

## Local theory of Banach spaces, convexity and their applications

Ecole Doctorale de Sciences Mathématiques de Paris Centre  
Trimester *Phenomena in High Dimensions*, Institut Henri Poincaré  
April 25 - June 2, 2006<sup>1</sup>

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**Summary**<sup>2</sup> (or a wish-list, subject to reality test)

1. Recalling fundamental notions and results from classical convexity. Emphasis on peculiarities of the theory of non-symmetric convex sets, the role of cones.
2. The John and the Löwner ellipsoids, the John theorem. Elements of “classical” proofs and a derivation from the Lewis’ lemma. Consequences for the structure of the Banach-Mazur compactum, including that of non-symmetric sets; open problems, some of which are quite elementary.
3. A selection of needed results from probability, particularly in the Gaussian setting: Slepian-Gordon inequalities, Sudakov inequality, Ehrhard inequality (with at most a sketch of the proof), isoperimetric inequality, . . .
4. Elements of approximation theory. Kolmogorov and Gelfand diameters, entropy numbers (of operators and of convex sets). Duality of metric entropy. (The last topic is complemented by lectures of Dr. Tomczak-Jaegermann on May 3, 10 and 17.)
5. Kashin decomposition, random spaces and convex bodies, possibly the saturation phenomenon.
6. Random and pseudo-random matrices. Elements of free probability.  
*Note: Topics 4-6 will be covered not-necessarily sequentially.*
7. Elements of information theory: classical, non-commutative et quantum; links to convexity theory.

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<sup>1</sup>Generally on Tuesdays 13:30-15:30 and Fridays 8:30-10:30, but sometimes on Wednesdays 10:45-12:45; see <http://ceb-math.univ-mlv.fr/schedule.html> for the precise schedule.

<sup>2</sup>This is May 4 version; see <http://www.institut.math.jussieu.fr/projets/af/DEA/szarek/> for further updates

## References

- [1] G. Pisier, *The volume of convex bodies and Banach space geometry*, Cambridge University Press 1989.
- [2] *Handbook on the Geometry of Banach spaces*, W. B. Johnson, J. Lindenstrauss eds., Elsevier Science 2001. In particular, the chapters “Local operator theory, random matrices and Banach spaces” by K. R. Davidson, S. J. Szarek, Vol. 1, p. 317-366 and “Quotients of finite-dimensional Banach spaces; random phenomena” by P. Mankiewicz, N. Tomczak-Jaegermann, Vol. 2, p.1201-1246
- [3] S. Artstein, V. Milman, S. J. Szarek and N. Tomczak-Jaegermann, On convexified packing and entropy duality. *Geom. Funct. Anal.* 14 (2004), no. 5, 1134-1141.
- [4] S. J. Szarek and N. Tomczak-Jaegermann, Saturating Constructions for Normed Spaces. *Geom. Funct. Anal.* 14 (2004), no. 6, 1352-1375.
- [5] Hiai, Fumio, Petz, Dnes, *The semicircle law, free random variables and entropy*. *Mathematical Surveys and Monographs*, 77. American Mathematical Society, Providence, RI, 2000. x+376 pp.
- [6] Dembo, A., Cover, T. M. and Thomas, J. A., *Information Theoretic Inequalities*, *IEEE Transactions on Information Theory* 37, (1991), No. 6, 1501–1518.
- [7] Nielsen, Michael A.; Chuang, Isaac L., *Quantum computation and quantum information*. Cambridge University Press, Cambridge, 2000. xxvi+676 pp.