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Homework 4

Due October 1st

Problem 1 :
Let $\mathcal{L}$ be the language consisting of a unary function symbol f, binary functions symbol g, and two binary predicate R. Let us consider the following $\mathcal{L}$-structures:

- N_1 whose underlying set is $\mathbb{N}$, where f is interpreted as the function $x \mapsto x+1$, g is interpreted as the addition, and R as the equality;
- N_2 whose underlying set is $\mathbb{Z}$, where f is interpreted as the function $x \mapsto x+1$, g is interpreted as the addition, and R as the equality;
- N_3 whose underlying set is $\mathbb{Z}$, where f is interpreted as the function $x \mapsto -x$, g is interpreted as the multiplication and R as the strict order.

For each of the following formulas, say in which of the above structures they are satisfied:

1. $\forall x \forall y\, g(x, f(y)) R f(g(x,y))$;
2. $\forall x \exists y\, (xRy \wedge \neg(yRx))$;
3. $\forall x \forall y\, (xRy \to f(x)Rf(y))$;
4. $\exists x\, \neg(\exists y\, f(y)Rx)$;
5. $\exists x\, ((\forall y\, g(x,y)Rx) \wedge f(x)Rx)$.

Problem 2 :

1. Find a sentence φ which is true in $(\mathbb{Z}, <)$ but false in $(\mathbb{N}, <)$;
2. Find a sentence φ which is true in $(\mathbb{Z}, <)$ but false in $(\mathbb{Q}, <)$;
3. Let M be the structure $(\mathbb{Z}, =, +)$. Find a formula φ with a unique free variable x such that for any assignement δ, $\varphi_\delta^M = 1$ if and only if $\delta(x)$ is odd.