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Midterm (Lecture 002)

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To do a later question in a problem, you can always assume a previous question even if you have not answered it.

Problem 1 (Cyclic groups of order p^2) **:**
Let p be a prime number.

1. Let G be a cyclic group of order p^2. Show that there are $p(p-1)$ elements in G that generate it.
2. Let G be a finite group, n be the number of elements of order p^2, m be the number of cyclic subgroups of G of order p^2, show that

$$n = p(p-1)m.$$

Problem 2 (Groups of order $2p$) **:**
Let G be a group of order $2p$ for some prime p.

1. Show that there exists a and $b \in G$ such that a is order 2, b has order p and $G = \langle a, b \rangle$.
2. Show that $aba \in \langle b \rangle$.
3. Show that $aba = b$ or $aba = b^{-1}$.
4. Show that, if $aba = b$, then $G \cong \mathbb{Z}/2p\mathbb{Z}$.
5. Show that, if $aba = b^{-1}$, then $G \cong D_{2p}$.