

Analyse Complexe, Géométrie Complexe et Applications Colloque en mémoire de Gennadi M. Henkin

Organisateurs : Tien-Cuong Dinh et Andrei Iordan

Bo Berndtsson (Göteborg)

Remarks on superforms and supercurrents

Mardi 12/12/2017 - 15:30-16:20

This is basically a survey of Lagerberg's work on superforms and supercurrents, with some additions. We will illustrate the formalism with a proof of Weyl's tube formula and state a conjecture related to the Alexandrov-Fenchel inequality.

Jean-Michel Bismut (Orsay)

Hypoelliptic deformation, self-adjointness, and analytic torsion

Mardi 12/12/2017 - 9:30-10:20

The purpose of the talk is to explain the construction of non self-adjoint Hodge Laplacians, which naturally deform classical Hodge theory.

If X is a compact Riemannian manifold, let \mathcal{X} be the total space of its tangent bundle. The deformed Hodge Laplacian is constructed over \mathcal{X} . It is a hypoelliptic operator on \mathcal{X} , which is essentially the sum of a harmonic oscillator and of the generator of the geodesic flow. In the real case, the symplectic form of X is used in its construction.

Applications to analytic torsion, real and holomorphic, will be given. Time permitting, connections with Selberg's trace formula will be explained.

Maxim Kontsevich (IHES)

Twistor families of categories

Lundi 11/12/2017 - 11:0-11:50

I will give a definition of a twistor family $(\mathcal{C}_\zeta), \zeta \in \mathbb{C}P^1$ of triangulated categories. The prototypical example is the family of derived categories of coherent sheaves on compact hyperkähler manifold, endowed with complex structures parametrized by twistor parameter ζ . Another basic example comes from Simpson's non-abelian Hodge theory. In a joint work (in progress) with Y. Soibelman we propose a general approach to twistor families using Fukaya categories associated with holomorphic symplectic manifolds.

The most clear case is the product of an elliptic curve and \mathbb{C}^\times . For $\zeta \neq 0, \infty$ the corresponding category has a description in terms of elliptic difference equations. Harmonic objects are solutions of Bogomolony equations on 3-dimensional torus with isolated singularities. The universal family of categories in this example is parametrized by the non-Hausdorff quotient $(\mathbb{C}P^2 - \mathbb{R}P^2)/GL(3, \mathbb{Z})$.

George Marinescu (Cologne)

Equivariant Kodaira embedding for CR manifolds with circle action

Lundi 11/12/2017- 15:00-15:50

In this talk we will discuss some results about the embedding of CR manifolds with circle action endowed with a positive CR line bundle. We prove that a certain weighted Fourier-Szegő kernel of CR sections admits an asymptotic expansion and as a consequence we establish an equivariant Kodaira embedding theorem. This is based on a joint work with C.-Y. Hsiao and X. Li.

Ngaiming Mok (Hong Kong)

Geometric substructures, uniruled projective subvarieties, and applications to Kähler geometry

Lundi 11/12/2017 - 14:00-14:50

In a series of articles with Jun-Muk Hwang starting from the late 1990s, we introduced a geometric theory of uniruled projective manifolds based on the variety of minimal rational tangents (VMRT), i.e., the collection of tangents to minimal rational curves on a uniruled projective manifold (X, \mathcal{K}) equipped with a minimal rational component. This theory provides differential-geometric tools for the study of uniruled projective manifolds, especially Fano manifolds of Picard number 1. Associated to (X, \mathcal{K}) is the fibered space $\pi : \mathcal{C}(X) \rightarrow X$ of VMRTs called the VMRT structure on (X, \mathcal{K}) . I will discuss germs of complex submanifolds S on (X, \mathcal{K}) inheriting geometric substructures, to be called sub-VMRT structures, obtained from intersections of VMRTs with tangent subspaces, i.e., from $\varpi : \mathcal{C}(S) \rightarrow S$, $\mathcal{C}(S) := \mathcal{C}(X) \cap \mathbb{P}\mathbb{T}(S)$. Central to our study is the characterization of certain classical Fano manifolds of Picard number 1 or special uniruled projective subvarieties on them in terms of VMRTs and sub-VMRTs. As applications I will relate the theory to the existence and uniqueness of certain classes of holomorphic isometries into bounded symmetric domains. For uniqueness results parallel transport (holonomy), a notion of fundamental importance both in Kähler geometry and in the study of sub-VMRT structures, will play an important role.

Roman G. Novikov (École Polytechnique de Paris)

Multidimensional inverse scattering problem

Mardi 12/12/2017 - 14:14:50

We give a review of old and recent results on the multidimensional inverse scattering problem related with works of G.M. Henkin. This talk is based, in particular, on the following references:

- G.M. Henkin, R.G. Novikov, The $\bar{\partial}$ -equation in the multidimensional inverse scattering problem, *Russ. Math. Surv.* 42(3), 109-180, 1987;
- G.M. Henkin, N.N. Novikova, The reconstruction of the attracting potential in the Sturm-Liouville equation through characteristics of negative discrete spectrum, *Stud. Appl. Math.* 97, 17-52, 1996;
- R.G. Novikov, The $\bar{\partial}$ -approach to monochromatic inverse scattering in three dimensions, *J. Geom. Anal.* 18, 612-631, 2008;
- R.G. Novikov, Formulas for phase recovering from phaseless scattering data at fixed frequency, *Bull. Sci. Math.* 139, 923-936, 2015.

Duong Hong Phong (Columbia)

New curvature flows in non-Kähler geometry

Lundi 11/12/2017 - 9:30-10:20

We begin with some reminiscences about Henkin's 1969 solution of the $\bar{\partial}$ equation, and its influence on kernel representations for the $\bar{\partial}$ problem, especially the $\bar{\partial}$ -Neumann problem. This turned out to have wide ramifications in Fourier analysis, notably on the development of the theory of singular Radon transforms.

We turn next to a topic of current interest in complex geometry, which is curvature flows. A well-known flow is the Kähler-Ricci flow, which is a flow of $(1, 1)$ -forms preserving the closedness property. Current problems from both physics and non-Kähler geometry lead to a flow which still preserves the closedness property, but which is rather a flow of $(2, 2)$ -forms, called the Anomaly flow. Such flows pose some major challenges, and their study has barely begun. We discuss some illustrative cases, including the flows on fibrations over K3 surfaces, on fibrations over Riemann surfaces, and on unimodular Lie groups. This last case is noteworthy as it requires Bismut connections rather than Chern connections. The emphasis will be on the various PDE techniques to be developed.

Nessim Sibony (Orsay)

Unique ergodicity for foliation

Lundi 11/12/2017 - 16:30-17:20

Consider the polynomial differential equation in \mathbb{C}^2

$$\frac{dz}{dt} = P(z, w), \quad \frac{dw}{dt} = Q(z, w).$$

The polynomials P and Q are holomorphic, the time is complex. In order to study the global behavior of the solutions, it is convenient to consider the extension as a foliation in the projective plane \mathbb{P}^2 .

I will discuss some recent results around the following questions. What is the ergodic theory of such systems ? How do the leaves distribute in a generic case ? What is the topology of generic leaves ?

Yum-Tong Siu (Harvard)

Pluri-Hodge Decomposition and Associated Jacobian

Mardi 12/12/2017 - 11:00-11:50

The deformational invariance of the m -genus, the dimension of $H^0(X; mK_X)$, is known for the case of a compact complex algebraic manifold X and is conjectured for the case of a compact Kähler manifold. For $m = 1$, the deformational invariance in the Kähler case follows from the Hodge decomposition. The question arises whether $H^0(X; mK_X)$ for $m \geq 2$ is naturally a direct summand of the cohomology group of some flat bundle so that the deformational invariance of the m -genus can be explained in terms of such a "pluri-Hodge decomposition". The talk will discuss the question, starting with the case of a compact Riemann surface, and study the construction of Jacobians associated to such a "pluri-Hodge decomposition" for a compact Riemann surface.

Lundi 11 décembre Amphi Herpin

- 08:30-09:15 Accueil des participants / Reception of participants
- 09:15-09:30 J. CHAMBAZ J: Ouverture
- 09:30-10:20 D. H. PHONG : **New cutvature flows in non-Kähler geometry**
- 10:30-11:00 Pause café
- 11:00-11:50 N. MOK, : **Geometric substructures, Uniruled projective subvarieties, and applications to Kähler geometry**
- 14:00-14:50 M. KONTSEVICH : **Twistor families of categories**
- 15:00-15:50 G.MARINESCU : **Equivariant Kodaira embedding for CR manifolds with circle action**
- 16:00-16:30 Pause café
- 16:30-17:20 N. SIBONY : **Unique ergodicity for foliation**

Mardi 12 décembre Amphi Astier

- 09:30-10:20 J.-M. BISMUT : **Hypoelliptic deformation, self-adjointness, and analytic torsion**
- 10:30-11:00 Pause café
- 11:00-11:50 Y-T. SIU : **Decomposition and Associated Jacobian**
- 14:00-14:50 R. NOVIKOV : **Multidimensional inverse scattering problem**
- 15:00-15:30 Pause café
- 15:30-16:20 B. BERNDTSSON : **Remarks on superforms and supercurrents**