ERRATUM FOR "GENERIC RANK OF BETTI MAP AND UNLIKELY INTERSECTIONS"

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1. In Theorem 1.3 and Theorem 10.1, hypotheses (b) should be changed to: For each $s \in S(\mathbb{C})$, X_s generates $\mathcal{A}_s$. This is to guarantee that $\mathcal{A}_{X^{[m]}} = \mathcal{A}^{[m]}$ for all $m \geq 1$ above the proof of Theorem 10.1.(i).

In Theorem 1.3.(ii) of the paper, one more assumption needs to be added: "if ι is quasi-finite". Theorem 1.3.(ii) is proved by applying Theorem 10.1.(ii) to $t = 0$, which implies $\mathrm{rank}_{\mathbb{R}}(\mathrm{d}b_{\Delta}^{[m]}|_{\mathscr{D}_m^{\mathcal{A}}(X^{[m+1]})}) \geq 2\dim \iota^{[m]}(\mathscr{D}_m^{\mathcal{A}}(X^{[m+1]}))$ for all $m \geq \dim X$. If ι is quasi-finite, then $\iota^{[m]}|_{\mathscr{D}_m^{\mathcal{A}}(X^{[m+1]})}$ is also quasi-finite, so $\dim \mathscr{D}_m^{\mathcal{A}}(X^{[m+1]}) = \dim \iota^{[m]}(\mathscr{D}_m^{\mathcal{A}}(X^{[m+1]}))$.

In summary, Theorem 1.3.(ii) should read

$$\mathrm{rank}_{\mathbb{R}}(\mathrm{d}b_{\Delta}^{[m]}|_{\mathscr{D}_m^{\mathcal{A}}(X^{[m+1]})}) = 2\dim \mathscr{D}_m^{\mathcal{A}}(X^{[m+1]}) \text{ for all } m \geq \dim X \text{ if } \iota \text{ is quasi-finite.}$$

This does not affect the applications of Theorem 1.3.(ii) in this paper (Theorem 1.2') and in [DGH20] ([DGH20, Thm.6.2]). Indeed, in both cases ι is the identity map (or a quasi-finite morphism according to convention) and a curve generates its Jacobian.

2.[1] Remove Theorem 1.1.(ii), because Theorem 1.7 should be weakened to be: For each integer $l \leq \dim \iota(X)$, we have

$$\mathrm{rank}_{\mathbb{R}}(\mathrm{d}b_{\Delta}|_X) < 2l \Leftrightarrow X^{\deg}(l - \dim X) \text{ is Zariski dense in } X. \tag{1}$$

These modifications do not change the rest of the results stated in the Introduction or Theorem 10.1: First of all, these changes have no impact on Theorem 1.8. So they do not change the major result of the paper, which is the criterion to characterize the generic Betti rank (previously Theorem 1.1.(i), currently Theorem 1.1), because the proof of this criterion in §9.3 is unchanged (it uses Theorem 1.8 and this weaker version of Theorem 1.7). Thus the consequences of this criterion (equation (1.4), Theorem 1.2, Theorem 1.2', Theorem 1.3, Theorem 1.4, Theorem 10.1) remain unchanged. Finally the proof of Proposition 1.10 in §11 is unchanged as it does not use Theorem 1.7.

The reason for this modification of Theorem 1.7 lies in Proposition 6.1: the inclusion $\mathbf{u}(X_{<2l}) \subseteq X^{\deg}(l-d)$ does not hold in general. However, the statement in "In particular" ("Conversely" in the current version) still holds true, and this statement together with the other inclusion $X^{\mathrm{sm}}(\mathbb{C}) \cap X^{\deg}(l-d) \subseteq \mathbf{u}(X_{<2l})$ imply the equivalence (1) above; see the proof of Theorem 1.7 in §9.2.

In the proof of this "In particular" statement of Proposition 6.1, equation (6.1) should be changed to

$$(\dim_{\mathbb{R}})_{\widetilde{x}}(\widetilde{b}^{-1}(r) \cap \widetilde{X}) > 2(d-l) \text{ for all } \widetilde{x} \text{ in a non-empty open subset } \widetilde{U} \text{ of } \widetilde{X}.$$

Notice that $\mathbf{u}(\widetilde{U})$ contains a non-empty open subset (in the usual topology) of $X^{\mathrm{sm,an}}$, so $\mathbf{u}(\widetilde{U})$ is Zariski dense in X. The rest of the original proof of Proposition 6.1 then shows that $\mathbf{u}(\widetilde{U}) \subseteq X^{\deg}(l-d)$. Thus this establishes the statement in "In particular".

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References

[DGH20] V. Dimitrov, Z. Gao, and P. Habegger. Uniformity in Mordell–Lang for curves. *Annals of Mathematics*, **194**:237–298, 2021.

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