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Homework 10

Due December 1st

Problem 1 :
Let $I = \lambda x\, x$ and $W = \lambda x \lambda y ((x)y)y$.

1. Give all $t \in \Lambda$ such that $(\lambda x\, ((I)x)x)\lambda x\, ((I)x)x \to_\beta^\star t$.
2. Give all $t \in \Lambda$ such that $((W)I)(W)I \to_\beta^\star t$.

One good way of doing that is to draw the graph of reductions as we did in class.

Problem 2 :
Let $t_n \in \Lambda$ be defined by induction on n as follows:

- $t_0 = x$;
- $t_{n+1} = (f)t_n$.

Let $\overline{n} = \lambda f \lambda x\, t_n$.

1. Show that $((\overline{m})f)((\overline{n})f)x \to_\beta^\star t_{m+n}$;
2. Find a λ-term S such that for all $n \in \mathbb{N}$, $(S)\overline{n} \to_\beta^\star \overline{n+1}$.
3. Find a λ-term A such that for all $n \in \mathbb{N}$, $((A)\overline{m})\overline{n} \to_\beta^\star \overline{m+n}$.
4. Find a λ-term M such that for all $n \in \mathbb{N}$, $((M)\overline{m})\overline{n} \to_\beta^\star \overline{m \cdot n}$.
5. Find a λ-term E such that for all $n \in \mathbb{N}$, $((E)\overline{m})\overline{n} \to_\beta^\star \overline{m^n}$.

Problem 3 :

1. Show that $\vdash \overline{n} : (A \to A) \to (A \to A)$ for any type A.
2. Let $T = \lambda x \lambda y \lambda z\, ((x)z)(y)z$. Show that $\vdash T : (A \to (B \to C)) \to ((A \to B) \to (A \to C))$.
3. Let $K = \lambda x \lambda y\, x$. Is the term $((T)K)K$ typable?