1. **Quick Reminders (in modern mathematics' forms)**

a) Hippocrates' theorem

Let a and d two magnitudes with $2d = a$. If there exist b and c such that:

$a/b = b/c = c/d$, alors $c^3 = 2d$

in other words, the cube of side c is the double of the cube of side d (it is Hippocrates' theorem). Quick proof:

Let us multiply successively the three ratios which is equivalent to multiply any of them three times. We get in particular:

$(c/d)^3 = (a/b)\times(b/c)\times(c/d) = (a/d)$, d'où (puisque $a/d = 2$), $(c/d)^3 = 2$.

b) Geometric form for the duplication of the cube.

We want to get the following situation:

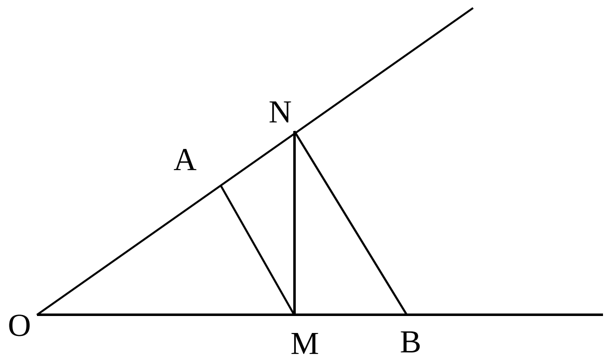

Figure 1

which means to find a point M such that

i) ONB is a right angle

ii) OMB is a right angle

iii) OAM is a right angle with OB = 2OA.

Since if we have found M, we get (an easy corollary from Pythagoras' theorem):

$OM \times OB = ON^2$ et $OA \times ON = OM^2$

which is equivalent to:

OB/ON = ON/OM et ON/OM = OM/OA i.e. OB/ON = ON/OM = OM/OA.

From a) above, OM is the side of a volume twice the volume of the cube of side OA. CQFD.

2. Geometric approximation for the duplication of the cube.

We start for the below Figure 2 where:

The line OA is arbitrary, then the line OB de is twice the segment OA, then D is the middle of OB, and last the circle OABD, is of centre D and diameter OB = 2DB = 2OA.

We begin by drawing AM the height of the triangle OAB, then MN the height of the triangle OAM.

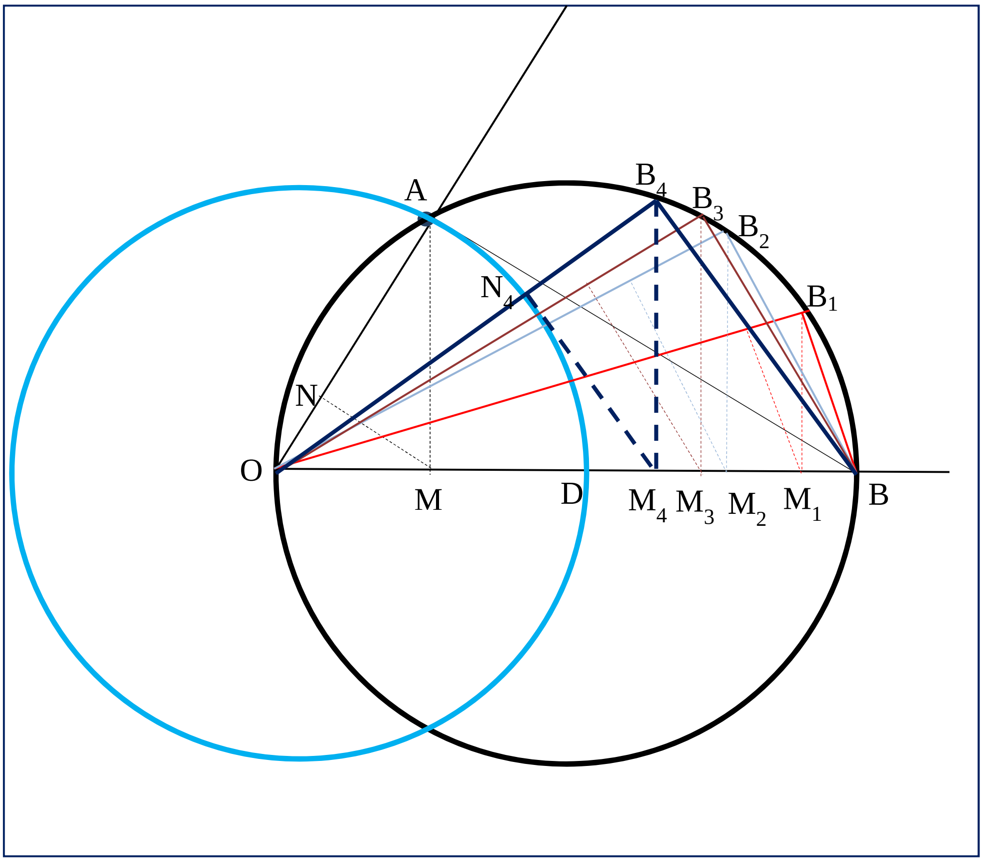

Figure 2

Approximation to find (approximatively) the point to double the cube of side OA.

We begin by making vary on the above Figure 2 the point B along the circle OABD, so that we get successively the points B_1, B_2, B_3, B_4, ….

The problem consists to find the N (denoted successively N_1, N_2, N_3, N_4, …) so that the lenght of ON would be equal to OA. Then the point we want is the point M_i such that:

i) B_iM_i is the height of the triangle rectangle OB_iB

ii) If $N_i$is the height of the triangle OB_iM_i, then ON_i = OA.

For any B_i on the circle AOBD, we consider the point M_i so that B_iM_i is the height of the (right) triangle OB_iB. Then we draw N_i so that M_iN_i is the height of the (right) triangle OB_iM_i cf. Figure 2 above).

Thus we get a sequence of points N_1, N_2, N_3, N_4,… (we have just indicated on the drawing the point N_4). Thus $OA = ON_i$ means N_i is on the circle OAD.

On the Figure 2 above, we see that N_4 is (approximately) on the circle OAD so that the cube of sde OM_4 has a volume very close from the double of the volume of the cube of side OA.

Such an approximation is sufficient if we have to build really a cube of twice the volume of a given cube (for instance according to the Delian story), so it would certainly satisfy an engineer, as Archytas is said to have been (in addition to be a mathematician, statesman, musician, …).

Since we have nothing about the origins of Archytas' solution (as almost any other mathematical work before Euclid), this construction is necessarily hypothetical. But

i) It is the most evident and elementary approach of the problem of doubling a cube

ii) It is different for all the (approximated) solutions known in the Antiquity. They were much more subtle and complex, so that the chances are strong this one was well-known and commentators did not think it needs to be accounted for, especially since

iii) It is the plane projection of Archytas (exact) spatial solution (cf. our global study of Archytas' construction: *Archytas' duplication of the cube – An easy construction*).

As we see, the problem is a purely planar geometric problem. To pass to the spatial geometry is an extremely ingenious idea, but nothing shows Archytas was the first to use this idea to solve a geometrical problem. Though ingenious, it supposes no special mathematical result. It is indeed a mechanical idea applied to mathematics which would explain Plato's criticism (true or false this reported criticism reported by some authors is then not completely baseless).