Selfish Mining and Dyck Words in Bitcoin and Ethereum Networks

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Block withholding strategies

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- *"Selfish Mining in Ethereum"*, ArXiv:1904.13330, May 2019.

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And the asymptotic $P\&L$ per unit of time

$$P\&LT_{\infty} = \lim_{T \to +\infty} \frac{R(T) - C(T)}{T}$$

Repetition games

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Proof. Strong law of large numbers.

Comparing repetition games with equal costs

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Definition (Revenue and Cost Ratio)

The Revenue Ratio, resp. Cost Ratio, of a Repetition Strategy ξ is

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Theorem (Comparing profitabilities of equal cost strategies)

We consider two integrable strategies with $\Upsilon(\xi) = \Upsilon(\xi')$. Then ξ is more profitable than strategy ξ' in the long run if and only if

$$\Gamma(\xi') \leq \Gamma(\xi) \, .$$

Block withholding mining

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- Key observation:

Block withholding and honest strategies have exactly the same cost

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- The Cost depends on external factors like energy prize, hardware costs, etc These costs are independent of the strategy as long as the strategy is a full time intensive computation.

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N and N' two independent Poisson processes with parameters α' and α with $\alpha' < \alpha$ and $N(0) = N'(0) = 0$.

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$$\mathbb{E}[\tau] = \frac{1}{\alpha - \alpha'} , \quad \mathbb{E}[N'(\tau)] = \frac{\alpha'}{\alpha - \alpha'} , \quad \mathbb{E}[N(\tau)] = \frac{\alpha}{\alpha - \alpha'} .$$

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- We present a new, very direct, and elementary approach to compute the Revenue Ratio without using Martingales, nor Markov Chains.
- It uses Dyck words, Catalan numbers, and Catalan distributions.

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- We denote by $C(x) = \frac{1-\sqrt{1-4x}}{2x}$ its generating series, and by $\mathcal{D}$ the space of all Dyck words.
- Catalan distributions: For $(p, q) \in [0, 1]^2$ with $q < p$ and $p + q = 1$:

$$\sum_{n\geq 0} p(pq)^n C_n = 1$$

$$\sum_{n\geq 0} np(pq)^n C_n = \frac{q}{p-q}$$

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Corollary

Let L be the number of official blocks added to the blockchain after an attack cycle. Then, $\mathbb{P}[L = 1] = p$, $\mathbb{P}[L = 2] = p + pq^2$ and

$$\mathbb{P}[L = n] = pq^2(pq)^n C_{n-2}$$

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- We express a dimensionless Revenue Ratio in b/τ_0 units where $b =$ is the coinbase and $\tau_0 =$ Bitcoin interblock time.

Theorem (Revenue Ratio of SM strategy)

The Revenue Ratio of SM is

$$\Gamma_B = \frac{[(p-q)(1+pq)+pq]q-(p-q)(1-\gamma)p^2q}{pq^2+p-q}$$

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Sketch of proof.

We have $\mathbb{E}[T] = \mathbb{E}[L]\tau_0$, $\mathbb{E}[Z] = \mathbb{E}[L] - \left(p + (2-\gamma)p^2q\right)$ and $\mathbb{E}[L] = 1 + \frac{p^2q}{p-q}$. □

Other classical block witholding strategies

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Theorem (Revenue Ratio of LSM strategy)

The Revenue Ratio of LSM is

$$\Gamma_{LSM} = \frac{q(p+pq-q^2)}{p+pq-q} - \frac{pq(p-q)(1-\gamma)}{\gamma} \cdot \frac{1-p(1-\gamma)C((1-\gamma)pq)}{p+pq-q}$$

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Theorem (Revenue Ratio of EFSM strategy)

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$$\Gamma_{EFSM} = \frac{q}{p} - \frac{(1-\gamma)(p-q)}{\gamma p}\left(1-pC\left((1-\gamma)pq\right)\right)$$

Comparison of strategies

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We color the region $(q, \gamma) \in [0, 0.5] \times [0, 1]$ according to which strategy is more profitable.

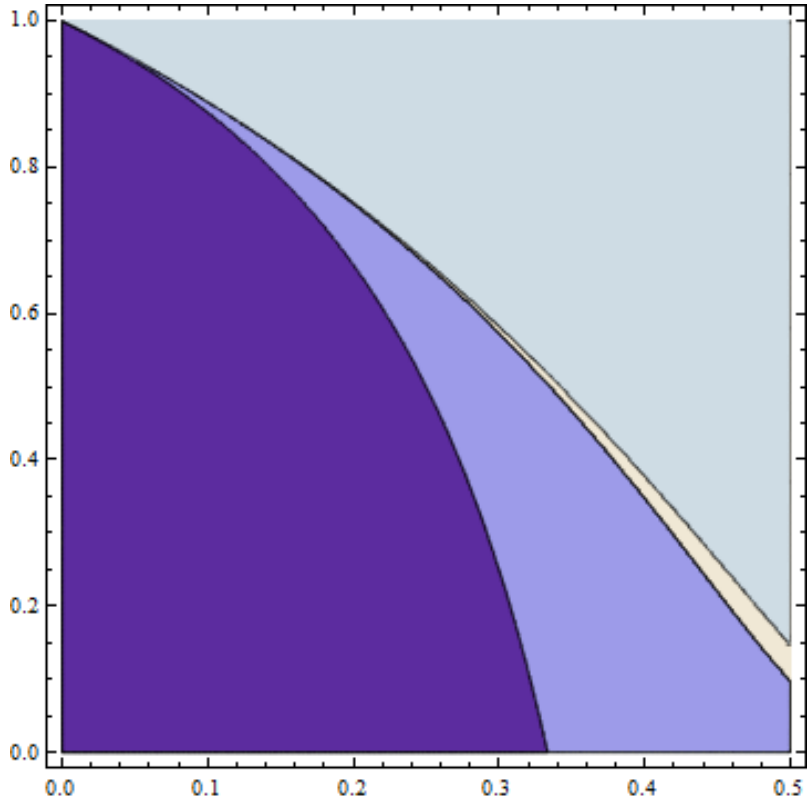

Figure: From left to right: HM, SM, LSM, EFSM.

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- SM2A: Minimum belligerence and signals all uncles.
- SM2B: Minimum belligerence and signals no uncle. Goal: Minimize the difficulty parameter.

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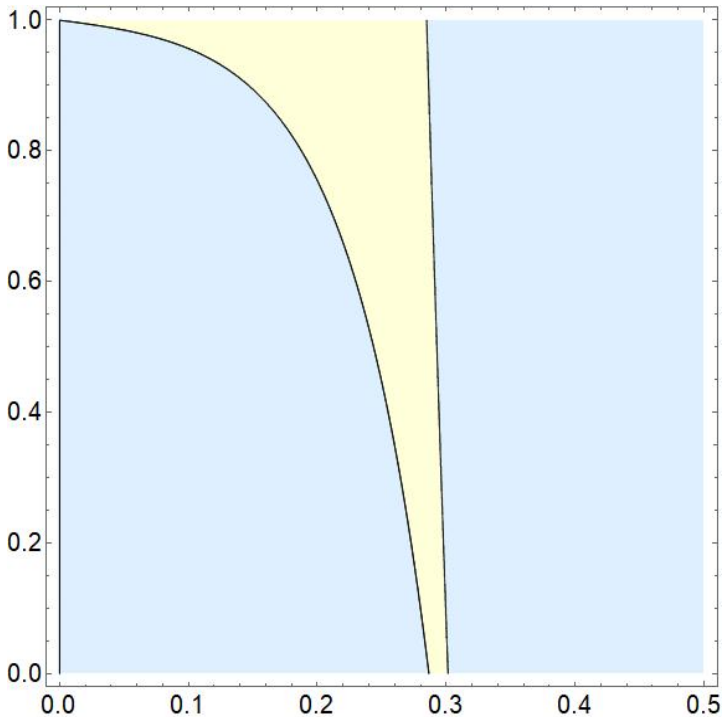
Theorem (Comparison of SM strategies in Ethereum)

For any (q, γ), $\Gamma(SM1) < \Gamma(SM2A)$ *and for* $q > 30.1\%$, $\Gamma(SM2A) < \Gamma(SM2B)$



Threshold of profitabilities Bitcoin/Ethereum

Ethereum more resilient than Bitcoin iff $q < q_0$ with $q_0 \approx 25\%$.

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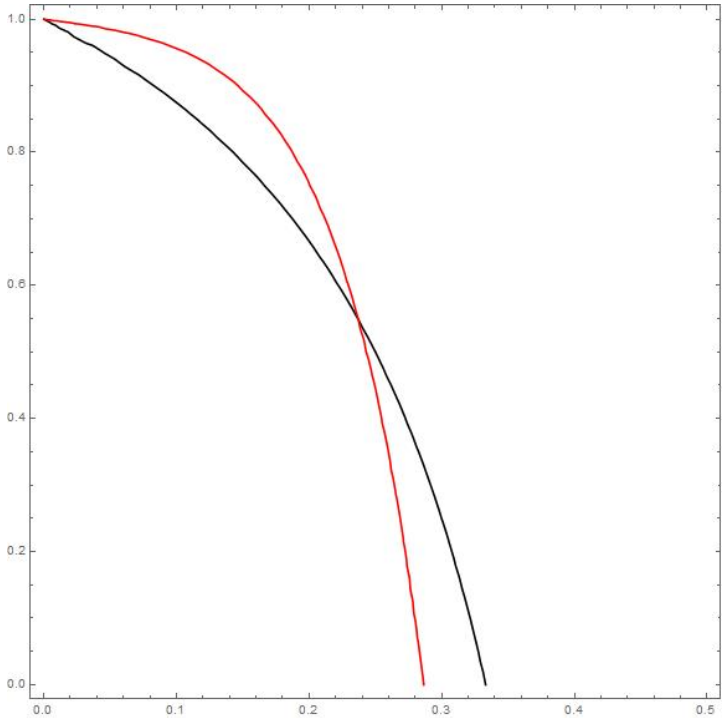

Figure: Blue: Bitcoin, Red: SM2A

Conclusions

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- Best strategy (for $q > 25\%$): Avoid competitions and ignore uncles.

Thank you for your attention!