On instabilities of the Bitcoin protocol

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Breaking Bitcoin

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On instabilities of the Bitcoin protocol

1. Dynamical instability
3. Hard forks
4. Double spend attacks
5. Catch-up mining instability
The Mandelbrot set

Iteration of $x \mapsto x^2 + c$
Details of the Mandelbrot set
The Lorentz Attractor

Simple meteorological model.
Details of the Lorentz Attractor
The Butterfly Effect

Butterfly effect

A tiny change on initial conditions may produce large deviations on the long term behavior.
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Example

Something as small as the flutter of a butterfly’s wing can cause a tornado in another part of the Earth.
Dynamical Instability

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This “change” can have different natures: Catastrophic, mildly disfunctioning, operating abnormaly,...
Some examples

- The stability of the Solar System.
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- Hydrodynamical stability and turbulence.
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The Bitcoin Dynamical System

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S. Nakamoto, 11/1/2008
Basic stability question:

How likely is the Bitcoin network be able to survive over the long run?
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There are some general arguments that give an useful insight.
Lindy Effect

A quantitative version of the “test of time”.
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Goldman (1964), Mandelbrot (1984), Taleb (2007),...
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Example: Bitcoin’s life expectancy is 4 times larger than ETH life expectancy.
Idea for a definite BIP proposal

Corollary

Resilience improves with time.
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If it ain’t broke, don’t fix it!
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Definite BIP

Don’t touch the Bitcoin protocol unless there is an obvious bug or a direct threat.
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Definite BIP
Don’t touch the Bitcoin protocol unless there is an obvious bug or a direct threat.

And this is a BIP since Bitcoin improves along with its lifetime...
Antifragility

Antifragile System (Taleb, 2012)

An evolving Dynamical System that increases in capability, resilience, or robustness as a result of stressors, shocks, volatility, noise, mistakes, faults, attacks, or failures
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Decentralized systems are more antifragile than centralized systems that have a “central point of failure”.

“What doesn’t kill you makes you stronger”
Good and bad hard forks

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Are hard forks a disruption or a feature of the Bitcoin protocol?
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But things are far more subtle...
Hard forks and prize

- The market price should reflect if a hard fork is bad.
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- What’s going on?

R. Pérez-Marco

On instabilities of the Bitcoin protocol
Uncertainty and prize

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- There is a hidden cost for conflict. The price is high.
- The fork resolves the tension when each camp gets their choice blockchain with the version of the protocol they like.
- Once the tension is resolved, the uncertainty leaves and the market price adjusts.
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A classical instability

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- Computation of the probability of a double spend attack.

The recipient waits until the transaction has been added to a block and $z$ blocks have been linked after it. He doesn't know the exact amount of progress the attacker has made, but assuming the honest blocks took the average expected time per block, the attacker's potential progress will be a Poisson distribution with expected value:

$$\lambda = z \frac{q}{p}$$

To get the probability the attacker could still catch up now, we multiply the Poisson density for each amount of progress he could have made by the probability he could catch up from that point:

$$\sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} \begin{cases} (q/p)^{(z-k)} & \text{if } k \leq z \\ 1 & \text{if } k > z \end{cases}$$

Rearranging to avoid summing the infinite tail of the distribution...

$$1 - \sum_{k=0}^{z} \frac{\lambda^k e^{-\lambda}}{k!} (1-(q/p)^{(z-k)})$$

“assuming the honest blocks took the average expected time per block”
Correct computation


Theorem

Let \( 0 < q < \frac{1}{2} \) be the relative hash power of the group of the attackers, and \( p = 1 - q \). After \( z \) blocks have been validated by the honest miners, the probability of success of the attackers is

\[
P(z) = I_\frac{pq}{2}(z, \frac{1}{2}),
\]

where \( I_{x}(a, b) \) is the Regularized Incomplete Beta Function

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I_{x}(a, b) = \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1}(1-t)^{b-1} \, dt.
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P(z) = I_{4pq}(z, 1/2),
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l_x(a, b) = \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1}(1 - t)^{b-1} dt.
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Corollary

A Corollary (that everyone knew)
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The probability of a double spend decreases exponentially with the number of confirmations.
A more precise probability

The parameter $\kappa$ measures the deviation from average time validation of honest blocks ($\kappa = 1$ is Satoshi’s assumption).
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![Graph showing probability of success of a double-spend attack as a function of $\kappa$.]
A new type of instability

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- If a miner just mined a block, but he realizes than just before another block has propagated on the network, will he start mining on top of the network blockchain?
A new type of instability


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• Example: A miner must mine on top of the largest work blockchain.

• If a miner just mined a block, but he realizes than just before another block has propagated on the network, will he start mining on top of the network blockchain?

• The answer depends on the miner’s hashrate.
Catch-up mining

- Let $E^m_n(q, v)$ be the EV (Expected Value) of the optimal strategy to overcome $m$ blocks delay in $n$ steps for a reward $v$ (includes block rewards, plus fees, plus a possible double spend). When $E^m_n(q, v) > 0$ the strategy is profitable.
Catch-up mining

- Let $E_n^m(q, \nu)$ be the EV (Expected Value) of the optimal strategy to overcome $m$ blocks delay in $n$ steps for a reward $\nu$ (includes block rewards, plus fees, plus a possible double spend). When $E_n^m(q, \nu) > 0$ the strategy is profitable.

- The map $\nu \mapsto E_n^m(q, \nu)$ is a continuous increasing convex affine by pieces function.
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**Theorem**

If $q > 0.42$, $m = 2$, and $b > 0$ is the block reward, then

$$\lim_{n \to +\infty} E^2_n(q, 3b) > 0.$$
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$$\lim_{n \to +\infty} E_n^2(q, 3b) > 0.$$ 

- If $\nu > 3b$, considering block fees or possible rewards for double spends, the minimal value for $q$ is lower.
Why this happens?

• If the hashrate is over 42\% of the total hashrate, following the optimal strategy and with unlimited resources, he has a positive return expectation of catch-up, mining on top of his mined block.

• Quick explanation: If the miner mines 2 consecutive blocks before the rest of the network, although it is less probable, he will cash the reward corresponding to 3 blocks: The 2 mined blocks plus the invalidate one.
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Catch-up mining instability

When a miner refuses to mine on top of the network blockchain and mines on top on blocks that he has secretly validated.
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and thank you for your attention!!