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Definable convex and henselian valuations on ordered fields

A valuation v on a field K is said to be definable (in a specified language) if its corresponding valuation ring is a definable subset of K. Historically, the study of definable valuations on certain fields was motivated by the general analysis of definable subsets of fields and related decidability questions, but has also re-emerged lately in the context of classifying NIP fields.

In my talk, I will present some recent progress in the study of definable valuations on ordered fields ([1] to [4]), where definability is considered in the language of rings as well as the richer language of ordered rings. Within this framework, the focus lies on convex valuations, that is, valuations whose valuation ring is convex with respect to the linear ordering on the field. The most important examples of such valuations are the henselian ones, which are convex with respect to any linear ordering on the field.

I will present topological conditions on the value group and the residue field ensuring the definability of the corresponding valuation. Moreover, I will outline some definability and non-definability results in the context of specific classes of ordered fields such as t-henselian, almost real closed, and strongly dependent ones.

[1] P. Dittmann, F. Jahnke, L. S. Krapp and S. Kuhlmann, Definable valuations on ordered fields, to appear in Model Theory, 17 pp.

[2] L. S. Krapp, S. Kuhlmann and G. Lehéricy, Ordered fields dense in their real closure and definable convex valuations, Forum Math. 33 (2021) 953972.

[3] L. S. Krapp, S. Kuhlmann and G. Lehéricy, Strongly NIP almost real closed fields, MLQ Math. Log. Q. 67 (2021) 321328.

[4] L. S. Krapp, S. Kuhlmann and M. Link, Definability of henselian valuations by conditions on the value group, to appear in J. Symb. Log., 19 pp.