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Around NIP Noetherian domains

Which Noetherian integral domains are NIP? This is a natural question to ask, given the prominence of Noetherian rings in commutative algebra. We cannot hope to answer this question in full generality any time soon, as it includes other hard problems such as the conjectures on stable fields and NIP fields. Nevertheless, we present some interesting partial results which begin to paint a picture of NIP Noetherian rings. Let R be a Noetherian domain which is NIP. Then either R is a field, or R is a semilocal domain of Krull dimension 1 and characteristic 0. Assuming the henselianity conjecture on NIP valued fields, R is a henselian local ring. In the dp-minimal case, one can give a complete classification. Specifically, every dp-minimal Noetherian domain is a finite index subring of a dp-minimal discrete valuation ring. The situation in dp-rank 2 seems to be significantly worse, but a classification may still be possible in terms of differential valued fields.