Sylvy Anscombe

Transfer of decidability for existential theories of (valued) fields

In previous work with Fehm we found that the existential theory of an equicharacteristic henselian valued field is axiomatised using the existential theory of its residue field. From this we deduced a transfer of decidability: for a complete theory T of residue fields, the existential consequences of T are decidable if and only if the existential consequences of the theory $H(T)$ are decidable, where $H(T)$ is 'equicharacteristic, henselian, and residue field models T'. In more recent work with Dittmann and Fehm we considered a similar problem in which $H(T)$ is expanded to a theory that distinguishes a uniformizer, using an additional constant symbol. In this case Denef and Schoutens gave a transfer of existential decidability conditional on Resolution of Singularities. We introduce a consequence of Resolution and prove that it implies a similar transfer of existential decidability.

In this talk I'll explain these results and describe ongoing work with Fehm in which we broaden the above setting to obtain versions of these transfer results that allow incomplete theories T. Consequently we find several existential theories Turing equivalent to the existential theory of $\mathbb{Q}$, including the existential theory of large fields.