{\mathrm{d}\mathbf{P}}\right|_{\mathcal{F}_T} =: L_T^{\mathbf{Q}}, \quad \mathbf{P}-\text{a.s.},$$

where

$$L_t^{\mathbf{Q}} := \mathcal{E}\left(-\int_0^{\cdot} \frac{\mu_S P_{s-\tau} - r(s)}{\sigma_S \sqrt{P_{s-\tau}}} \mathrm{d}W_s - \int_0^t \gamma_s \mathrm{d}Z_s\right)_t, \quad t \in [0, T], \tag{2}$$

which is parametrized by a suitable $\mathbb{F}$-progressively measurable process $\gamma = \{\gamma_t,\ t \in [0, T]\}$, governing the change of drift of the $(\mathbb{F}, \mathbf{P})$-Brownian motion Z.

Minimal Martingale measure

One simple example of a candidate equivalent martingale measure is the so-called *minimal martingale measure*, denoted by $\widehat{\mathbf{P}}$, which corresponds to the choice $\gamma \equiv 0$ in (2). In such case, by Girsanov's theorem, we have two independent $(\mathbb{F}, \widehat{\mathbf{P}})$-Brownian motions defined respectively by

$$\widehat{W}_t := W_t + \int_0^t \frac{\mu_S P_{s-\tau} - r(s)}{\sigma_S \sqrt{P_{s-\tau}}} \mathrm{d}s, \quad t \in [0, T],$$
$$\widehat{Z}_t := Z_t, \quad t \in [0, T].$$

The dynamics of the attention index P does not change while for the discounted BitCoin price S_t^* we get:

$$S_t^* = s_0 \exp\left\{ \left(\sigma_S \int_0^t \sqrt{P_{u-\tau}} \mathrm{d}\widehat{W}_u - \frac{\sigma_S^2}{2} \int_0^t P_{u-\tau} \mathrm{d}u \right) \right\}, \quad t \in [0, T].$$

Option Pricing

Let $H = \varphi(S_T)$ be the payoff a European-type contingent claim with date of maturity T and set $A^{\tau}_{t,T} := A^{\tau}_{T} - A^{\tau}_{t}$, for each $t \in [0, T)$.

Theorem

The risk-neutral price $\Phi_t(H)$ at time t of H is given by

$$\Phi_t(H) = \mathbb{E}^{\widehat{\mathbf{P}}}\left[\psi(t, S_t, A^{\tau}_{t,T})\middle| S_t\right], \quad t \in [0, T). \tag{3}$$

Here $\psi : [0, T) \times \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}$ is a Borel-measurable function such that

$$\psi(t, S_t, A^{\tau}_{t,T}) = B_t \mathbb{E}^{\widehat{\mathbf{P}}}\left[\frac{1}{B_T} G\left(t, S_t, A^{\tau}_{t,T}, Y_{t,T}\right)\middle| \mathcal{F}^W_t \vee \mathcal{F}^P_{T-\tau}\right],$$

for a suitable function G depending on the contract and such that $G\left(t, S_t, A^{\tau}_{t,T}, Y_{t,T}\right)$ is $\widehat{\mathbf{P}}$-integrable.

Price computation in practice

It is worth noticing that the result in (3) only depends on the distribution of $A^{\tau}_{t,T}$ which is the same both under measure $\widehat{\mathbf{P}}$ and $\bar{\mathbf{P}}$. Then, for every $t \in [0, T)$ it may be written as:

$$\Phi_t(H) = \int_0^{+\infty} \psi(t, S_t, a) f_{A^{\tau}_{t,T}}(a) \mathrm{d}a,$$

where $f_{A^{\tau}_{t,T}}(a)$ denotes the density function of $A^{\tau}_{t,T}$, for each $t \in [0, T)$.

In order to numerically compute prices we approximate $f_{A^{\tau}_{t,T}}(a)$ of $A^{\tau}_{t,T}$ as above by applying otcomes in Levy (1992).

Plain Vanilla and Binary CALL options

Plain Vanilla options and Binary Cash-or-Nothing options have started to be traded in the BitCoin system.
Under our model assumptions the price of these contracts is given by

$$C_0 = \frac{1}{T-\tau} \int_0^{+\infty} C^{BS}(0, S_0, a) \mathcal{LN}pdf_{\alpha,\nu^2}\left(\frac{a}{T-\tau}\right) \mathrm{d}a,$$

for the plain vanilla Call and

$$CoN_0 = \frac{X}{T-\tau} \int_0^{+\infty} \mathcal{N}\left(d_2(T-t, S_0, a)\right) \mathcal{LN}pdf_{\alpha,\nu^2}\left(\frac{a}{T-\tau}\right) \mathrm{d}a.$$

for a Cash-or-Nothing binary Call paying X if exercised. Note that these integrals may be computed numerically.

Model pricing performance

In order to assess pricing performance, our model prices are compared with the mid-value of the Bid-Ask prices on *Deribit* platform by computing the Root Mean Squared Error (RMSE) and using the Black & Scholes pricing formula as a benchamark.

K	Bid	Ask	CFP-Volume	CFP-Google	BS
2200	0.1662	0.2318	0.1981	0.2052	0.1967
2300	0.1670	0.2072	0.1674	0.1765	0.1655
2400	0.1390	0.1845	0.1429	0.1501	0.1369
2500	0.1142	0.1638	0.1182	0.1264	0.1112
2600	0.0922	0.1376	0.0965	0.1053	0.0887
2700	0.0749	0.1202	0.0776	0.0868	0.0695
2800	0.0572	0.1047	0.0616	0.0709	0.0535
2900	0.0442	0.0983	0.0483	0.0573	0.0405

Table: Option Prices with $S_t = 2710\$$, t=30 July and T=25 August: source `http://www.deribit.com` prices in BTC, strikes in USD

Model pricing performance

Model performance when options are aggregated with moneyness or time to maturity

Table: Root mean squared error (RMSE) between model and market prices

Options	N	CFP Volume	CFP Google	BS
All	144	**0.0257**	0.0290	0.0407
Very Shorts	16	0.0225	**0.0130**	0.0204
1 Months	32	0.0200	**0.0110**	0.0256
2 Months	32	**0.0231**	0.0426	0.0310
ITM	54	**0.0209**	0.0268	0.0356
ATM	36	0.0281	**0.0277**	0.0437
OTM	54	**0.0283**	0.0318	0.0432

where $$RMSE^2 = \frac{1}{N}\sum_{i=1}^{N}\left(C_i^{mod} - C_i^{mk}\right)^2$$

Concluding Remarks

- We introduced a bi-variate model to describe the joint dynamics of the BitCoin price and an attention index which affects the price dynamics directly through drift and diffusion coefficients;.
- A closed form approximation for the joint density and a statistical estimation for the model have been provided.
- The model has been fitted using real data of BitCoin price with Volume of transactions and SVI index alternatively. Parameters have been estimated via the Profile Quasi Maximum Likelihood method.
- A closed form approximation for the valuation of european derivatives has been introduced.
- Option pricing performance has been assessed on options traded on `https://deribit.com`.

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