WHAT DID THEODORUS DRAW?

In Plato’s *Theaetetus*, the youth Theaetetus describes his tutor’s use of a diagram to demonstrate a point about incommensurability. Having just returned from Theodorus’s mathematics class with his friend—the young Socrates, namesake of the philosopher asking the questions—Theaetetus wonders whether a mathematical proof qua *diagram* might be a good model for answering Socrates’s *What is knowledge?* question. (The full text is on your handout.) Theodorus was making a point about *powers* that Theaetetus attempts to define as ‘oblong numbers’. To understand this, we need to recognize at least that the mathematics of the time does without ‘irrational’ numbers; that it therefore must address what we call the ‘roots’ of non-square numbers in a special way; that this special way is geometrical in form. The whole exercise is not just about numbers, but geometry too.

The passage has raised a number of issues that remain alive and in dispute. They include: (first) Plato’s use of the term *dunamis* and its cognates; (second) the method of proof—if proof it was—used by Theodorus to establish the incommensurability of certain square roots; and (third) the *reason* Theodorus stopped with $\sqrt{17}$. I will examine these overlapping issues in pursuit of an appropriate diagram that Theodorus could *in fact* have drawn in Socrates’s lifetime.

A prior question, however, is whether Theodorus should be assumed to have been showing the boys the actual *proofs* of incommensurability, or rather drawing something less rigorous to display *results* of his own previous investigations. I will offer reasons to prefer the latter, while arguing that one particular drawing could well be *the* drawing that Plato imagined Theodorus producing for the boys, one that graphically displays the relationship among the square roots of the first seventeen whole numbers, and supports my view of those three living issues.

First, some background. The need for an understanding of irrationals arose in the context of quantifying geometric magnitudes associated with two- and three-dimensional objects. As quantification in ancient Greece was historically restricted to the notion of whole numbers, there was no established terminology to apply to quantities that were *not* expressible using whole numbers. The term *dunamis*—which has the everyday meaning of ‘power’ ‘capacity’, or ‘ability’—was adapted to refer to an aspect of a range of mathematical notions that included two-dimensional space, plane geometry, and the magnitudes for measuring areas.

The importance of translating the Greek term *dunamis* correctly in mathematical contexts cannot be overestimated. Høyrup calls *dunamis* "among the most debated single terms in ancient Greek mathematics." More recent commentators and translators translate 'square' rather than 'square root' which was often used in the earlier part of the twentieth century—but the controversies run deeper. Complaining that "immense heat has been generated over the terminology of the passage through failure to make any distinction between meaning and application," Burnyeat cites the scores of authors, articles, and books articulating positions on *dunamis* in the *Theaetetus*; I recommend it for getting up-to-date on two thousand years of scholarship to 1978. For *today's* purposes, however, I take up the discussion earlier and later, that is, with Plato's *text* in the context of ancient Greek mathematics.

In the quoted passage, we can start by examining the context of the first appearance of the term *dunamis* in the passage. Literally translated, *dunamis … tēs tripodos* is "power of three feet," but both 'three feet' and 'power' can have more than one meaning. The most straightforward meaning of *tripodos* is "measuring three feet"; but LSJ cites *Theaetetus* 147d as its paradigm for the mathematical sense: "the side of a square three feet *in area*" because a foot was a unit of linear measurement in ancient Greek. Theaetetus says, "three feet" is not commensurable with "one foot" (*podiaios*), which only makes sense if he is referring to square figures with *areas* of three feet and one foot, respectively. Thus part of the passage's ambiguity stems from the use of terms such as *tripodos* and *podiaios*, which can be used to designate both linear and area measurements. (The same ambiguity occurs at *Meno* 82c5–8.)

But we do not yet have an explanation for why *dunamis* could refer in *this* passage to either the square root of a number or the resulting product of multiplying two equal factors. This brings us to another puzzle: unravelling the meaning of 'power' or *dunamis* in the expression "power of three feet."

Plato's use of 'foot' to refer to a measurement of length or a measurement of area has not been controversial, but the reader must depend on *context* to determine which is meant. I contend that a possible resource for disambiguating *dunamis* lies in a parallel relation between length-area and square root-squared number. There is a complementary nature to the notions of square roots and squared numbers in their *geometric* form. While in modern mathematics these notions are abstract and without spatial connotations, in Greek mathematics they were conceptualized geometrically as, respectively, the side and areas of actual square figures. And unlike modern mathematics, where the concept of number has a broad meaning, in Plato's time, *arithmos* designated positive integers.

One of the uses of *dunamis* in this passage is to distinguish sides of square figures that are irrational magnitudes, and so not expressible in whole numbers, from sides that *are* expressible in whole numbers. A quantity such as √2 is irrational and, consequently, would not fit in the category of number. The realization that many geometric magnitudes could not be expressed by rational numbers (*arithmoi*), non-commensurability, was problematic. The formulation of a solution required conceiving of certain lengths, such as a magnitude equaling √2, as *geometric* entities. The need to address magnitudes of such lengths naturally emerged in geometry: the diagonal of the one unit square was easily constructed, but it was impossible to assign it an exact magnitude by measurement.

The fact that the construction of a linear magnitude equal to √2 is dependent on the construction of a unit square suggests that there is an organic connection between such a length and a square figure and, by extension, an area measured in square units. Thus, in the case of our search for the correct meaning of *dunamis* in these passages, it can be said that the identity of a √2 length is inextricably tied to that of a two-foot² *area*. The geometer must work using a two-dimensional construction to produce this one-dimensional object, as shown here [Figure 1], where a line (AB) of √2 unit length is constructed as the diagonal of a unit square. This contrasts with a √4 length, which is not inextricably tied to a four-foot² area since a √4 length can be exactly expressed by the rational number 2, a magnitude that can be constructed simply by extending the length of a line from one unit to two using a compass and straightedge as shown here [Figure 2] where a line of 2-unit length is constructed with a compass. The geometer need not reach into two dimensions to construct a figure to determine such a length.

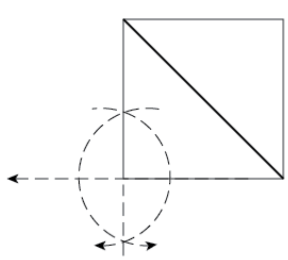

Figure 1

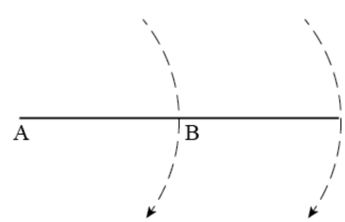

Figure 2

Thus the measurement assigned to areas is not only in terms of square units but can also be thought of as another order of quantification, 'plane numbers' in the broader modern sense of number. To work with quantities inexpressible by rational numbers required an initial move to a more complex notion of number that could include the operation of finding square roots as well as ways to conceptualize mathematical objects that fall into the category of quadratic equations in modern algebraic thinking. In their absence, mathematicians developed a geometric conception of quadratic equations and finding square roots. The role of algebraic equations would be filled by proportional magnitudes: ratios and proportions of the sides of rectilinear figures. I return now to Plato's text.

Typically, *dunamis* occurs in the dative (*dunamei*)—'by its ability' (or 'by its square'). But, in the *Theaetetus* passage, we find it in the genitive because it is governed by *peri* (concerning [the powers] …). Still, *dunamis* in our passage must have the dative meaning because, as we have seen, the *dunamis* (nominative case) of three feet, that is, three squared or three to the second *power*, is commensurable with the *dunamis* (nominative case) of one foot, that is, 1^2. The entities being declared incommensurable are lengths, not areas. But they are a special kind of length whose very identity, I have argued, is tied to the two-dimensional figure that *must* be constructed for the length to be produced.

I am saying that the geometrical notion of *dunamis* (power) covers both the finding of a square root and the squaring of a magnitude. While square roots and squares are usually stated as magnitudes, these magnitudes are viewed as the result of specific mathematical operations. These two operations are related in just the way that subtraction and addition, or division and multiplication, are related: the one operation is the inverse of the other. Given the tight connection between inverse operations, it might have seemed natural in early times to use a single term to refer to both. Thus *dunamis* refers to *both* of the directions of the relationship between a square and its sides, the inverse relation between squares *and* square roots. When *dunamis* is translated 'powers', therefore, the notions of a squared number and a square root of a number share a *commonality* in that they are manifested by the inverse operations of squaring, and extracting a square root.

The relationship between the side and diagonal of a square is an acknowledged constant in mathematics as the ratio of $1n : \sqrt{2}n$ that is invariant for a square of any size. The notion of side and diagonal numbers figured in ancient Greek mathematics in the investigation of irrational numbers. It was known to Plato. Thus it is easily supposed that a similar relationship was theorized between a square and its square root.

The connection between the two operations becomes even more apparent when their values are graphed in co-ordinate geometry. [Figure 3] The foci of their points trace similar conic curves; n^2 generates a parabola along the y axis while $\sqrt{n}$ generates one of the same shape on the x axisThis insight into the nature of the mathematical concept of *dunamis* will prove useful in

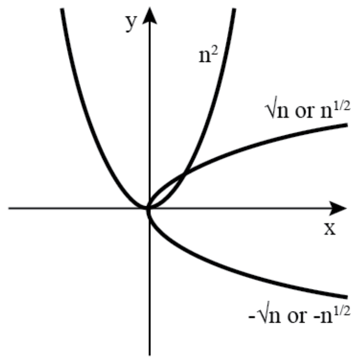

Figure 3

determining *what* diagram Plato envisioned Theodorus drawing for the boys. Any plausible reconstruction must meet five requirements set by the text and by then-current geometrical practice, while avoiding more mathematical complexity than is justified by the text.

Requirement 1. Theodorus must draw a figure worthy of his reputation as an expert in geometry, credited with expertise "in astronomy, arithmetic, music, and everything else that goes to make an educated person." His friendship with Protagoras marks him as an intellectual of his era, shown also by his return the following day to participate in the discussions of the *Sophist* and its sequel, the *Statesman*, where Socrates remarks that Theodorus is "the best arithmetician and geometer."

Requirement 2. The drawing should be something Theodorus could reasonably have drawn while the boys were present for their lesson. If a proposed diagram would take Theodorus several hours to draw, it could not fulfill requirement 2.

Requirement 3. The proposed diagram should be one we have good reason to think *Plato* would have been able to conceive—a drawing he could have had in mind as he dictated to his scribe. The history of Greek mathematics tells us that Euclid, Archimedes, and Apollonius *canonized* mathematics long after Plato wrote his dialogues, but Greek mathematicians had long been utilizing the mathematics.

Requirement 4. The diagram must somehow account for Theaetetus's report that Theodorus ran into difficulties or stopped when he reached the $\sqrt{17}$—an assertion about which there has been considerable speculation.

Requirement 5. The diagram should suggest to the boys what Theaetetus reports: (a) that all of the square roots of whole numbers form a class or are somehow one in kind, and (b) that this class can in fact be divided into two subclasses.

Each of my requirements narrows the possible candidates from the literature. I must show that there is a diagram that really does meet all the requirements. I will treat requirements 1 and 2 together since we are looking for a diagram that is both something a renowned geometer is likely to have drawn (requirement 1), but *in the presence of* others (requirement 2).

It should not be *presupposed* that Theodorus's drawing constitutes a rigorous diagrammatic *proof:* a drawing with the elements of each step constructed according to enumerable geometric principles. The text only states that he used a drawing (*egraphe*) to show that these square roots were "not commensurable in length with a figure of one square foot." A *proof* would require that each step be constructed according to *geometric* principles using only a compass and straightedge. This would require a great number of *auxiliary* lines to justify the steps. By contrast, a drawing could *illustrate* the final result of the constructed proof without including all its steps and their auxiliary construction lines. That the diagram(s) Theodorus drew need not have reached the level of *proof* is reasonable given that there is, first, his discovery of a proof rigorous enough to be mathematically convincing and, second, a drawing that *displays* his results, with the aim of informing other geometers of his discovery.

Netz observes that, in Greek mathematics, proof via construction of a figure is a precise practice requiring the drawing of *auxiliary* circles and lines. He rightly points out that it is impossible to construct a figure such as the isosceles triangle of proposition I.1 of Euclid's *Elements without* auxiliary circles. The geometer may assume that auxiliary circles have been constructed in increasingly more complex constructions, but in that case he must "acknowledge the shadow of a possible construction without actually performing it."

Consider for example [Figure 4] the difference between the construction a geometer would produce to *prove* that a pentagon is constructible with compass and straightedge versus the mere construction of a pentagon with compass and straightedge [Figure 5]. The first [Figure

4] shows the rather messy result of a proof that carries within itself what must be *known* to draw the second, cleaner figure. Yet one need not include all the auxiliary construction lines every time one wants to draw a pentagon because the cleaner method comes to light from the process of proving that a pentagon can be constructed with the geometric method, which is in effect a simplification of the *synthesis* part of an ancient Greek mathematical proof.

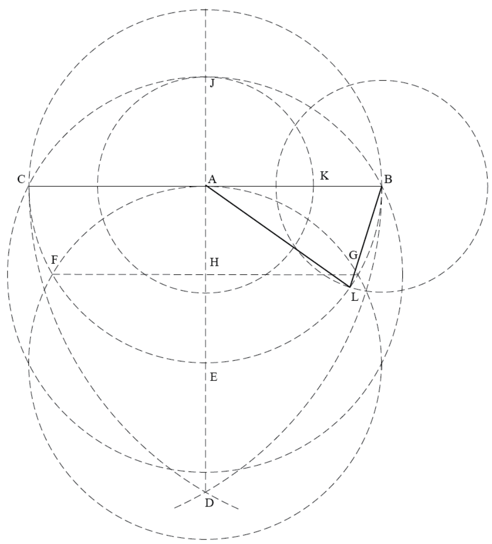

Figure 4

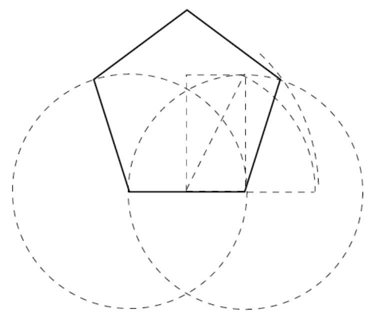

Figure 5

A much discussed candidate diagram is the Euclidean algorithm in its geometric guise of anthyphairesis found at Euclid X.2, which is applied to magnitudes (rather than numbers as in VII.2). Contemporaries are more likely to be familiar with the term ‘continued fractions’ or ‘reciprocal subtraction’. Here, for example, is an anthyphairetic proof for √5. [Figure 6]

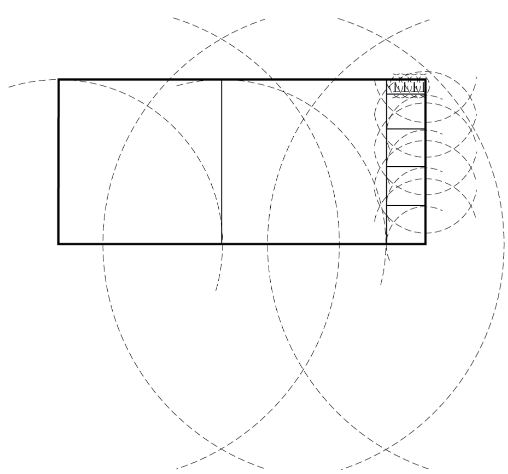

Figure 6

Zeuthern first proposed the method of anthyphairesis for Theodorus in 1896; Artmann illustrated its steps, Fowler *described* the process, and Knorr catalogued other anthyphairetic diagrams showing a special problem at √17. They attempt *proofs*, however, and *none* succeed in *all* my requirements, so I will not rehearse them all here. In 2017, Ksenia's used his computer to give more exact values for the irrationals, showing numerically why some are more difficult to construct by the anthyphairetic method than others.

It is unlikely that Theodorus would have used anthyphairesis to display his results to geometry students—not only because a proof is not *necessary* for such a display, but because a well-known method would not have exalted Theodorus as a preeminent geometer. Besides, drawing the diagrams themselves would have taken an inordinate amount of time. The labor involved in producing a *single* diagram of geometric anthyphairetic proof is illustrated in the figure before you, so the view that Theodorus was engaged in the process of anthyphairesis with the boys *fails* to fulfill requirements 1–2.

Any geometric anthyphairetic proof would require a considerable number of steps because each of the squares and rectangles dividing the interior of the initial rectangle would need to be constructed individually. This would entail the dropping of an arc to mark off the side distance on the base of the rectangle and constructing a line perpendicular to the base to establish the side of the square, using the method that Euclid canonizes at I.11. The construction involves drawing two similar arcs that intersect on either side of the line on which the perpendicular line is to be constructed (here as dashed lines). The process would have to be repeated until no more squares could be added to the base, leaving a rectilinear area as a remainder. The process would again be applied to this smaller rectangle using the side length of the initial triangle as the base, and the remainder distance of the initial base as the new side.

[Figure 7] Now consider that in addition to √2, which is already understood to be irrational, the method of anthyphairesis would require *twelve separate diagrams* to prove the irrationality of the remaining twelve irrational square roots between √1 and √17. The technique involved in the proof is the same in each case; yet the resultant figures are distinct, with each requiring the construction of at least eight individual lines using eighteen construction arcs. This is after the construction of rectangles with two sides equaling the appropriate irrational square roots. If Theodorus drew such diagrams for the boys, laying out each individual anthyphairesis, the boys would have stood around watching him while he repeated the procedure until it became apparent that the process would not terminate—in *each case*. While it is true that anthyphairesis can be used to find the greatest common denominator between two numbers, it is also included in *Elements* X as a method of proof that a quantity or magnitude is irrational. Despite the attention it has attracted, no one wants to commit to saying that Theodorus really drew it. Nevertheless, continuing with my requirements, let us ask how the process fares. Requirement 3, that the diagram could have been conceived and constructed by Plato is easily met.

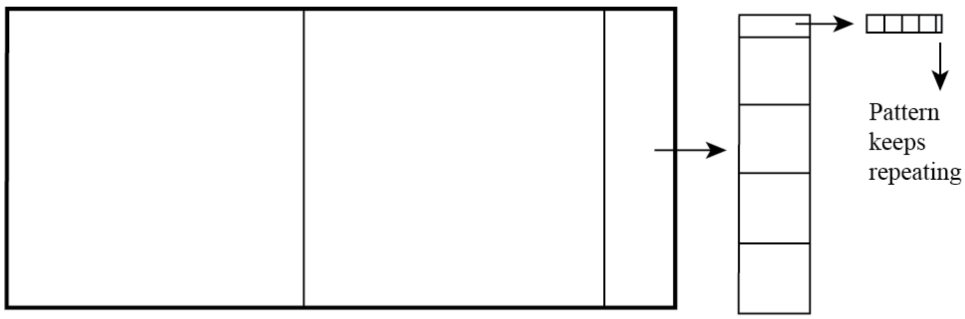

Figure 7

Requirement 4, that there be some *special* difficulty that would prevent Theodorus from continuing after √17 is more difficult to assess. Difficulties at √10 and √13 are reported by Artmann, who did not use auxiliary construction lines, suggesting that he used a ruler rather than the Euclidean straightedge. Fowler's sequence of proofs "snarls up" he says, at the √19. Unguru agreed. Trouble at the √19 has been supposed to satisfy the text, assuming Theodorus stopped at √17 because he could foresee becoming entangled at √19.

A philological point leads other commentators to suppose that something special needs to occur at exactly V17: Hackforth cites commentators who "take the Greek ἐνέσχετο to mean stopped" at 147d8, but points out that LSJ quotes only the Theaetetus passage for "came to a standstill" and cites precedent for "entangled by" which could give the sense that Theodorus had difficulties at V17. Let's say the jury is still out on how well the anthyphairesis diagrams fulfill requirement 4.

According to requirement 5, the diagram must suggest the two implications that the boys take from the lesson: that all of the square roots of whole numbers form a class; and that this class can be *divided* into two subclasses. Scholars agree that the boys could easily have generalized to a definition of 'incommensurability' and divided the class into 2 subclasses.

Ultimately then, proofs by anthyphairesis meet about half of my requirements.

Since I am concerned here with *what Theodorus drew,* and not especially with whether or how he may have proved the irrationality of the square root of a whole number, I will move in a different direction by investigating the possibility that a diagram short of a proof is what we are looking for. [Figure 8] Here is the so-called spiral of Theodorus, widely quoted in mathematical textbooks and on the internet—though rarely by philosophers. The serial construction of square roots generating a Theodoran spiral diagram provides both a simple construction of each root by the same method, and a means to show that there are two classes of magnitudes in the series of *all the square roots* from 1 to 17—a construction based on the Pythagorean theorem.

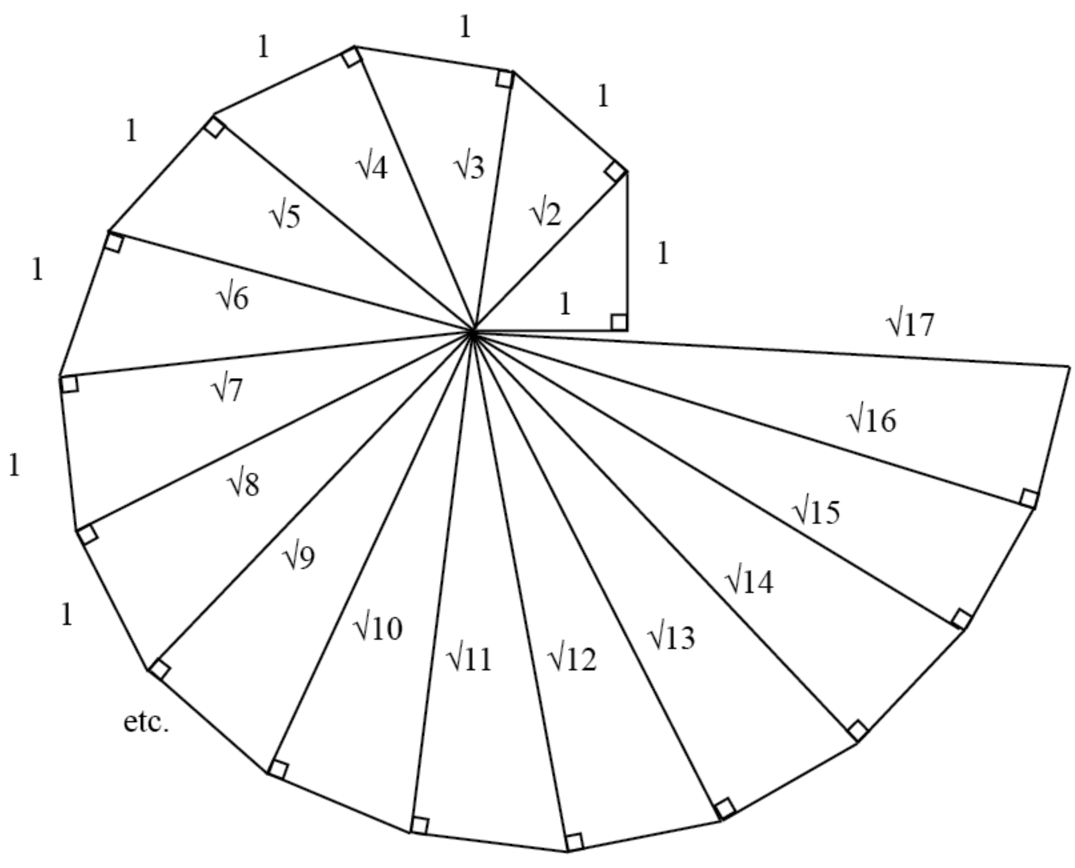

Figure 8

Before defending it, I digress briefly to acknowledge the obscure history of the now-popular diagram. [Figure 9] In 1877, Schmidt published the first known ancestor. Although it is not right in its details, one can see that Schmidt had the seminal idea for how Theodorus might have proceeded. In 1883, Campbell offered this diagram [Figure 10], which clarifies Schmidt's version without rigor.

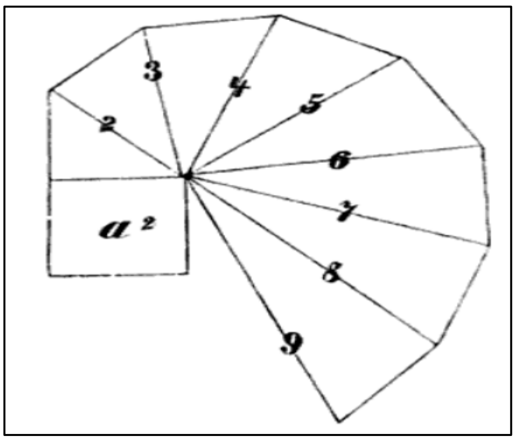

Figure 9

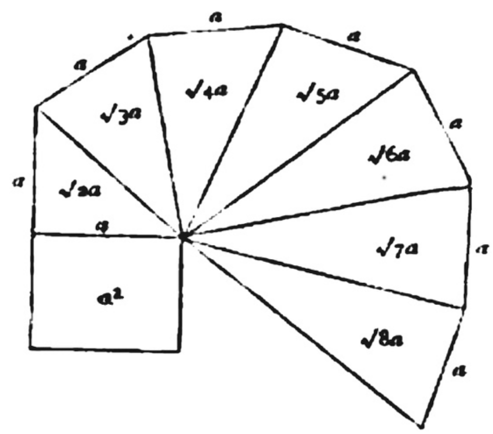

Figure 10

Anyone who has attempted to chase down the original source of the *rigorous* version of Theodorus's spiral will have found that an accurate citation for Anderhub 1941 was lacking in most texts that utilized the diagram. Complete information was surprisingly hard to find because as it turned out because Anderhub's *accurate* version of what may have been Theodorus's drawing [Figure 11] appears at the back of a collection of "cheerful stories" titled "Joca-Seria," privately published as a Christmas present for friends of Anderhub's publishing house at the beginning of World War II, and we hear no more of him after the war. Jakob Heinrich Anderhub was a jurist and commercial publisher whose hobbies were fine arts and Plato. Because Anderhub's chapter is in German gothic script, it has defied search engines; but it is filled with calculations and interpretive figures based on *Theaetetus*, and chronicles previous efforts by fifty-five scholars, starting with Ficino, to address the problem occasioned by the very phrase Hackforth focused on. It was Anderhub's recognition that *there really is a special problem* at √17. He draws attention to that problem, with his dotted line, to the *overlap* that would have resulted if Theodorus had continued drawing. [1]

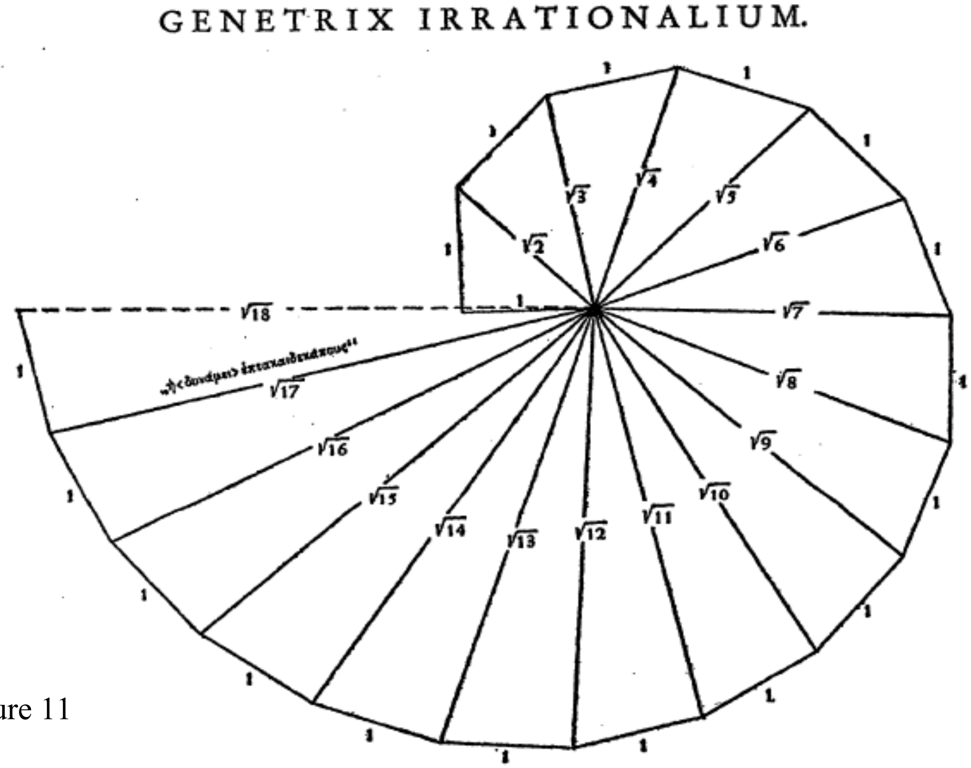

Figure 11

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[1] Holger Thesleff p.153 "Theaitetos and Theodoros," *ARCTOS*, Acta Philosophica Fennica xxiv (1990)
"He stopped at √17 because the √18 triangle would have intruded into his first triangle. [18] I am sure Anderhub was right. There is an additional indication of this, never observed in this connection as far as I know. The only evidence we have of Theodoros' mathematical studies which is seemingly independent of this Platonic passage, is a somewhat cryptic statement on spirals in Proklos. A *helix*, Proklos says, is a mixed line which does not consist of parts, so Theodoros the mathematician wrongly took it to be a '*krasis* based on lines'. [19] Modern scholars do not seem to have noticed the connection with *Theaetetus.* Without knowing it, Anderhub drew the relevant figure illustrating what Proklos meant. Proklos probably had access to an old tradition about the historical Theodoros having studied triangle-based spirals and Plato having referred to such figures orally and in the dialogue. The Anonymous Commentator on *Theaetetus*, as usual, is not so well informed. [20]
I also find it important to note that throughout the dialogue Theodoros is depicted as an adherent of *phainomena*, and indeed of Protagoras, who is known to have opposed theoretical geometry. [21] We should definitely not expect any 'proofs' from Theodoros. And shall I add that I am not a believer in the legend of Plato receiving instruction from him in Cyrene*?* [22]
[18] The sum of the inner angles of the √*2* - √18 triangles would amount to 364.783 degrees, the √17 one reaching 351.150 (and certainly somewhat further, if drawn in sand). I am indebted to Henrik Segercrantz for these calculations.
[19] Proklos, In Eucl. *Elem.* I, p. 117.25-118.8 Friedl., discussing the nature of curves. I understand *krasis epi ton grammon* to mean a mixture made 'on the basis of' or 'out of' straight lines."

Returning now to the present purpose of defending the spiral, note in its favor that its construction would not take all day. The time taken to draw a accurate spiral amounts to constructing, at most, two anthyphairetic proofs, and the simplicity of the process recommends it. [Figure 12] Starting with a right-angled triangle with legs of one unit, each succeeding triangle is constructed by extending a line of unit length from the outer end of the hypotenuse of the previous triangle. The square-root generating spiral satisfies the first two requirements: it displays the results of Theodorus's important discovery concerning powers, reflecting actual geometric practice, but without going through laborious and time-consuming anthyphairetic demonstrations to show each non-perfect square is irrational.

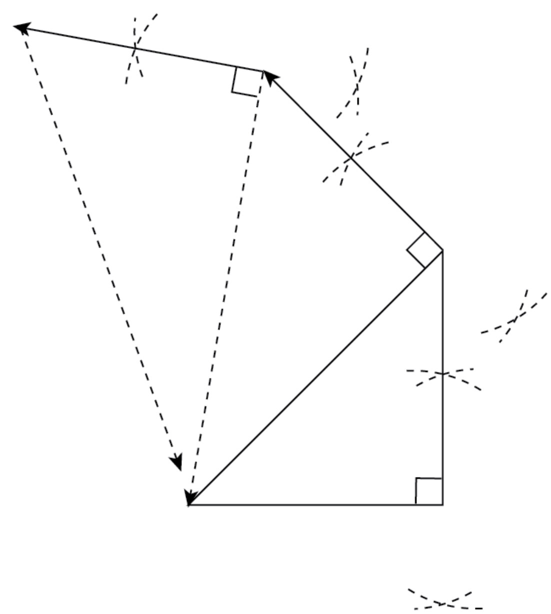

Figure 12

The third requirement, that the proposed diagram must be something we have good reason to think would lie within Plato's range of mathematical knowledge, can be shown by examining a pun in the *Statesman*, understood since ancient times and well described by Szabó, that removes any doubt that Plato was aware that a geometric relationship involving square roots could produce a spiral figure. The full text is on the handout.

In the pun, the two-footed humans are characterized by the diagonal of the unit square, while the four-footedness of some animals is characterized by a diagonal constructed on a square whose *side* is the diagonal of the unit square as shown here [Figure 13]. This diagonal produces the numerical value of our human *dunamis* (ability) to walk on two feet. The diagonal of the second square is a linearly commensurable quantity that can be described as having a length of two feet. Yet the Visitor's pun calls for this second diagonal to be measured "by its square" (*dunamei* or 'by its ability'); and when measured by its square, it has twice two feet, or an area of four. Giving the magnitude of a line by the square that can be constructed on it is much like using the square root sign where it would usually be considered redundant, for example, writing 4 as $(\sqrt{4})^2$.

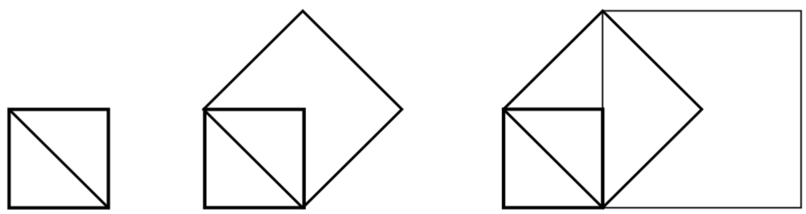

Figure 13

The diagonal-square procedure in the *Statesman* mirrors the hypotenuse-triangle procedure of the *Theaetetus* that generates the spiral. [Figure 14] By extending the *Statesman* diagram through the continuation of the same procedure, as shown here, a repetition of the diagonal-of-a-diagonal figure, it naturally starts to look spiral-like.

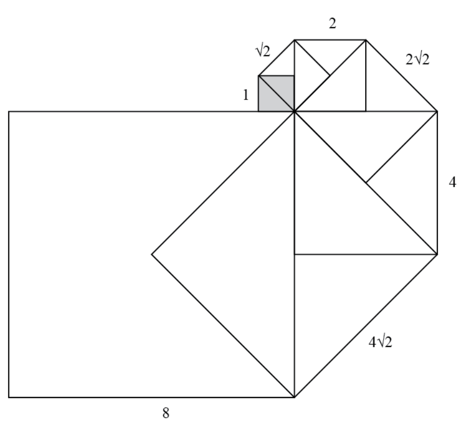

Figure 14

The spiral diagram [Figure 8] satisfies requirement 4 by providing a straightforward reason for Theodorus to stop after 17 that emerges naturally from its construction. The overlap illustrated here would make it more difficult with each additional triangle to distinguish the triangles. It is true that stopping at 17 works for the method of anthyphairesis as well, but only if one accepts that 19 is the real problem, merely anticipated at 17, a solution both Fowler and Artmann ultimately reject.

Requirement 5—that the diagram suggest both (a) that the square roots of whole numbers form a class and (b) that this class contain two distinct subclasses—is also fulfilled by the square root generating the spiral diagram. The boys watching Theodorus perform repeated iterations of the same operation while constructing the spiral diagram might well surmise that the square roots of whole numbers form a single class. The sequence of square roots, 1–17, have a unity as a mathematical progression formed by repeating the same operation; but repeating the triangle construction gives this class a concrete representation that might be apparent if their numerical values were compared. The diagram itself shows only how the roots can be seen as a single class. The square roots of whole numbers fall into two distinct kinds of magnitudes: those commensurable in square and line ($\sqrt{4}$, $\sqrt{9}$, and $\sqrt{16}$) and those commensurable in square only ($\sqrt{3}$ and $\sqrt{5}$). Theaetetus notices that that there are two kinds of shapes that follow: square or and oblong.

The spiral diagram itself, however, suggests that there are two classes of lines among the square roots of whole numbers. All one needs to do to see this is to add circles with radii having whole-number values—something a geometer might do. [Figure 15] A modern geometer might easily be imagined as going to the board and adding lines and circles to show additional relationships, marking a discovery by playing with Theodorus's diagram. Adding circles with radii in whole numbers that increase by one unit clearly marks off the square roots that are commensurable *in line* from the intermediate, incommensurable ones. The right angle of the triangle of each commensurable in-line root touches one of the circles; ones that do not are incommensurable.

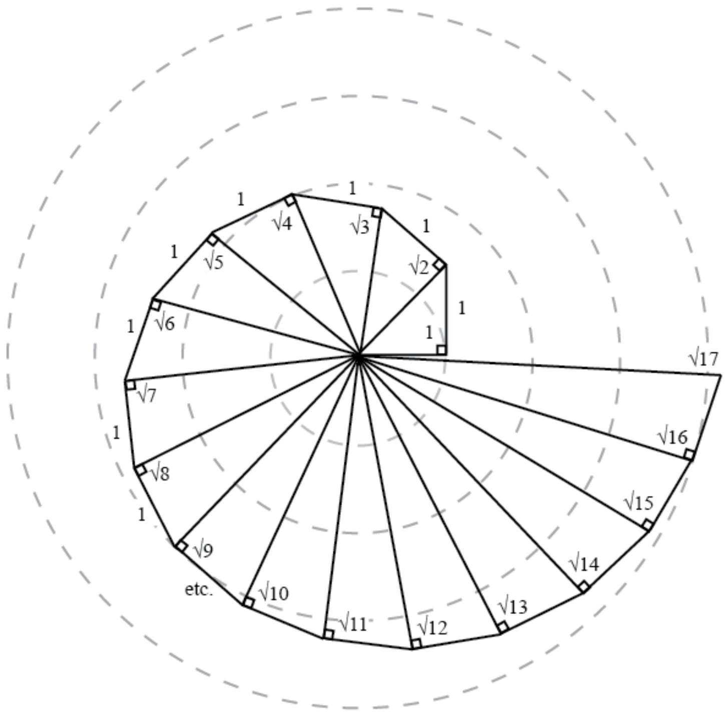

Figure 15

An important aspect of Theaetetus's discovery is his method; it involves making an irrational side of a square into a rational side of a rectangle. One needs to work in two dimensions to construct irrational one-dimensional lengths and understand that these lengths are not rational magnitudes. This fact connects the examples in the *Meno* and *Theaetetus* to the *Republic* and gives support to my view published elsewhere that the divided line diagram is a two-dimensional diagram, and not simply a line. The move from arithmetic to two dimensions points to the next step of understanding magnitudes in three dimensions. This is a theme for Plato: just as we needed to move from one dimension to two dimensions in mathematical thinking, so we need to develop our mathematical thinking to work with three dimensions.

Plato uses mathematical examples that show how to cross the boundaries between geometry and arithmetic. In a sense, he is arithmetizing geometry, though not in the objectionable sense where arithmetic is considered sufficient for working with the full spectrum of real numbers. Notions such as doubling and halving are essentially arithmetical, not geometrical. Plato is using geometry to get the right kind of units, ones that can be expressed as rational numbers, so he can understand and operate arithmetically. In the *Theaetetus*, the square root of three is not an *arithmos*, but geometrically it is the side of a square with an area equal to three (as √3 x √3). It can be transformed into an oblong rectangle with an area of one by three, both of which are rational *arithmoi*. This is a way of turning a shape with irrational sides into a shape with rational sides—so that something that could only be understood geometrically (√3) can now be understood arithmetically (3 and 1)—an operation familiar from the *Meno*. Plato's mathematical understanding across the dialogues is, I shall argue on another occasion, consistent and coherent. Thank you.

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